Mechanics
Oscillations
Simple Harmonic Motion

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Dec 3, 2018

Last time

- gravity
- Newton's universal law of gravitation
- gravitational field
- gravitational potential energy

Overview

- oscillations
- simple harmonic motion (SHM)
- springs and SHM
- energy in SHM
- pendula and SHM

Oscillations and Periodic Motion

Many physical systems exhibit cycles of repetitive behavior.

After some time, they return to their initial configuration.

Examples:

- clocks
- rolling wheels
- a pendulum
- bobs on springs

Oscillations

oscillation

motion that repeats over a period of time

amplitude

the magnitude of the vibration; how far does the object move from its average (equilibrium) position.

period

the time for one complete oscillation.

After 1 period, the motion repeats itself.

Oscillations

frequency

The number of complete oscillations in some amount of time. Usually, oscillations per second.

$$f = \frac{1}{T}$$

Units of frequency: Hertz. $1 \text{ Hz} = 1 \text{ s}^{-1}$

If one oscillation takes a quarter of a second (0.25 s), then there are 4 oscillations per second. The frequency is $4 \text{ s}^{-1} = 4 \text{ Hz}$.

Simple Harmonic Motion

The oscillations of bobs on springs and pendula are very regular and simple to describe.

It is called simple harmonic motion.

simple harmonic motion (SHM)

any motion in which the acceleration is proportional to the displacement from equilibrium, but opposite in direction

The force causing the acceleration is called the "restoring force".

SHM and Springs

If a mass is attached to a spring, the force on the mass depends on its displacement from the spring's natural length.

Hooke's Law:

$$\mathbf{F} = -k\mathbf{x}$$

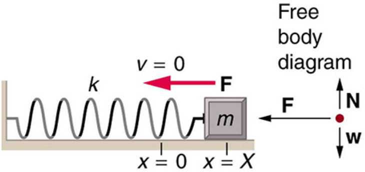
where k is the spring constant and x is the displacement (position) of the mass.

Hooke's law gives the force on the bob $\Rightarrow$ SHM.

The spring force is the *restoring force*.

SHM and Springs

How can we find an equation of motion for the block?



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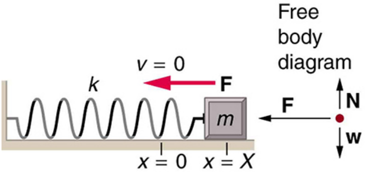

Newton's second law:

$$\mathbf{F}_{\text{net}} = m\mathbf{a}$$

Considering the x direction

$$\begin{aligned} F_s &= ma_x \\ -kx &= ma_x \end{aligned}$$

SHM and Springs

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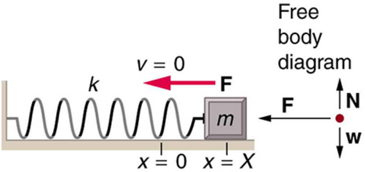

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Considering the x direction

$$\begin{aligned} F_s &= ma_x \\ -kx &= ma_x \end{aligned}$$

using the definition of acceleration, $a_x = \frac{\mathrm{d}^2 x}{\mathrm{d}t^2}$,

$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = -\frac{k}{m} x$$

SHM and Springs

$$\frac{\mathrm{d}^2 x}{\mathrm{dt}^2} = -\frac{k}{m}x$$

Define

$$\omega = \sqrt{\frac{k}{m}}$$

and we can write this equation as:

$$\frac{\mathrm{d}^2 x}{\mathrm{dt}^2} = -\omega^2 x$$

SHM and Springs

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Define

$$\omega = \sqrt{\frac{k}{m}}$$

and we can write this equation as:

$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = -\omega^2 x$$

This is the equation of motion for the block. The block is in SHM because $a \propto -x$.

SHM and Springs

This is a second order linear differential equation.

$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = -\omega^2 x$$

A solution $x(t)$ to this equation has the property that if we take its derivative twice, we get the same form of the function back again, but with an additional factor of $-\omega^2$.

Candidate: $x(t) = A\cos(\omega t)$, where A is a constant.

SHM and Springs

This is a second order linear differential equation.

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

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Candidate: $x(t) = A\cos(\omega t)$, where A is a constant.

