Mechanics
Angular Momentum
Angular Momentum Conservation

Lana Sheridan

De Anza College

Nov 28, 2018

Last time

- parallel axis theorem for rotational inertia
- rotational kinetic energy
- angular momentum

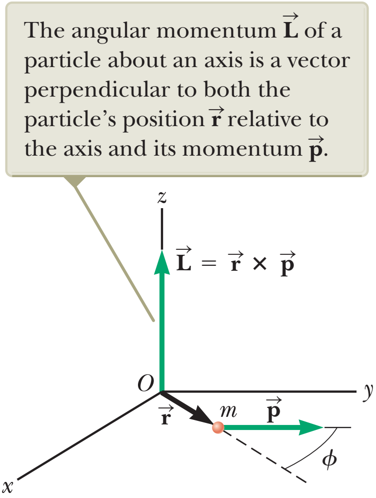
Overview

- angular momentum of rigid objects
- angular momentum conservation

Angular Momentum

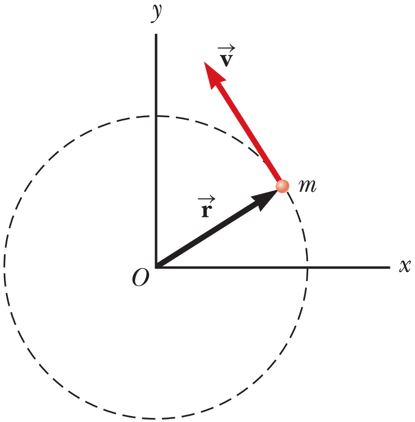
For a particle:

$$\mathbf{L} = \mathbf{r} \times \mathbf{p}$$



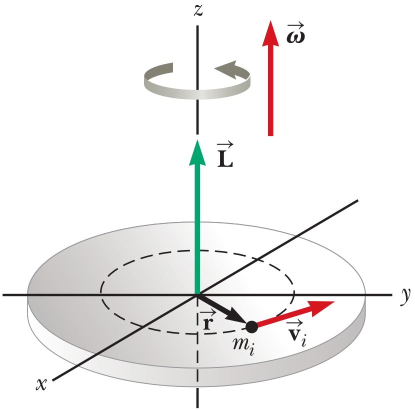
Angular Momentum of a Particle in Circular Motion

For a particle moving in a circle:



$$L_O = mrv$$

Angular Momentum of Rigid Object



All parts of the object have the same angular velocity ω.

Angular Momentum of Rigid Object

(Let $\hat{\mathbf{n}}$ be a unit vector in the direction of $\boldsymbol{\omega}$.)

For a rigid object made of many particles:

$$\begin{aligned}
\mathbf{L}_{\text{tot}} &= \sum_i \mathbf{L}_i \\
&= \sum_i m_i r_i v_i \, \hat{\mathbf{n}}
\end{aligned}$$

Notice that for each particle $v_i = \omega r_i$

$$\begin{aligned}
&= \left(\sum_i m_i r_i^2 \right) \omega \, \hat{\mathbf{n}} \\
&= I \boldsymbol{\omega}
\end{aligned}$$

Angular Momentum of Rigid Object

For a rigid object:

$$\mathbf{L}_{\text{tot}} = I\boldsymbol{\omega}$$

where I is the moment of inertia and $\boldsymbol{\omega}$ is the angular velocity.

Question

Quick Quiz 11.3[1] A solid sphere and a hollow sphere have the same mass and radius. They are rotating with the same angular speed. Which one has the higher angular momentum?

(A) the solid sphere

(B) the hollow sphere

(C) both have the same angular momentum

(D) impossible to determine

[1] Serway & Jewett, page 343.

Question

Quick Quiz 11.3[1] A solid sphere and a hollow sphere have the same mass and radius. They are rotating with the same angular speed. Which one has the higher angular momentum?

