Mechanics
Gravity

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Nov 29, 2018

Last time

- angular momentum of rigid objects
- conservation of angular momentum
- examples

Overview

- Newton's law of gravitation
- gravitational field
- gravitational potential energy

Motion of the stars, planets, and falling apples

Part of Newton's genius was to realize that the same force that dictates the motion of the stars and planets is what holds us on the Earth.

This realization is called the **Newtonian synthesis**.

The planets are also falling, but they are constantly falling around the Sun.

Gravitation

Newton's Universal Law of Gravitation states that any two massive object in the universe interact with each other according to the same rule.

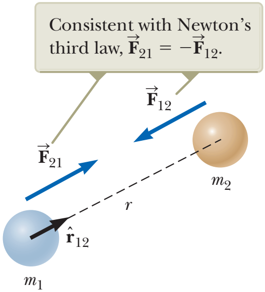
Newton's Law of Universal Gravitation

$$F_G = \frac{Gm_1 m_2}{r^2}$$

for two objects, masses m_1 and m_2 with a distance r between their centers.

$G = 6.67 \times 10^{-11}\ \mathrm{Nm^2\ kg^{-2}}$.

Gravitation



$$\mathbf{F}_{G,1\to 2} = -\frac{G m_1 m_2}{r^2}\,\hat{\mathbf{r}}_{1\to 2}$$

for two objects, masses m_1 and m_2 at a distance r.

$G = 6.67 \times 10^{-11}\ \mathrm{Nm}^2\ \mathrm{kg}^{-2}$.

The Universal Gravitational Constant, G

$$F = G\,\frac{m_1 m_2}{r^2}$$

G sets the scale of the force due to gravity (and makes the units come out correctly).

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$$G = 6.67 \times 10^{-11} \text{N m}^2/\text{kg}^2$$

This could also be written:

$$G = 0.0000000000667 \text{N m}^2/\text{kg}^2$$

so it is quite a small number.

The Universal Gravitational Constant, G

The fact that G is so small indicates that gravity is a weak force.

Force	$\sim$ Rel. strength	Range (m)	Attract/Repel
Gravitational	10^{-38}	∞	attractive
Electromagnetic	10^{-2}	∞	attr. & rep.
Weak Nuclear	10^{-13}	$< 10^{-18}$	attr. & rep.
Strong Nuclear	1	$< 10^{-15}$	attr. & rep.

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Despite the fact that gravity is a weak force, it is the only one that (typically) matters on large scales.

Acceleration due to Gravity

This force in that it gives objects weight, F_g.

For an object of mass m near the surface of the Earth:

$$F_g = mg$$

and

$$g = \frac{GM_{\mathrm{E}}}{R_{\mathrm{E}}^2}$$

where

- $M_{\mathrm{E}} = 5.97 \times 10^{24}$ kg is the mass of the Earth and
- $R_{\mathrm{E}} = 6.37 \times 10^{6}$ m is the radius of the Earth.

The force $\mathbf{F}_g$ acts downwards towards the center of the Earth.

Acceleration due to Gravity

The acceleration due to gravity, g, can vary with height!

$$F_G = \frac{GM_\mathsf{E}m}{r^2} = m\left(\frac{GM_\mathsf{E}}{r^2}\right) = mg$$

Depends on r the distance from the center of the Earth. Suppose an object is at height h above the surface of the Earth, then:

$$g = \frac{GM_\mathsf{E}}{(R_\mathsf{E} + h)^2}$$

g decreases as h increases.

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g is the not just the acceleration due to gravity, but also **the magnitude of the gravitational field**.

Fields

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Fields were first introduced as a calculation tool. A force-field can be used to identify the force a particular particle will feel at a certain point in space and time based on the other objects in its environment that it will interact with.

We do not need a description of the sources of the field to describe what their effect is on our particle.

Gravitational force:

$$\mathbf{F}_G = m\left(-\frac{GM\hat{\mathbf{r}}}{r^2}\right) = m\mathbf{g}$$

Electrostatic force:

$$\mathbf{F}_E = q\mathbf{E}$$

Fields

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Gravitational field:

$$\mathbf{g} = \frac{\mathbf{F}_G}{m}$$

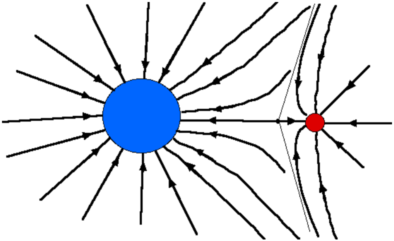
Electric field:

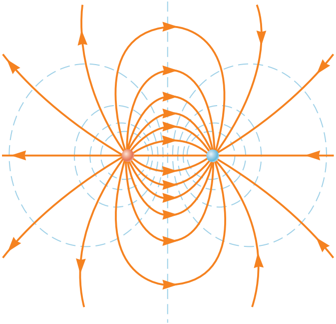
$$\mathbf{E} = \frac{\mathbf{F}_E}{q}$$

The field tells us what force a test particle of mass m (in the gravitational case) or charge q (in the electrostatic case) would feel at that point in space and time.

