Mechanics
Acceleration
The “Kinematics Equations”

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Last time

- kinematic quantities
- graphs of kinematic quantities

Overview

- acceleration
- the kinematics equations (constant acceleration)
- applying the kinematics equations

Question: Average Velocity vs Average Speed

Quick Quiz 2.1[1] Under which of the following conditions is the magnitude of the **average velocity** of a particle moving in one dimension **smaller** than the **average speed** over some time interval?

A A particle moves in the $+x$ direction without reversing.

B A particle moves in the $-x$ direction without reversing.

C A particle moves in the $+x$ direction and then reverses the direction of its motion.

D There are no conditions for which this is true.

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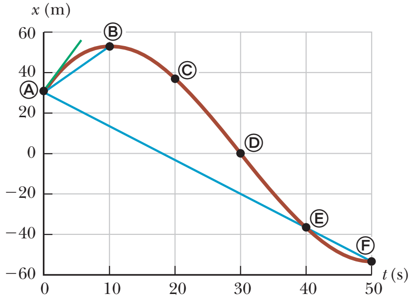
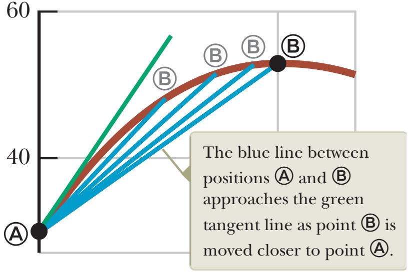
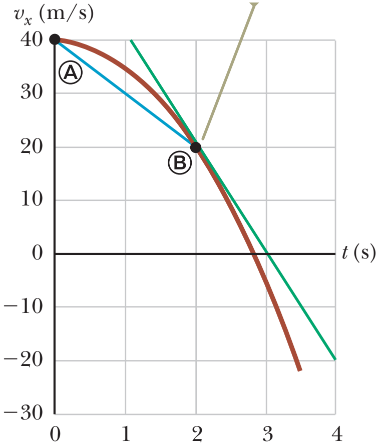
Instantaneous Velocity and Position-Time Graphs



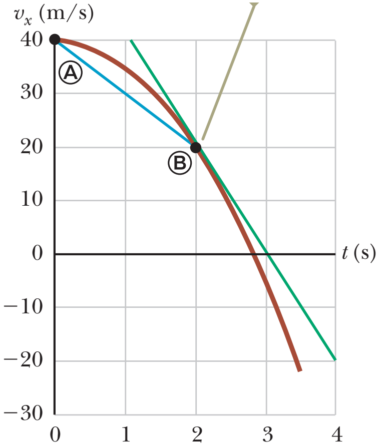


$$\mathbf{v} = \lim_{\Delta t \to 0} \frac{\mathbf{x}(t + \Delta t) - \mathbf{x}(t)}{t + \Delta t - t} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{x}}{\Delta t} = \frac{\mathrm{d}\mathbf{x}}{\mathrm{dt}}$$

Velocity vs. Time Graphs



Velocity vs. Time Graphs



The slope at any point of the velocity-time curve is the **acceleration** at that time.

Acceleration

acceleration	$\mathbf{a} = \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = \frac{\mathrm{d}^2\mathbf{x}}{\mathrm{d}t^2}$
average acceleration	$\mathbf{a}_{\mathbf{avg}} = \frac{\Delta \mathbf{v}}{\Delta t}$

Acceleration is also a vector quantity.

Acceleration

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Acceleration is also a vector quantity.

If the acceleration vector is pointed in the **same** direction as the velocity vector (*ie.* both are positive or both negative), the particle's **speed is increasing**.

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Acceleration is also a vector quantity.

If the acceleration vector is pointed in the **same** direction as the velocity vector (*ie.* both are positive or both negative), the particle's **speed is increasing**.

If the acceleration vector is pointed in the **opposite** direction as the velocity vector (*ie.* one is positive the other is negative), the particle's **speed is decreasing**. (It is "decelerating".)