(A) the solid sphere

(B) the hollow sphere $\leftarrow$

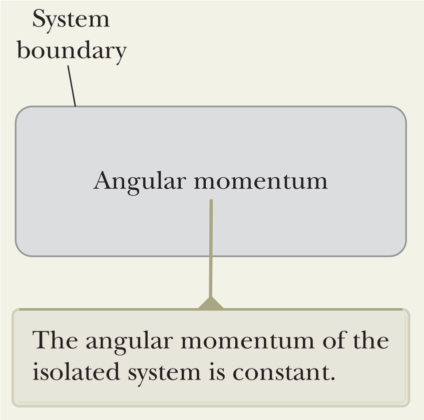
(C) both have the same angular momentum

(D) impossible to determine

[1] Serway & Jewett, page 343.

Conservation of Angular Momentum

For an *isolated* system, *ie.* a system with no external torques, total angular momentum is *conserved*.



$$\Delta L_{\text{total}} = 0$$

[1] Figure from Serway & Jewett.

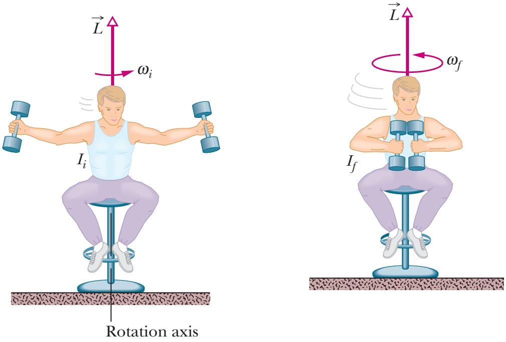
Angular Momentum is Conserved

The angular momentum of a system does not change unless it acted upon by an external torque. $L_i = L_f$.

Angular Momentum is Conserved

The angular momentum of a system does not change unless it acted upon by an external torque. $L_i = L_f$.

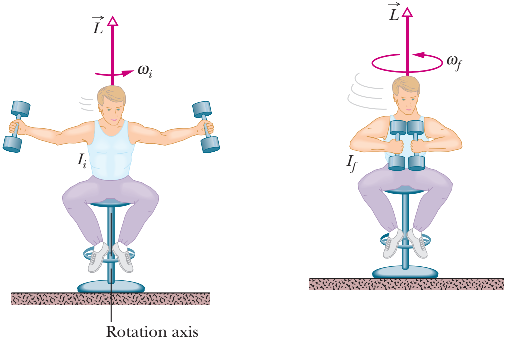
Suppose an object is changing shape, so that its moment of inertia gets smaller: $I_f < I_i$.



Angular Momentum is Conserved

The angular momentum of a system does not change unless it acted upon by an external torque. $L_i = L_f$.

Suppose an object is changing shape, so that its moment of inertia gets smaller: $I_f < I_i$.

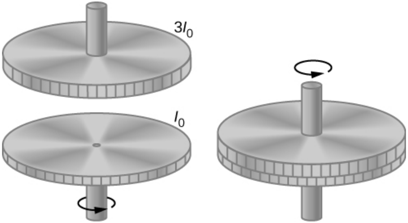


$$L_i = L_f \;\rightarrow\; I_i\omega_i = I_f\omega_f$$

That means the angular speed increases! $\omega_f > \omega_i$

Conservation of Angular Mtm: Collision Example

A flywheel rotates without friction at an angular velocity $\omega_0 = 600$ revs/min on a frictionless, vertical shaft of negligible rotational inertia. A second flywheel, which is at rest and has a moment of inertia three times that of the rotating flywheel, is dropped onto it.



Friction acts between the surfaces, and the flywheels end up spinning together with the same rotational velocity. (a) What is the angular velocity ω of the combination. (b) What fraction of the initial kinetic energy is lost in the coupling of the flywheels?

[1] Openstax "University Physics", page 562 ch11

Conservation of Angular Mtm: Collision Example

(a) System of 2 flywheels (disks) is isolated $\Rightarrow$ angular momentum is conserved.

$$
\begin{aligned}
\mathbf{L}_i &= \mathbf{L}_f \\
I_0\boldsymbol{\omega}_0 + 0 &= (I_0 + 3I_0)\boldsymbol{\omega}_f \\
\boldsymbol{\omega}_f &= \frac{1}{4}\boldsymbol{\omega}_0 \\
\boldsymbol{\omega}_f &= 150 \text{ rev/min} \quad \text{counterclockwise (as shown in diag.)} \\
\boldsymbol{\omega}_f &= \underline{15.7 \text{ rad/s} \quad \text{counterclockwise}}
\end{aligned}
$$

Conservation of Angular Mtm: Collision Example

(a) System of 2 flywheels (disks) is isolated $\Rightarrow$ angular momentum is conserved.

