Mechanics
Torque
Static Equilibrium

Lana Sheridan

De Anza College

Nov 20, 2018

Last time

- rotational kinematics
- introduced torque

Overview

- torque
- net torque
- static equilibrium

Torque

Torque is a measure of force-causing-rotation.

It is not a force, but it is related. It depends on a force vector and its point of application relative to an axis of rotation.

Torque is given by:

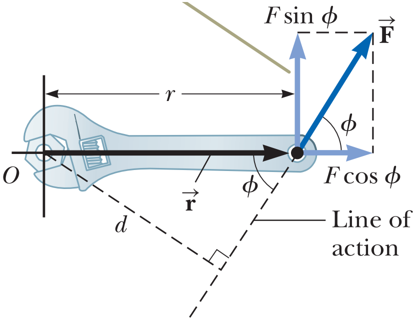
$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$$

That is: the cross product between

- a vector **r**, the displacement of the point of application of the force from the axis of rotation, and
- an the force vector **F**

Units: N m Newton-meters. These are not Joules!

Torque

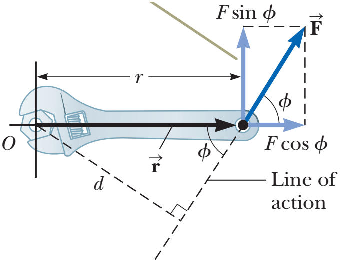


$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} = rF \sin\phi \, \hat{\mathbf{n}}$$

where ϕ is the angle between **r** and **F**, and $\hat{\mathbf{n}}$ is the unit vector perpendicular to **r** and **F**, as determined by the right-hand rule.

Torque

Diagram also illustrates two points of view about torque:



$$\boldsymbol{\tau} = r(F \sin \phi)\,\hat{\mathbf{n}}$$

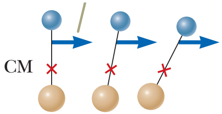
or

$$\boldsymbol{\tau} = F(r \sin \phi)\,\hat{\mathbf{n}} = Fd\,\hat{\mathbf{n}}$$

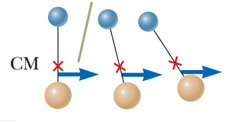
In the diagram, the distance $d = r \sin \phi$ and is called the "moment arm" or "lever arm" of the torque.

Torque

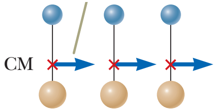
Torque:



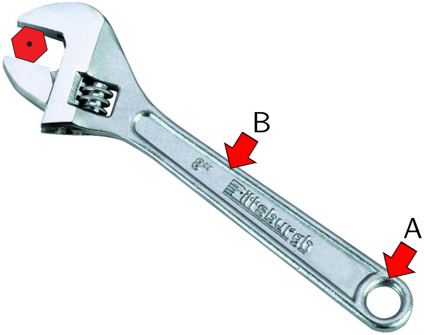
Torque:



No torque:



Question



A torque is supplied by applying a force at point A. To produce the same torque, the force applied at point B must be:

(A) greater

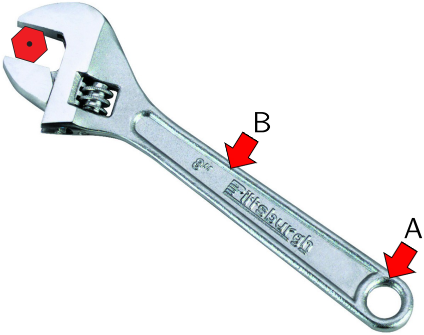
(B) less

(C) the same

---

[1]Image from Harbor Freight Tools, www.harborfreight.com

Question



A torque is supplied by applying a force at point A. To produce the same torque, the force applied at point B must be:

(A) greater ←

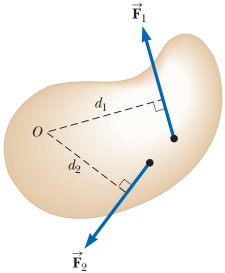
(B) less

(C) the same

[1] Image from Harbor Freight Tools, www.harborfreight.com

Net Torque

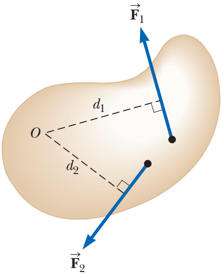
Object that can rotate about an axis at O:



There are two forces acting, but the two torques produced, $\boldsymbol{\tau}_1$ and $\boldsymbol{\tau}_2$ point in opposite directions.

$\boldsymbol{\tau}_1$ would produce a counterclockwise rotation
$\boldsymbol{\tau}_2$ would produce a clockwise rotation

Net Torque



The *net torque* is the sum of the torques acting on the object:

$$\boldsymbol{\tau}_{\text{net}} = \sum_i \boldsymbol{\tau}_i$$

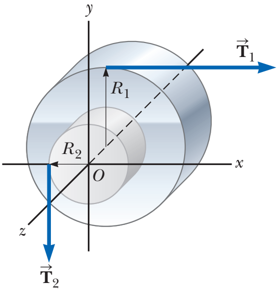
In this case, with $\hat{\mathbf{n}}$ pointing out of the slide:

$$\boldsymbol{\tau}_{\text{net}} = \boldsymbol{\tau}_1 + \boldsymbol{\tau}_2 = (F_1 d_1 - F_2 d_2)\hat{\mathbf{n}}$$

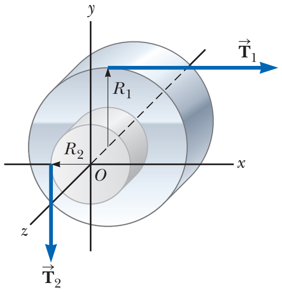
Example - Net Torque on a Cylinder

A one-piece cylinder is shaped as shown, with a core section protruding from the larger drum. The cylinder is free to rotate about the central z axis shown in the drawing. A rope wrapped around the drum, which has radius R_1, exerts a force T_1 to the right on the cylinder. A rope wrapped around the core, which has radius R_2, exerts a force T_2 downward on the cylinder.