Acceleration and Velocity-Time Graphs

Acceleration is the slope of a velocity-time curve.

Units: meters per second per second, m/s^2

Acceleration and Velocity-Time Graphs

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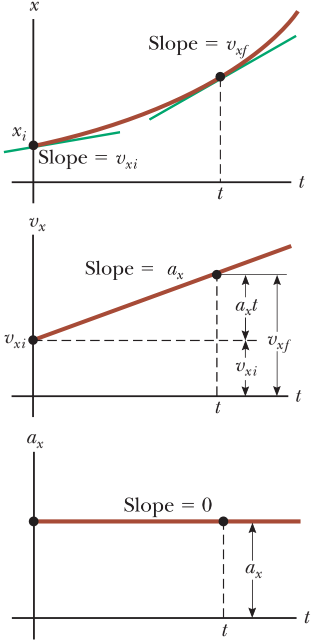
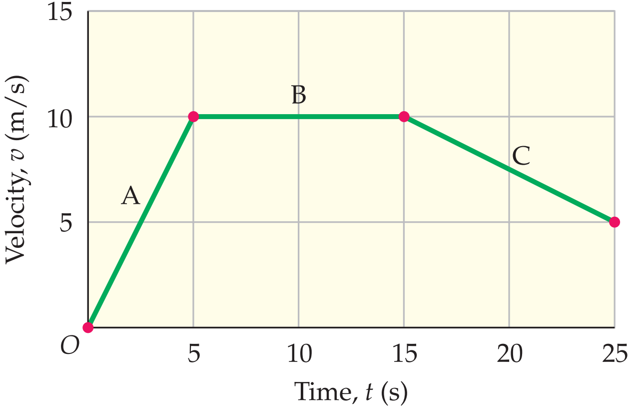
In general, acceleration can be a function of time $\mathbf{a}(t)$.

Acceleration Graphs



Returning to Velocity vs Time Graphs

The area under a velocity-time graph has a special interpretation: it is the **displacement** of the object over the time interval considered.



$$\boldsymbol{\Delta x} = \mathbf{v}_{\text{avg}}\,\Delta t$$

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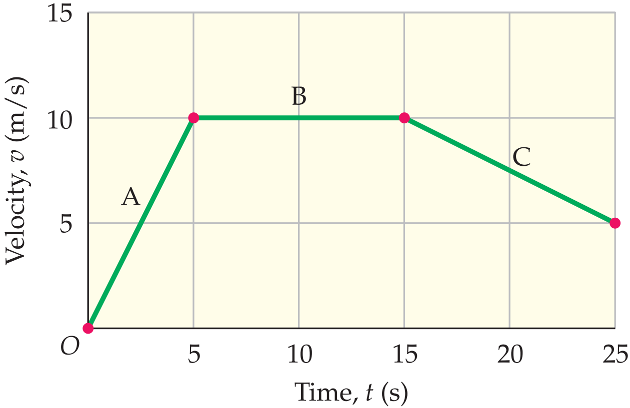
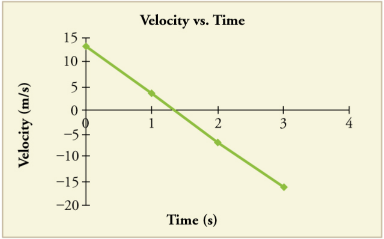


$$\Delta \boldsymbol{x} = (25 \text{ m} + 100 \text{ m} + 75 \text{ m})\mathbf{i} = 200 \text{ m } \mathbf{i}$$

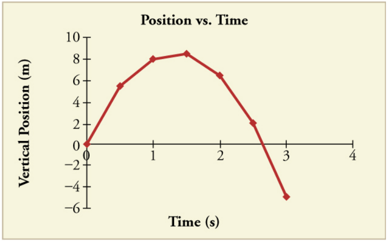
Area under Velocity vs. Time Graphs

v-t and x-t graphs for the same object:

