Mechanics
Vectors

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Oct 4, 2018

Last time

- free fall

Overview

- representing vectors
- vector properties

Math you will need for 2-Dimensions

Before going into motion in 2 dimensions, we will review some things about vectors.

Vectors

scalar

A scalar quantity indicates an amount. It is represented by a real number. (Assuming it is a physical quantity.)

vector

A vector quantity indicates both an amount and a direction. It is represented more than one real number. (Assuming it is a physical quantity.)

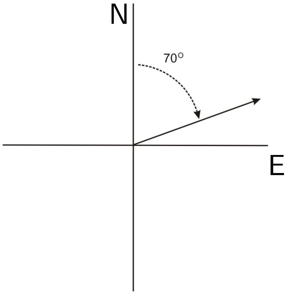
There are many ways to represent a vector.

- a magnitude and (an) angle(s)
- magnitudes in several perpendicular directions

Representing Vectors: Angles

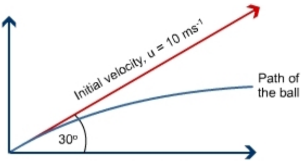
Bearing angles

Example, a plane flies 750 km h^{-1} at a bearing of 70°



Generic reference angles

A baseball is thrown at 10 m s^{-1}, 30° above the horizontal.



Representing Vectors: Unit Vectors

Magnitudes in several perpendicular directions: using *unit vectors.*

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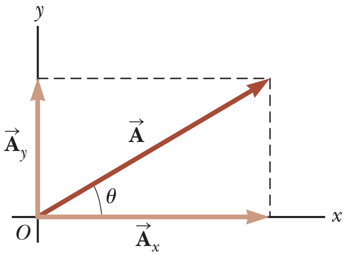
In two dimensions, a pair of perpendicular unit vectors are usually denoted $\mathbf{i}$ and $\mathbf{j}$ (or sometimes $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$).

Components

Consider the 2 dimensional vector $\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j}$, where A_x and A_y are numbers.

We then say that A_x is the *i*-component (or *x*-component) of $\mathbf{A}$ and A_y is the *j*-component (or *y*-component) of $\mathbf{A}$.



Notice that $A_x = A\cos\theta$ and $A_y = A\sin\theta$.

Components vs Magnitude-and-Angle Notation

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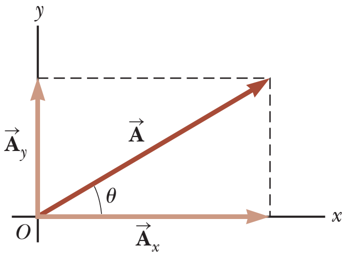

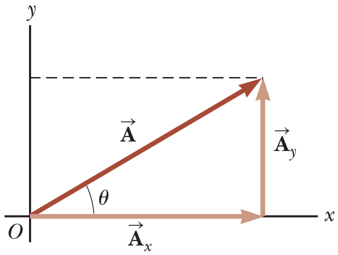

Also notice,

$$A = |\mathbf{A}| = \sqrt{A_x^2 + A_y^2}$$

and

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$

if the angle is given as shown.

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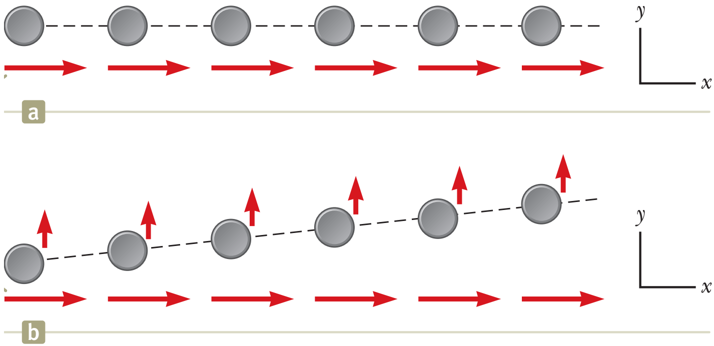
This makes life much easier: we will be able to solve for motion in the x direction separately from motion in the y direction.

Visualizing Motion in 2 Dimensions

Imagine an air hockey puck moving with horizontally constant velocity:



If it experiences a momentary upward (in the diagram) acceleration, it will have a component of velocity upwards.
The horizontal motion remains unchanged!