What is the net torque acting on the cylinder about the rotation axis (which is the z axis)?



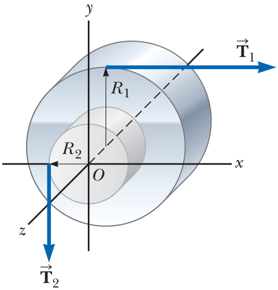
Example - Net Torque on a Cylinder



First: Find an expression for the net torque acting on the cylinder about the rotation axis.

Second: Let $T_1 = 5.0$ N, $R_1 = 1.0$ m, $T_2 = 15$ N, and $R_2 = 0.50$ m. What is the net torque? Which way is the rotation?

Example - Net Torque on a Cylinder



First: Find an expression for the net torque acting on the cylinder about the rotation axis.

$$\boldsymbol{\tau}_{\text{net}} = (T_2 R_2 - T_1 R_1)\mathbf{k}$$

Second: Let $T_1 = 5.0$ N, $R_1 = 1.0$ m, $T_2 = 15$ N, and $R_2 = 0.50$ m. What is the net torque? Which way is the rotation?

2.5 Nm counterclockwise, or 2.5 Nm $\mathbf{k}$

Static Equilibrium: System in Equilibrium (Ch 12)

Knowing that an object is in equilibrium can give a lot of information about the forces on the object.

Previously: a point-like system was in *equilibrium* if the net force was zero.

equilibrium $\iff$ constant velocity or at rest

Also, acceleration is zero.

Static Equilibrium: Extended System in Equilibrium

Now we consider extended rigid objects.

Static Equilibrium: Extended System in Equilibrium

Now we consider extended rigid objects.

Forces can cause rotations (torques).

Static Equilibrium: Extended System in Equilibrium

Now we consider extended rigid objects.

Forces can cause rotations (torques).

Force Equilibrium

$$\mathbf{F}_{\text{net}} = \sum_i \mathbf{F}_i = 0$$

Rotational Equilibrium

$$\boldsymbol{\tau}_{\text{net},O} = \sum_i \boldsymbol{\tau}_i = 0$$

about *any* axis O.

Static Equilibrium: Extended System in Equilibrium

Rigid Object in Equilibrium

A rigid object is said to be in equilibrium if
$\mathbf{F}_{\text{net}} = \sum_i \mathbf{F}_i = 0$ and
$\boldsymbol{\tau}_{\text{net},O} = \sum_i \boldsymbol{\tau}_{i,O} = 0$ about *any* axis O.

equilibrium $\iff$ $v = \text{const.}$ *and* $\omega = \text{const.}$

$\Rightarrow \quad a = 0$ and $\alpha = 0$

Static Equilibrium

Static Equilibrium is the special case that the object is also at rest:

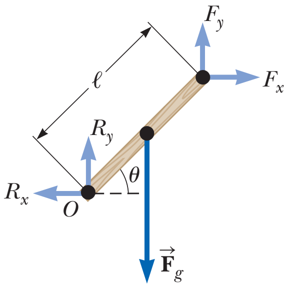
$$v_{CM} = 0$$

$$\omega = 0$$

Example

#1, page 379

1. What are the necessary conditions for equilibrium of the object shown in Figure P12.1? Calculate torques about an axis through point *O*.



Example

Conditions for equilibrium?

Example

Conditions for equilibrium?

$$\sum F_x = 0 \Rightarrow F_x = R_x$$

Example

Conditions for equilibrium?

$$\sum F_x = 0 \Rightarrow F_x = R_x$$

$$\sum F_y = 0 \Rightarrow F_y + R_y = F_g$$

Example

Conditions for equilibrium?

$$\sum F_x = 0 \Rightarrow F_x = R_x$$

$$\sum F_y = 0 \Rightarrow F_y + R_y = F_g$$

Torques about axis through O?

Example

Conditions for equilibrium?

$$\sum F_x = 0 \Rightarrow F_x = R_x$$

$$\sum F_y = 0 \Rightarrow F_y + R_y = F_g$$

Torques about axis through O?

$$\begin{aligned}\sum_i \tau_i &= \tau_{Fy} - \tau_{Fx} - \tau_{Mg} \\ &= F_y \ell \sin(90 - \theta) - F_x \ell \sin\theta - Mg\frac{\ell}{2}\sin(90 + \theta) \\ 0 &= F_y \ell \cos\theta - F_x \ell \sin\theta - Mg\frac{\ell}{2}\cos\theta\end{aligned}$$

Summary

- torque
- net torque
- static equilibrium

2nd Test tomorrow (finally).

Homework

- Ch 10 Probs: 45, 47
- Ch 12 Ques: 1, 3; Probs: 3, 17, 19, 23, 28, 37