Kinematics Equations and Examples

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Last time

- acceleration
- the kinematics equations (constant acceleration)

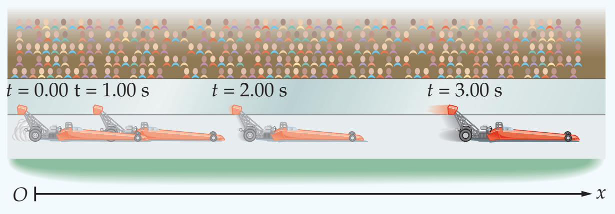
Overview

- the kinematics equations (constant acceleration), continued
- a harder kinematics example

Example 2-6, page 34

A drag racer starts from rest and accelerates at 7.40 m/s^2. How far has it traveled in (a) 1.00 s, (b) 2.00 s, (c) 3.00 s?

Sketch:



[1]Walker "Physics", pg 33.

Using the Kinematics Equations

Process:

1. Identify which quantity we need to find and which ones we are given.
2. Is there a quantity that we are not given and are not asked for?
   1. If so, use the equation that does *not* include that quantity.
   2. If there is not, more that one kinematics equation may be required or there may be several equivalent approaches.
3. Input known quantities and solve.

Example 2-6, page 34

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Asked for: Δx

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Strategy: Use equation

$$\boldsymbol{\Delta x} = \mathbf{x}(t) - \mathbf{x}_0 = \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2$$

(a) Letting the x-direction in my sketch be positive:

$$\begin{aligned} \Delta x &= \cancelto{0}{v_0}\, t + \frac{1}{2}at^2 \\ &= \frac{1}{2}(7.40\ \text{m/s}^2)(1.00\ \text{s})^2 \\ &= \underline{3.70\ \text{m}} \end{aligned}$$

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Example 2-6, page 34

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Use the same equation for (b), (c)

$$\boldsymbol{\Delta x} = \mathbf{x}(t) - \mathbf{x}_0 = \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2$$

$$\text{(b)} \qquad \begin{aligned} \Delta x &= \frac{1}{2}at^2 \\ &= \frac{1}{2}(7.40 \text{ m/s}^2)(2.00 \text{ s})^2 \\ &= \underline{14.8 \text{ m}} \end{aligned}$$

$$\text{(c)} \qquad \begin{aligned} \Delta x &= \frac{1}{2}at^2 \\ &= \frac{1}{2}(7.40 \text{ m/s}^2)(3.00 \text{ s})^2 \\ &= \underline{33.3 \text{ m}} \end{aligned}$$

Example 2-6, page 34

(a) 3.70 m, (b) 14.8 m, (c) 33.3 m

Analysis:
It makes sense that the distances covered by the car increases with time, and it makes sense that the distance covered in each one second interval is greater than the distance covered in the previous interval since the car is still accelerating.

The distance covered over 3 seconds is 9 times the distance covered in 1 second.

The car covers ~ 30 m in 3 s, giving an average speed of ~ 10 m/s. We know cars can go much faster than this, so the answer is not unreasonable.

[1] Walker "Physics", pg 33.

The Kinematics Equations: the “no-initial-velocity” equation

Exercise for you: try to prove this equation.

For constant acceleration:

$$\mathbf{x}(t) = \mathbf{x}_0 + \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2 \tag{4}$$

The Kinematics Equations: the “no-time” equation

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$$\boldsymbol{\Delta x} = \left(\frac{\mathbf{v}_0 + \mathbf{v}}{2} \right) t$$

We could also write this as:

$$(\Delta x)\, \mathbf{i} = \left(\frac{v_0 + v}{2} t \right) \mathbf{i}$$

where Δx, v_i, and v_f could each be positive or negative.

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We do the same for equation (1):

$$v\,\hat{\mathbf{i}} = (v_0 + at)\,\hat{\mathbf{i}}$$

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$$v = (v_0 + at)$$

Rearranging for t:

$$t = \frac{v - v_0}{a}$$

The Kinematics Equations: the "no-time" equation

$$t = \frac{v - v_0}{a} \; ; \qquad \Delta x = \left(\frac{v_0 + v}{2}\right) t$$

Substituting for t in our Δx equation:

$$\begin{aligned} \Delta x &= \left(\frac{v_0 + v}{2}\right)\left(\frac{v - v_0}{a}\right) \\ 2a\Delta x &= (v_0 + v)(v - v_0) \end{aligned}$$

so,

$$v^2 = v_0^2 + 2\,a\,\Delta x \tag{5}$$

The Kinematics Equations Summary

For constant acceleration:

$$\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t$$

$$\boldsymbol{\Delta x} = \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2$$

$$\boldsymbol{\Delta x} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$$

$$\boldsymbol{\Delta x} = \frac{\mathbf{v}_0 + \mathbf{v}}{2}t$$

$$v^2 = v_0^2 + 2\,a\,\Delta x$$

For zero acceleration:

$$\mathbf{x} = \mathbf{v}t$$

Example

A car driver sees an obstacle in the road and applies the brakes. It takes him 4.33 s to stop the car over a distance of 55.0 m. Assuming the car brakes with constant acceleration, what was the car's deceleration?