Examples of Fields

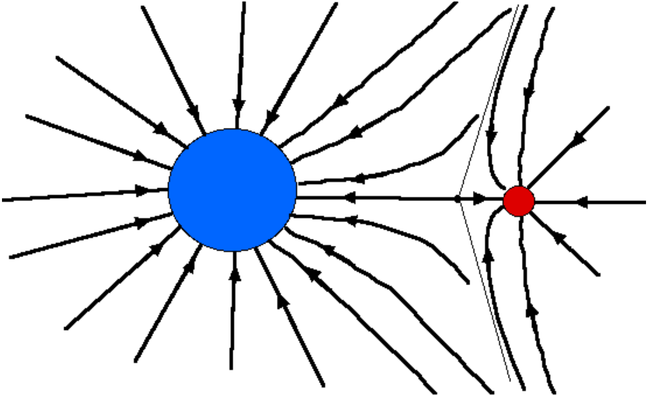
Fields are drawn with lines showing the direction of force that a test particle will feel at that point. The density of the lines at that point in the diagram indicates the approximate magnitude of the force at that point.





Examples of Fields

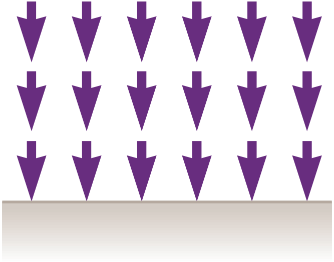
The gravitational field caused by the Sun-Earth system looks something like:



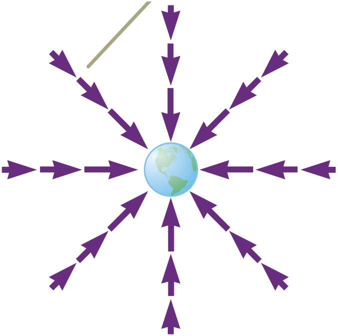
[1]Figure from http://www.launc.tased.edu.au

Gravitational Field of the Earth

Near the surface of the Earth:

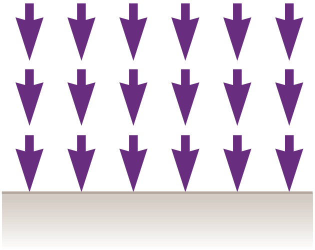


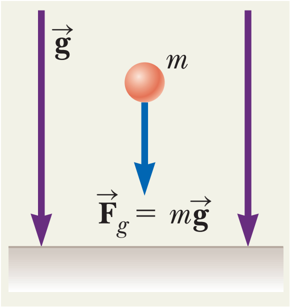
Farther out from the Earth:



Gravitational Field of the Earth

Uniform **g**:





A test mass m experiences a force $\mathbf{F}_g = m\mathbf{g}$, where $\mathbf{g}$ is the field vector.

Gravitational Potential Energy

The gravitational potential energy stored by two masses at a distance r from each other is given by:

$$U(r) = -\frac{Gm_1 m_2}{r}$$

Choosing $U(r) = 0$ at $r = \infty$.

This will *always* be a negative number.

Gravitational Potential Energy

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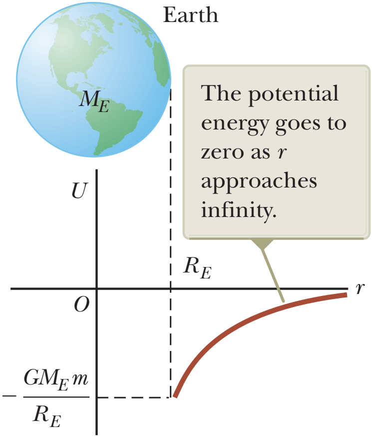
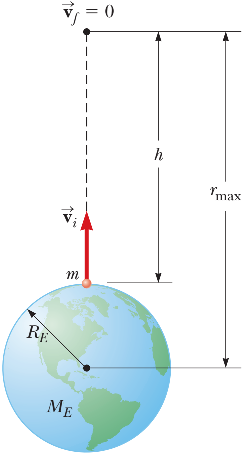


Example using Grav. PE: Escape Speed

How fast does an object need to be projected with to escape Earth's gravity?



Escape Speed

The object begins with kinetic energy $K = \frac{1}{2}mv_i^2$ and potential energy $U = -\frac{GM_E m}{R_E}$.

To "escape" Earth's gravity well the object needs to reach $U = 0$.
$\Rightarrow$ Trade kinetic for potential energy.

Find an expression for the speed the object must have at the surface of the Earth to escape.

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<u>system: object + Earth, isolated</u>

$$\Delta K + \Delta U = 0$$

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$$\begin{aligned} \Delta K + \Delta U &= 0 \\ (0 - \frac{1}{2}mv_i^2) + \left(0 - \left(-\frac{GM_E m}{R_E}\right)\right) &= 0 \\ \frac{1}{2}mv_i^2 &= \frac{GM_E m}{R_E} \\ v_i &= \underline{\sqrt{\frac{2GM_E}{R_E}}} \end{aligned}$$

Summary

- Newton's law of gravitation
- gravitational field
- gravitational potential energy

Homework

- Ch 13 Ques: 1; Probs: 1, 5, 17, 19, 29, 33