Mechanics
More Collisions
Rotational Quantites

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Last time

- collisions in 1-D
- elastic and inelastic

Overview

- ballistic pendulum
- 2-D collisions
- rotational quantities

1-D Elastic Collision Example

Ch 9, #62

Two titanium spheres approach each other head-on with the same speed and collide elastically. After the collision, one of the spheres, whose mass is 300 g, remains at rest.

(a) What is the mass of the other sphere?

(b) What is the speed of the two-sphere center of mass if the initial speed of each sphere is 2.00 m/s?

1-D Elastic Collision Example

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use:

$$\begin{cases} m_1 v_{1i} + m_2 v_{2i} = m_1 \cancelto{0}{v_{1f}} + m_2 v_{2f} \\ v_{1i} + \cancelto{0}{v_{1f}} = v_{2f} + v_{2i} \end{cases}$$

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(b) What is the speed of the two-sphere center of mass if the initial speed of each sphere is 2.00 m/s?

use:

$$\mathbf{v}_{\text{CM}} = \frac{1}{M} \sum_i m_i \mathbf{v}_i$$

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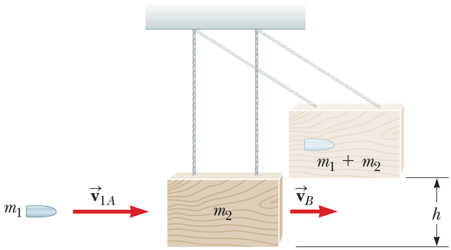
$$v_{\text{CM}} = 1.00 \text{ m/s}$$

Collisions and Energy

The Ballistic Pendulum



The ballistic pendulum is an apparatus used to measure the speed of a fast-moving projectile such as a bullet. A projectile of mass m_1 is fired into a large block of wood of mass m_2 suspended from some light wires. The projectile embeds in the block, and the entire system swings through a height h. How can we **determine the speed of the projectile** from a measurement of h?

[1]Serway & Jewett, page 262.

The Ballistic Pendulum

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Step 1: how does the speed of the block v_B depend on the bullet speed? Conservation of momentum, perfectly inelastic collision:

$$
\begin{aligned}
(m_1 + m_2) v_B &= m_1 v_1 + m_2 (0) \\
v_B &= \frac{m_1 v_1}{m_1 + m_2}
\end{aligned}
$$

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$$\begin{aligned}
\Delta K + \Delta U_g &= 0 \\
\left(0 - \frac{1}{2}(m_1 + m_2)v_B^2\right) + \left((m_1 + m_2)gh - 0\right) &= 0 \\
\frac{1}{2}(m_1 + m_2)v_B^2 &= (m_1 + m_2)gh
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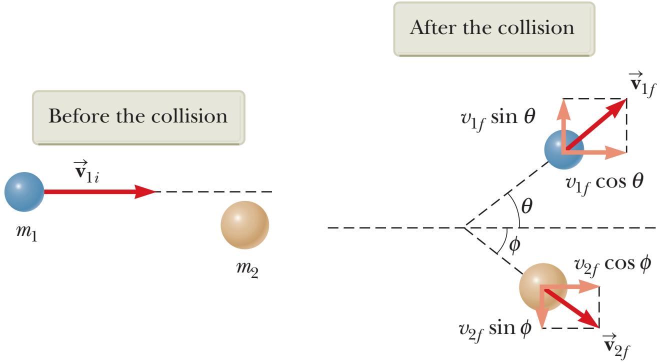
Replace $v_B = \frac{m_1 v_1}{m_1 + m_2}$:

$$\begin{aligned}
\frac{1}{2}(m_1 + m_2)\left(\frac{m_1 v_1}{m_1 + m_2}\right)^2 &= (m_1 + m_2)gh \\
\left(\frac{m_1^2 v_1^2}{m_1 + m_2}\right) &= 2(m_1 + m_2)gh \\
v_1 &= \underline{\left(\frac{m_1 + m_2}{m_1}\right)\sqrt{2gh}}
\end{aligned}$$

Collisions in 2 Dimensions

Collisions can take place in 2 dimensions.

As an example, consider the case of a *glancing* collision.