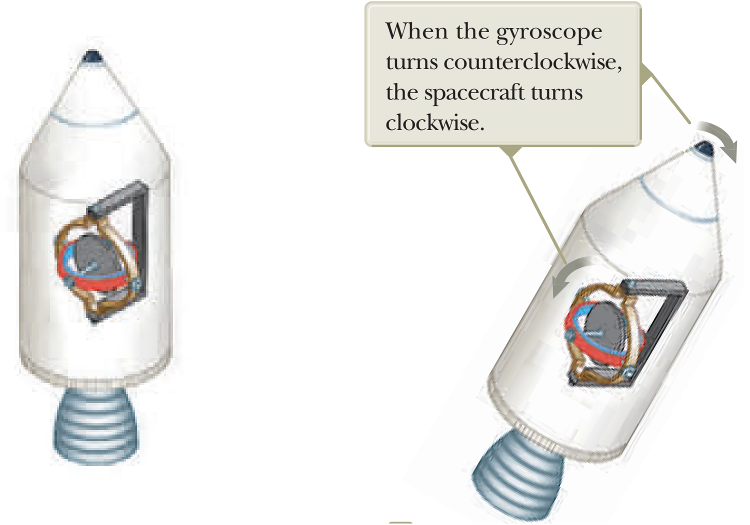
$$\begin{aligned}
\mathbf{L}_i &= \mathbf{L}_f \\
I_0\boldsymbol{\omega}_0 + 0 &= (I_0 + 3I_0)\boldsymbol{\omega}_f \\
\boldsymbol{\omega}_f &= \frac{1}{4}\boldsymbol{\omega}_0 \\
\boldsymbol{\omega}_f &= 150 \text{ rev/min} \quad \text{counterclockwise (as shown in diag.)} \\
\boldsymbol{\omega}_f &= \underline{15.7 \text{ rad/s} \quad \text{counterclockwise}}
\end{aligned}$$

(b) Collision is perfectly inelastic! Kinetic energy is not conserved.

Fraction KE lost $= \frac{|\Delta K|}{K_i} = 1 - \frac{K_f}{K_i}$

$$\begin{aligned}
1 - \frac{K_f}{K_i} &= 1 - \frac{\frac{1}{2}(4I_0)\omega_f^2}{\frac{1}{2}I_0\omega_0^2} = 1 - \frac{4(\omega_0/4)^2}{\omega_0^2} = 1 - \frac{1}{4} \\
&= \underline{\frac{3}{4}}
\end{aligned}$$

Application of Angular Momentum Conservation



A massive flywheel is driven to cause rotations in the entire rocket.

Gyroscope Application Example

42. A spacecraft is in empty space. It carries on board a gyroscope with a moment of inertia of $I_g = 20.0 \text{ kg} \cdot \text{m}^2$ about the axis of the gyroscope. The moment of inertia of the spacecraft around the same axis is $I_s = 5.00 \times 10^5 \text{ kg} \cdot \text{m}^2$. Neither the spacecraft nor the gyroscope is originally rotating. The gyroscope can be powered up in a negligible period of time to an angular speed of 100 rad/s. If the orientation of the spacecraft is to be changed by 30.0°, for what time interval should the gyroscope be operated?

Gyroscope Application Example

Time of gyroscope operation to achieve 30.0° rotation of craft?

Gyroscope Application Example

Time of gyroscope operation to achieve 30.0° rotation of craft?

Conservation of angular momentum:

$$0 = I_g \omega_g + I_s \omega_s$$

Gyroscope Application Example

Time of gyroscope operation to achieve 30.0° rotation of craft?

Conservation of angular momentum:

$$0 = I_g \omega_g + I_s \omega_s$$

$$\omega_s = 0.004 \text{ rad s}^{-1}$$

Gyroscope Application Example

Time of gyroscope operation to achieve 30.0° rotation of craft?