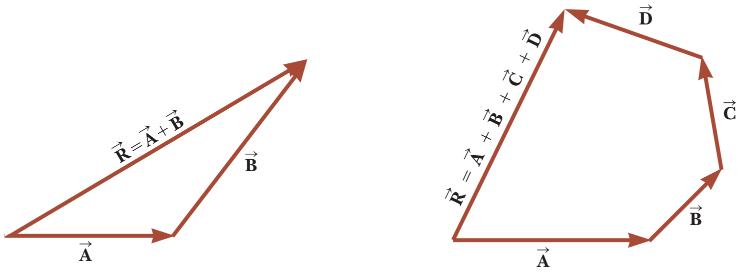
Vectors Properties and Operations

Equality

Vectors $\mathbf{A} = \mathbf{B}$ if and only if the magnitudes and directions are the same. (Each component is the same.)

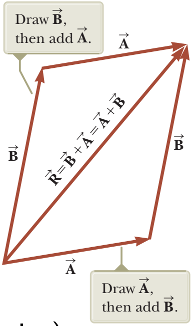
Addition

$\mathbf{A} + \mathbf{B}$

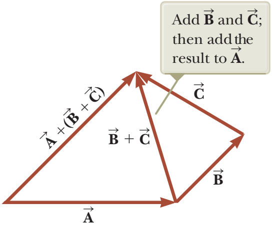


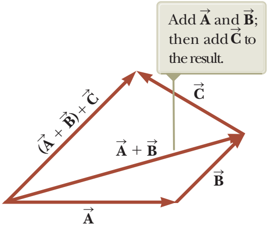
Vectors Properties and Operations
Properties of Addition

- $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$ (commutative)



- $(\mathbf{A} + \mathbf{B}) + \mathbf{C} = \mathbf{A} + (\mathbf{B} + \mathbf{C})$ (associative)

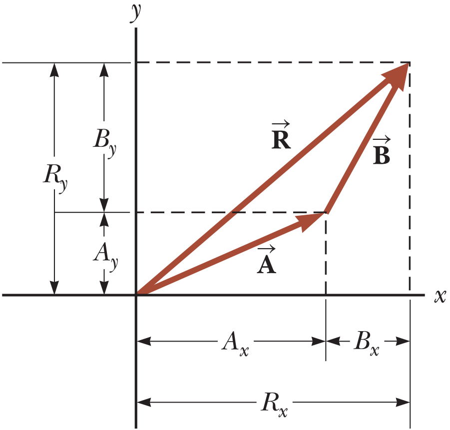




Vectors Properties and Operations

Doing addition:

Almost always the right answer is to break each vector into components and sum each component independently.



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$$\begin{aligned}\mathbf{w} + \mathbf{u} &= ?\\ &= (4+15)\mathbf{i} + (3+8)\mathbf{j}\\ &= (19\,\mathbf{i} + 11\,\mathbf{j})\text{ m}\end{aligned}$$

or 22.0 m at 30.1° above the horizontal.

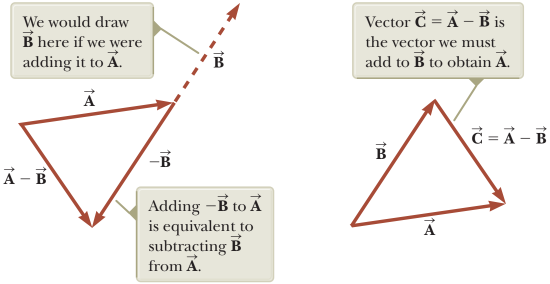
Vectors Properties and Operations

Negation

If $\mathbf{u} = -\mathbf{v}$ then $\mathbf{u}$ has the same magnitude as $\mathbf{v}$ but points in the **opposite** direction.

Subtraction

$\mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B})$



Summary

- vectors

Homework

- announced yesterday: Ch 3 Questions: 1, 4, 7; Problems: 1, 3, 5.
- new: Ch 3 Problems: 11, 15.
- new: Ch 4 Problem 76, 83 (relative motion - can wait to do).