Check by taking the derivative twice.

$$\frac{d^2x}{dt^2} = -\omega^2 \left(A \ \cos(\omega t)\right) = -\omega^2 x \qquad \checkmark$$

SHM and Springs

$$x = A\cos(\omega t + \phi)$$

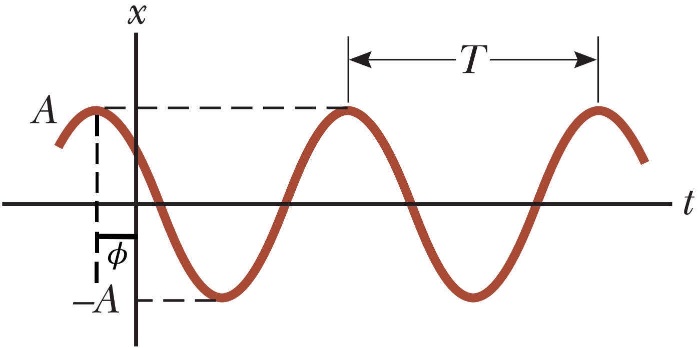
where $\omega = \sqrt{\frac{k}{m}}$ and A is the amplitude.

(In the textbook this is written as $x = x_m \cos(\omega t + \phi)$, with x_m the amplitude.)

This equation tells us what is the position of the block, x, at any point in time, t.

Waveform

$$x = A\cos(\omega t + \phi)$$



$$f = \frac{1}{T}$$

[1]Figure from Serway & Jewett, 9th ed, pg 453.

Oscillations

angular frequency

angular displacement per unit time in rotation, or the rate of change of the phase of a sinusoidal waveform

$$\omega = \frac{2\pi}{T} = 2\pi f$$

SHM and Springs

$$x = A\cos(\omega t + \phi)$$

ω is the angular frequency of the oscillation.

When $t = \frac{2\pi}{\omega}$ the block has returned to the position it had at $t = 0$. That is one complete cycle.

SHM and Springs

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Recalling that $\omega = \sqrt{k/m}$:

$$\text{Period, } T = 2\pi\sqrt{\frac{m}{k}}$$

Only depends on the mass of the bob and the spring constant.

SHM and Springs Question

A mass-spring system has a period, T. If the amplitude of the motion is quadrupled (and everything else is unchanged), what happens to the period of the motion?

(A) halves, $T/2$

(B) remains unchanged, T

(C) doubles, $2T$

(D) quadruples, $4T$

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(B) remains unchanged, T $\leftarrow$

(C) doubles, $2T$

(D) quadruples, $4T$

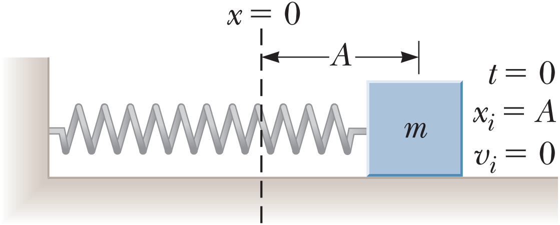
T does not depend on the amplitude.

SHM and Springs

The position of the bob at a given time is given by:

$$x = A\cos(\omega t + \phi)$$

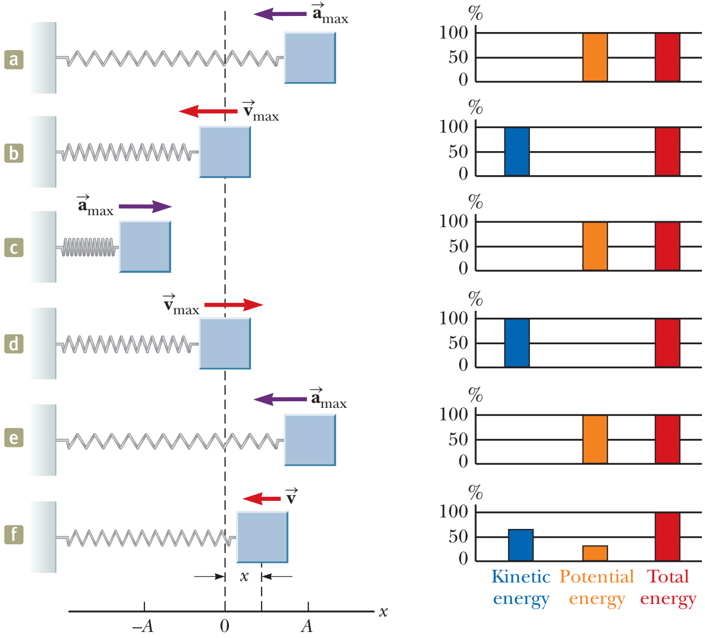
A is the amplitude of the oscillation. We could also write $x_{\text{max}} = A$.