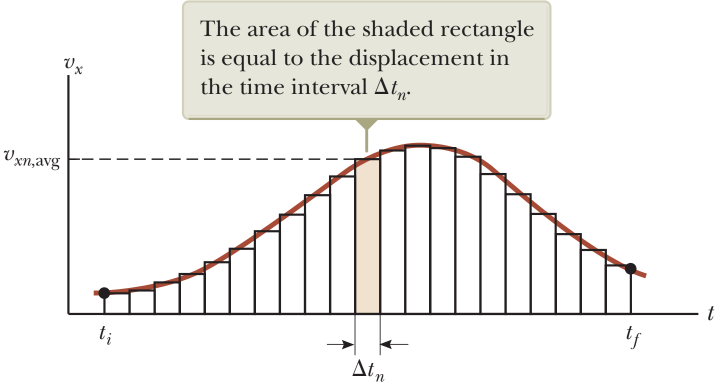
Area under v-t graph $= \Delta x$.



Slope of x-t curve $= v$.



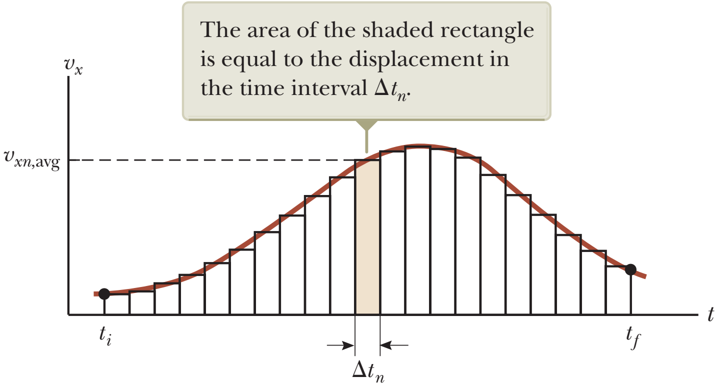
Velocity vs. Time Graphs



$$\Delta x = \lim_{\Delta t \to 0} \sum_n v_n \, \Delta t = \int_{t_i}^{t_f} v \, dt$$

where Δx represents the change in position (displacement) in the time interval t_i to t_f.

Velocity vs. Time Graphs



Or we can write

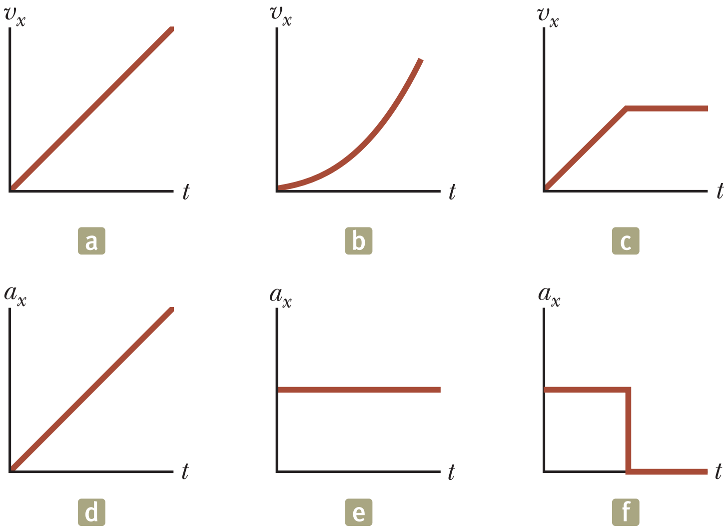
$$x(t) = \int_{t_i}^{t} v \, dt'$$

if the object starts at position $x = 0$ when $t = t_i$.

Question

What does the area under an **acceleration-time** graph represent?

Matching Velocity to Acceleration Graphs



Summary

- acceleration

Homework - CHANGED!

- Read Ch 2.
- Ch 2, Questions: 1, 2, 4, 5; Problems: 19, 21, 90
- (will be set on Monday: Ch 2, Problems: 23, 25, 31, 35, 41, 69, 73)