Conservation of angular momentum:

$$0 = I_g \omega_g + I_s \omega_s$$

$$\omega_s = 0.004 \text{ rad s}^{-1}$$

$\theta = \frac{\pi}{6}$

$$t = \frac{\theta}{\omega_s}$$

Gyroscope Application Example

Time of gyroscope operation to achieve 30.0° rotation of craft?

Conservation of angular momentum:

$$0 = I_g \omega_g + I_s \omega_s$$

$$\omega_s = 0.004 \text{ rad s}^{-1}$$

$\theta = \frac{\pi}{6}$

$$t = \frac{\theta}{\omega_s}$$

$$t = 131 \text{ s}$$

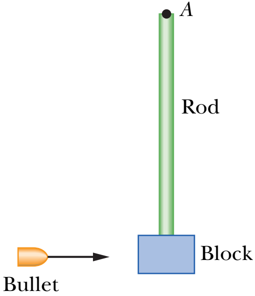
Another Example

#60, page 302[1]

A 1.0 g bullet is fired into a 0.50 kg block attached to the end of a 0.60 m nonuniform rod of mass 0.50 kg. The block-rod-bullet system then rotates in the plane of the figure, about a fixed axis at A. The rotational inertia of the rod alone about that axis at A is 0.060 kg m^2. Treat the block as a particle.

(a) What then is the rotational inertia of the block-rod-bullet system about point A?

(b) If the angular speed of the system about A just after impact is 4.5 rad/s, what is the bullet's speed just before impact?



---

[1]Halliday, Resnick, Walker, 9th ed, page 302.

Another Example

Let $r = 0.6$ m be the length of the rod.

(a) rotational inertia about A with bullet embedded?

$$
\begin{aligned}
I_A &= I_{\text{bullet}} + I_{\text{block}} + I_{\text{rod}} \\
&= m_{\text{bullet}} r^2 + m_{\text{blocks}} r^2 + I_{\text{rod}} \\
&= (0.001\text{kg})(0.6 \text{ m})^2 + (0.50\text{kg})(0.6 \text{ m})^2 + 0.060 \text{ kg m}^2 \\
&= 0.240 \text{ kg m}^2
\end{aligned}
$$

Another Example

Let $r = 0.6$ m be the length of the rod.

(a) rotational inertia about A with bullet embedded?

$$\begin{aligned}
I_A &= I_{\text{bullet}} + I_{\text{block}} + I_{\text{rod}} \\
&= m_{\text{bullet}} r^2 + m_{\text{blocks}} r^2 + I_{\text{rod}} \\
&= (0.001\text{kg})(0.6\text{ m})^2 + (0.50\text{kg})(0.6\text{ m})^2 + 0.060\text{ kg m}^2 \\
&= 0.240\text{ kg m}^2
\end{aligned}$$

(b) bullet's initial speed, v_i?

isolated system $\Rightarrow$ conserve angular momentum in collision

$$\begin{aligned}
L_i &= L_f \\
L_{\text{bullet},A,i} + L_{\text{block+rod},A,i} &= I_{A,f}\,\omega_f \\
m_{\text{bullet}} v_i r \quad + \quad 0 &= I_{A,f}\,\omega_f \\
v_i &= \frac{I_{A,f}\,\omega_f}{m_{\text{bullet}} r} = 1.80 \times 10^3\text{ m/s}
\end{aligned}$$

Summary

- angular momentum of rigid objects
- angular momentum conservation

Homework

- HW problem on the next slide/page
- Ch 11 Probs: 37, 45, 49, 51, 55, 61

HW Problem

A cylinder with rotational inertia $I_1 = 2.0$ kg m^2 rotates clockwise about a vertical axis through its center with angular speed $\omega_1 = 5.0$ rad/s. A second cylinder with rotational inertia $I_2 = 1.0$ kg m^2 rotates counterclockwise about the same axis with angular speed $\omega_2 = 8.0$ rad/s. If the cylinders couple so they have the same rotational axis (a) what is the angular speed of the combination? (b) What percentage of the original kinetic energy is lost to friction?

[1]Openstax "University Physics", ch 11, #54.