Conserve momentum in the x and y directions.

Collisions in 2 Dimensions

The conservation of momentum equation is a vector equation.

$$\mathbf{p}_i = \mathbf{p}_f \quad \Rightarrow \quad m_1 \mathbf{v}_{1i} + m_2 \mathbf{v}_{2i} = m_1 \mathbf{v}_{1f} + m_2 \mathbf{v}_{2f}$$

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We can write equations for each component of the momentum. In 2-d, with x and y components:

$$\begin{aligned}
\mathbf{x}: \ m_1 v_{1ix} + m_2 v_{2ix} &= m_1 v_{1fx} + m_2 v_{2fx} \\
\mathbf{y}: \ m_1 v_{1iy} + m_2 v_{2iy} &= m_1 v_{1fy} + m_2 v_{2fy}
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If it is an elastic collision:

$$K_i = K_f \quad \Rightarrow \quad \frac{1}{2}m_1(v_{1i})^2 + \frac{1}{2}m_2(v_{2i})^2 = \frac{1}{2}m_1(v_{1f})^2 + \frac{1}{2}m_2(v_{2f})^2$$

Rotation of Rigid Objects

Now we understand that while we *can* treat a collection of particles as a single point particle at the center of mass, we do not have to do that.

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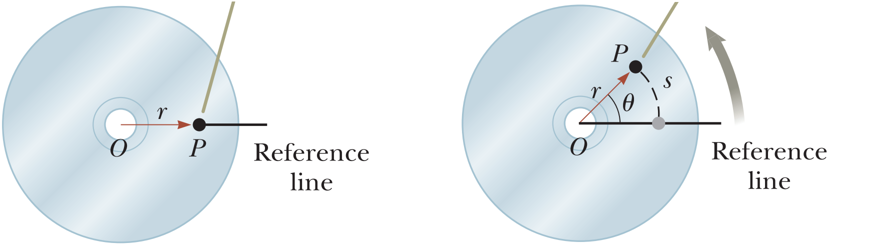
Begin with rotational kinematics.

Rotation of Rigid Objects

To begin, consider a rotating disc.



$$s = r\theta \; ; \quad \theta = \frac{s}{r}$$

r is constant in time for a rigid object.

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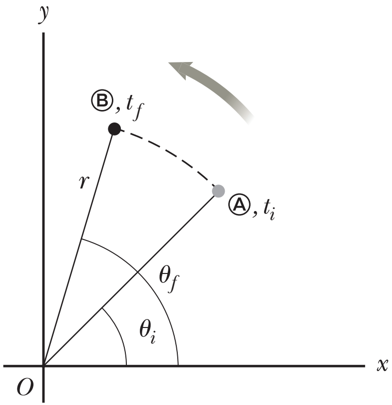
r is constant in time for a rigid object.

Units for θ: radians. Often written as "rad". But notice, that a dimensional analysis gives $\frac{[\mathrm{m}]}{[\mathrm{m}]} = 1$, unitless! The radian is an artificial unit. In fact, angles given in radians are dimensionless.

Rotation of Rigid Objects

How does the angle advance in time?



$$\Delta\theta = \theta_f - \theta_i$$

Angular Speed

Rate at which the angle advances is a speed: the angular speed, ω.

Average angular speed:

$$\omega_{\text{avg}} = \frac{\theta_f - \theta_i}{t_f - t_i} = \frac{\Delta\theta}{\Delta t}$$

Instantaneous angular speed:

$$\omega = \frac{d\theta}{dt}$$

Angular Acceleration

Rate at which the angular speed changes: the angular acceleration, α.

Average angular acceleration:

$$\alpha_{\text{avg}} = \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta\omega}{\Delta t}$$

Instantaneous angular acceleration:

$$\alpha = \frac{\mathrm{d}\omega}{\mathrm{d}t}$$

Summary

- ballistic pendulum
- 2-D collisions
- rotational quantities

2nd Test Thursday.

Homework

- Ch 9 Prob: 49, 52[1]
- Ch 10 Probs: 2, 7, 9, 13, 19, 29 (won't be on 2nd test - covered fully tomorrow)

[1] ans: 0.073 m