Mechanics
The "Kinematics Equations"

Lana Sheridan

De Anza College

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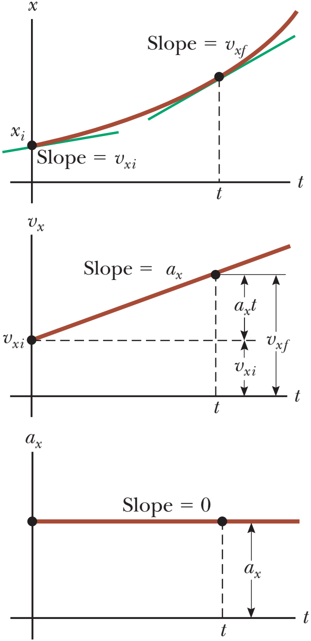
Last time

- acceleration
- graphs of kinematic quantities

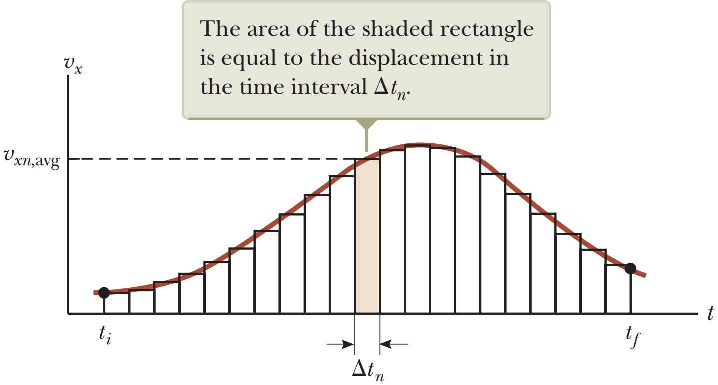
Overview

- finish discussion of graphs
- the kinematics equations (constant acceleration)
- applying the kinematics equations

Kinematics Graphs



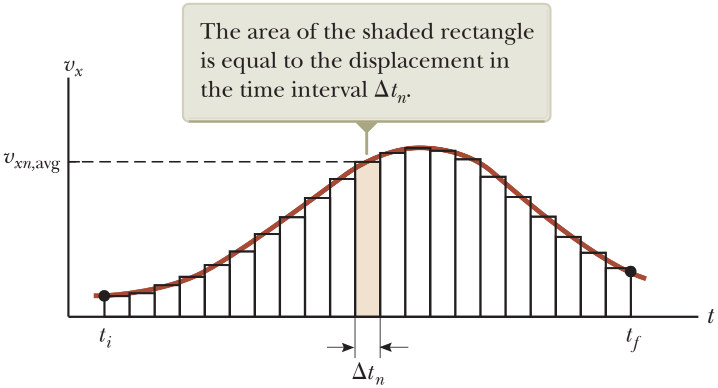
Velocity vs. Time Graphs



$$\Delta x = \lim_{\Delta t \to 0} \sum_n v_n \, \Delta t = \int_{t_i}^{t_f} v \, \mathrm{d}t$$

where Δx represents the change in position (displacement) in the time interval t_i to t_f.

Velocity vs. Time Graphs



Or we can write

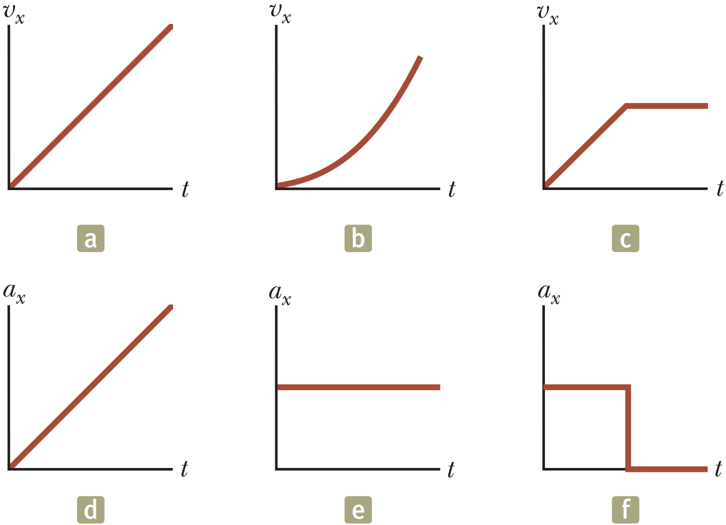
$$x(t) = \int_{t_i}^{t} v \, dt'$$

if the object starts at position $x = 0$ when $t = t_i$.

Question

What does the area under an **acceleration-time** graph represent?

Matching Velocity to Acceleration Graphs



The Kinematics Equations

This is a set of very useful equations for the case of **constant acceleration.**

Very often in real life accelerations are not constant, but one important case where acceleration is constant[1] is for falling objects.