The speed of the particle at any point in time is:

$$v = \frac{dx}{dt} = -A\omega\sin(\omega t + \phi)$$

Energy in SHM



Energy in SHM

Potential Energy:

$$U = \frac{1}{2}kx^2 = \frac{1}{2}kA^2\cos^2(\omega t + \phi)$$

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Kinetic Energy:

$$K = \frac{1}{2}mv^2 = \frac{1}{2}mA^2\omega^2\sin^2(\omega t + \phi)$$

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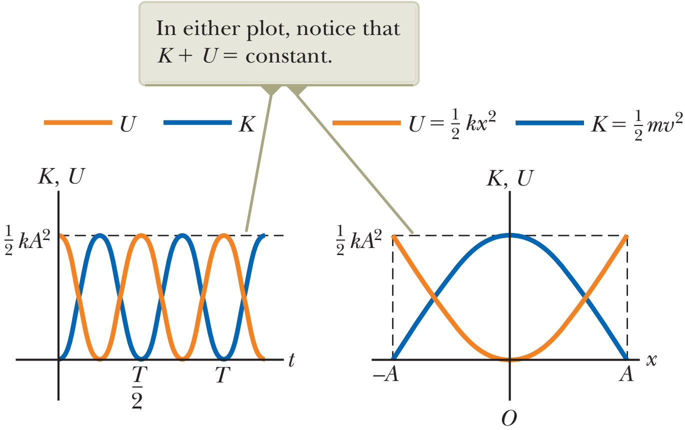
Using $\omega^2 = k/m$

$$\begin{aligned} K + U &= \frac{1}{2}kA^2\big(\sin^2(\omega t + \phi) + \cos^2(\omega t + \phi)\big) \\ &= \frac{1}{2}kA^2 \end{aligned}$$

This does not depend on time!

Energy in SHM



$$K + U = \frac{1}{2}kA^2$$

[1]Figure from Serway & Jewett, 9th ed.

Pendula and SHM

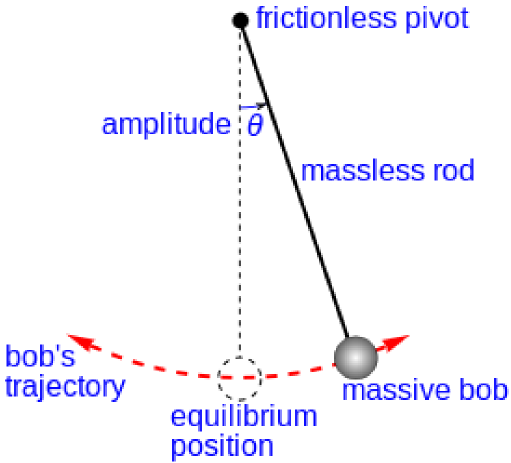
pendulum

a massive bob attached to the end rod or string that will oscillate along a circular arc under the influence of gravity

A pendulum bob that is displaced to one side by a small amount and released follows SHM to a good approximation.

Gravity and the tension in the string provide the *restoring force*.

Pendula and SHM



Pendula and SHM

Pendula also obey simple harmonic motion to a very good approximation, as long as the amplitude of the swing is small.

$$\theta = A\cos(\omega t + \phi)$$

Period of a pendulum:

$$\text{Period, } T = 2\pi\sqrt{\frac{L}{g}}$$

where L is the length of the pendulum and g is the acceleration due to gravity.

Problem

An astronaut on the Moon attaches a small brass ball to a 1.00 m length of string and makes a simple pendulum. She times 15 complete swings in a time of 75 seconds. From this measurement she calculates the acceleration due to gravity on the Moon. What is her result?[1]

[1] Hewitt, "Conceptual Physics", problem 8, page 350.

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1.58 m/s^2

[1] Hewitt, "Conceptual Physics", problem 8, page 350.

Summary

- SHM
- springs and pendula
- energy in SHM

Final Exam Tuesday Dec 11, 9:15-11:15am, S16.

Homework

- Ch 15 Ques: 2; Probs: 1, 5, 13, 17, 29, 33