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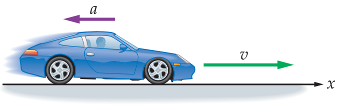

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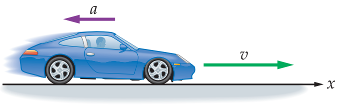

Given: $t = 4.33$ s, $\Delta x = 55.0$ m, $v = 0$ m/s
Asked for: **a**

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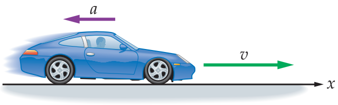

Given: $t = 4.33$ s, $\Delta x = 55.0$ m, $v = 0$ m/s
Asked for: $\mathbf{a}$
Strategy: use

$$\boldsymbol{\Delta x} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$$

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$$\begin{aligned}
\boldsymbol{\Delta x} &= \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2 \\
\frac{1}{2}\mathbf{a}t^2 &= \mathbf{v}t - \boldsymbol{\Delta x} \\
\mathbf{a} &= \frac{2(\mathbf{v}t - \boldsymbol{\Delta x})}{t^2}
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\mathbf{a} &= \frac{2(\mathbf{v}t - \boldsymbol{\Delta x})}{t^2} \\
&= \frac{2(0 - 55.0\ \mathrm{m}\mathbf{i})}{(4.33\ \mathrm{s})^2} \\
\mathbf{a} &= -5.87\ \mathrm{m/s}^2\,\mathbf{i}
\end{aligned}$$

Or, the car's acceleration is <u>5.87 m/s^2, opposite the direction of the car's travel</u>.

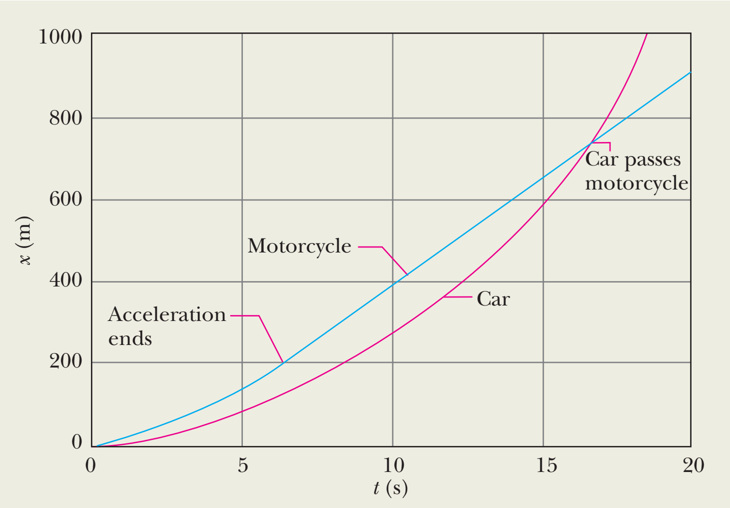
Harder Kinematics Equations Example

Sample Prob 2.04, HRW 10th ed

A popular web video shows a jet airplane, a car, and a motorcycle racing from rest along a runway. Initially the motorcycle takes the lead, but then the jet takes the lead, and finally the car blows past the motorcycle.

Consider just the car and motorcycle. The motorcycle first takes the lead because its (constant) acceleration $a_m = 8.40$ m/s^2 is greater than the car's (constant) acceleration $a_c = 5.60$ m/s^2, but it soon loses to the car because it reaches its greatest speed $v_m = 58.8$ m/s before the car reaches its greatest speed $v_c = 106$ m/s. How long does the car take to reach the motorcycle?

Harder Kinematics Equations Example



Harder Kinematics Equations Example

constant accelerations: $a_m = 8.40$ m/s^2, $a_c = 5.60$ m/s^2
top speeds: $v_m = 58.8$ m/s, $v_c = 106$ m/s

How long does the car take to reach the motorcycle?

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How long does the car take to reach the motorcycle?

Car catches up when their positions are the same.

$$x_c = x_m$$

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How long does the car take to reach the motorcycle?

Car catches up when their positions are the same.

$$x_c = x_m$$

The car we shall assume accelerates the entire time, starting from rest, so:

$$x_c = \frac{1}{2} a_c t^2$$

The motorcycle has two portions of motion: constant acceleration, and constant velocity

$$x_m = \frac{1}{2} a_m t_1^2 + v_m (t - t_1)$$

where t_1 is the time for the motorcycle to reach its maximum speed.

Harder Kinematics Equations Example

How long does the motorcycle take to reach its maximum speed, starting from rest?

$$\begin{aligned} v_m &= a_m t_1 \\ t_1 &= \frac{v_m}{a_m} \;\; (= 7.00\ s) \end{aligned}$$

$$\begin{aligned} x_c &= x_m \\ \frac{1}{2} a_c t^2 &= \frac{1}{2} a_m t_1^2 + v_m (t - t_1) \\ \frac{1}{2} a_c t^2 &= \frac{1}{2} \frac{v_m^2}{a_m} + v_m \left(t - \frac{v_m}{a_m}\right) \end{aligned}$$

Solving this quadratic equation gives:

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\end{aligned}
$$

Solving this quadratic equation gives:

$$
\begin{aligned}
t &= 16.6 \text{ s} \\
\text{and } t &= 4.44 \text{ s} \leftarrow \text{ rejected - why?}
\end{aligned}
$$

Harder Kinematics Equations Example

$$\underline{t = 16.6 \text{ s}}$$

Check: we assumed that the car accelerates during this whole time. Is that right?

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$$v = a_c t = 92.96 \text{ m/s} < v_c$$

Yes, the car has not yet reached its maximum speed it passes the motorcycle.

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Yes, the car has not yet reached its maximum speed it passes the motorcycle.

If the car's maximum speed was lower, say 85 m/s, would it still catch the motorcycle? Earlier or later?

Summary

- Kinematics equations

Quiz Thursday.

Homework

- previously set: Ch 2, Problems: 19, 21, 23, 25, 31, 35, 41, 69, 73
- Ch 2 Problems: 45, 49, 63, 89 (free fall - you can wait until tomorrow to do these if you like)