[1]At least nearly constant, neglecting air resistance and small variations in g near the Earth's surface.

The Kinematics Equations

For constant acceleration:

$$\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t$$

$$\Delta\boldsymbol{x} = \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2$$

$$\Delta\boldsymbol{x} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$$

$$\Delta\boldsymbol{x} = \frac{\mathbf{v}_0 + \mathbf{v}}{2}t$$

$$v^2 = v_0^2 + 2\,a\,\Delta x$$

For zero acceleration:

$$\Delta\boldsymbol{x} = \mathbf{v}t$$

Vector Equations vs Scalar Equations

I will write the kinematics equations in vector form, for example:

$$\boldsymbol{\Delta x} = \mathbf{v} t$$

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What we can do, is write this equation instead as a scalar equation by factoring out the unit vectors from each side:

$$\Delta x\mathbf{i} = (v\mathbf{i})t$$

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What we can do, is write this equation instead as a scalar equation by factoring out the unit vectors from each side:

$$\Delta x = vt$$

In that last expression, Δx and v are the *signed* magnitudes of the $\boldsymbol{\Delta x}$ and $\mathbf{v}$ vectors.

That is, Δx and v can be positive or negative.

The Kinematics Equations: the "no-displacement" equation

From the definition of average acceleration:

$$\mathbf{a}_{\text{avg}} = \frac{\Delta \boldsymbol{v}}{\Delta t}$$

$$\Delta \boldsymbol{v} = \mathbf{v} - \mathbf{v}_0$$

and starting at time $t = 0$ means $\Delta t = t - 0 = t$.

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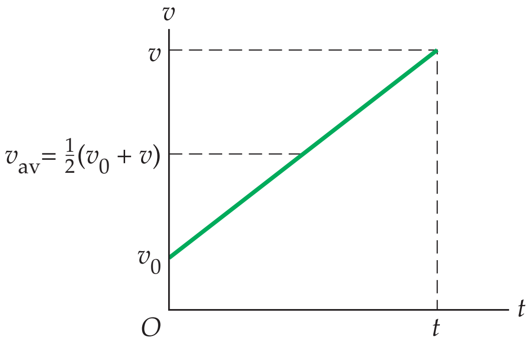
For constant acceleration $\mathbf{a}_{\text{avg}} = \mathbf{a}$, so $\mathbf{a} = \frac{\mathbf{v} - \mathbf{v}_0}{t}$

$$\mathbf{v}(t) = \mathbf{v}_0 + \mathbf{a}t \tag{1}$$

where v_0 is the velocity at $t = 0$ and $\mathbf{v}(t)$ is the velocity at time t.

Average Velocity

IF the acceleration of an object is constant, then the velocity-time graph is a straight line,



$$\mathbf{v}_{avg} = \frac{1}{2}(\mathbf{v}_0 + \mathbf{v})$$

The Kinematics Equations: the "no-acceleration" equation

From the definition of average velocity:

$$\mathbf{v}_{avg} = \frac{\boldsymbol{\Delta x}}{t}$$

and $\mathbf{v_{avg}} = \frac{\mathbf{v}_0 + \mathbf{v}}{2}$

Equating them, and multiplying by t:

$$\Delta \boldsymbol{x} = \left(\frac{\mathbf{v}_0 + \mathbf{v}}{2} \right) t \qquad (2)$$

The Kinematics Equations: the "no-final-velocity" equation

Using the equation

$$\boldsymbol{\Delta x} = \left(\frac{\mathbf{v}_0 + \mathbf{v}}{2}\right) t$$

and the equation

$$\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t$$

replace $\mathbf{v}$ in the first equation.

$$\begin{aligned} \boldsymbol{\Delta x} &= \left(\frac{\mathbf{v}_0 + (\mathbf{v}_0 + \mathbf{a}t)}{2}\right) t \\ &= \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2 \end{aligned}$$

For constant acceleration:

$$\mathbf{x}(t) = \mathbf{x}_0 + \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2 \tag{3}$$

Example 2-6, page 34

A drag racer starts from rest and accelerates at 7.40 m/s^2. How far has it traveled in (a) 1.00 s, (b) 2.00 s, (c) 3.00 s?

[1] Walker "Physics", pg 33.

Example 2-6, page 34

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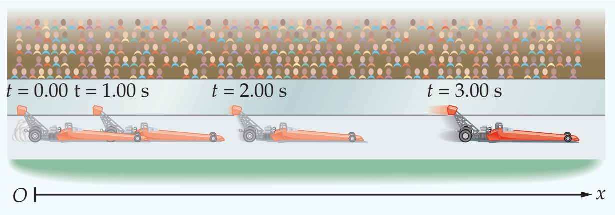
Sketch:



[1]Walker "Physics", pg 33.

Using the Kinematics Equations

Process:

1. Identify which quantity we need to find and which ones we are given.
2. Is there a quantity that we are not given and are not asked for?
    1. If so, use the equation that does *not* include that quantity.
    2. If there is not, more that one kinematics equation may be required or there may be several equivalent approaches.
3. Input known quantities and solve.

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Given: $a = 7.40$ m/s^2, $v_0 = 0$ m/s, t.
Asked for: Δx

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Asked for: Δx

Strategy: Use equation

$$\boldsymbol{\Delta x} = \mathbf{x}(t) - \mathbf{x}_0 = \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2$$

(a) Letting the x-direction in my sketch be positive:

$$\begin{aligned}\Delta x &= \cancelto{0}{v_0}\, t + \frac{1}{2}at^2 \\ &= \frac{1}{2}(7.40\ \text{m/s}^2)(1.00\ \text{s})^2 \\ &= \underline{3.70\ \text{m}}\end{aligned}$$

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A drag racer starts from rest and accelerates at 7.40 m/s^2. How far has it traveled in (a) 1.00 s, (b) 2.00 s, (c) 3.00 s?

Use the same equation for (b), (c)

$$\boldsymbol{\Delta x} = \mathbf{x}(t) - \mathbf{x}_0 = \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2$$

$$\text{(b)} \qquad \begin{aligned} \Delta x &= \frac{1}{2}at^2 \\ &= \frac{1}{2}(7.40\ \mathrm{m/s^2})(2.00\ \mathrm{s})^2 \\ &= \underline{14.8\ \mathrm{m}} \end{aligned}$$

$$\text{(c)} \qquad \begin{aligned} \Delta x &= \frac{1}{2}at^2 \\ &= \frac{1}{2}(7.40\ \mathrm{m/s^2})(3.00\ \mathrm{s})^2 \\ &= \underline{33.3\ \mathrm{m}} \end{aligned}$$

Example 2-6, page 34

(a) 3.70 m, (b) 14.8 m, (c) 33.3 m

Analysis:
It makes sense that the distances covered by the car increases with time, and it makes sense that the distance covered in each one second interval is greater than the distance covered in the previous interval since the car is still accelerating.

The distance covered over 3 seconds is 9 times the distance covered in 1 second.

The car covers ~ 30 m in 3 s, giving an average speed of ~ 10 m/s. We know cars can go much faster than this, so the answer is not unreasonable.

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[1] Walker "Physics", pg 33.

The Kinematics Equations: the “no-initial-velocity” equation

Exercise for you: try to prove this equation.

For constant acceleration:

$$\mathbf{x}(t) = \mathbf{x}_0 + \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2 \tag{4}$$

The Kinematics Equations: the “no-time” equation

The last equation we will derive is a scalar equation.

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$$\boldsymbol{\Delta x} = \left(\frac{\mathbf{v}_0 + \mathbf{v}}{2}\right) t$$

We could also write this as:

$$(\Delta x)\,\mathbf{i} = \left(\frac{v_0 + v}{2} t\right)\mathbf{i}$$

where Δx, v_i, and v_f could each be positive or negative.

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We do the same for equation (1):

$$v\,\hat{\mathbf{i}} = (v_0 + at)\,\hat{\mathbf{i}}$$

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where Δx, v_i, and v_f could each be positive or negative.
We do the same for equation (1):

$$v = (v_0 + at)$$

Rearranging for t:

$$t = \frac{v - v_0}{a}$$

The Kinematics Equations: the “no-time” equation

$$t = \frac{v - v_0}{a} \; ; \qquad \Delta x = \left(\frac{v_0 + v}{2}\right) t$$

Substituting for t in our Δx equation:

$$\begin{aligned} \Delta x &= \left(\frac{v_0 + v}{2}\right)\left(\frac{v - v_0}{a}\right) \\ 2a\Delta x &= (v_0 + v)(v - v_0) \end{aligned}$$

so,

$$v^2 = v_0^2 + 2\,a\,\Delta x \tag{5}$$

The Kinematics Equations Summary

For constant acceleration:

$$\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t$$

$$\boldsymbol{\Delta x} = \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2$$

$$\boldsymbol{\Delta x} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$$

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$$v^2 = v_0^2 + 2\,a\,\Delta x$$

For zero acceleration:

$$\mathbf{x} = \mathbf{v}t$$

Summary

- acceleration
- the “kinematics equations”
- applying the kinematics equations

Homework

- previous: Ch 2, Questions: 1, 2, 4, 5; Problems: 19, 21, 90
- new: Ch 2, Problems: 23, 25, 31, 35, 41, 69, (73 - can wait to do)