Mechanics
Relative Motion
Projectiles

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Last time

- vectors

Overview

- motion in 2 dimensions
- relative motion
- introducing projectile motion

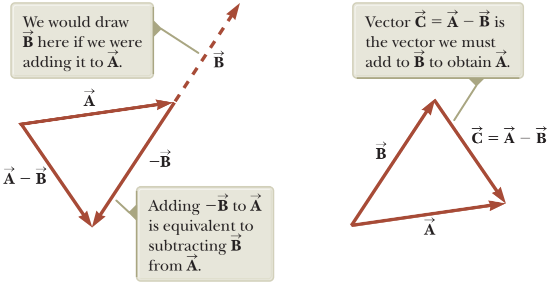
Vectors Properties and Operations

Negation

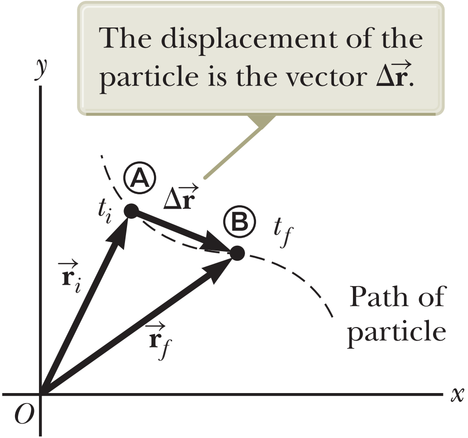
If $\mathbf{u} = -\mathbf{v}$ then $\mathbf{u}$ has the same magnitude as $\mathbf{v}$ but points in the **opposite** direction.

Subtraction

$\mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B})$



Motion in 2 Dimensions



$$\mathbf{r} = x\mathbf{i} + y\mathbf{j}$$

$$\Delta \boldsymbol{r} = \mathbf{r}_f - \mathbf{r}_i$$

Motion in 2 Dimensions

All other kinematic quantities generalize in a straightforward way.

$$\mathbf{v} = \frac{\mathrm{d}\mathbf{r}}{\mathrm{dt}}$$

where $\mathbf{v} = v_x\mathbf{i} + v_y\mathbf{j}$

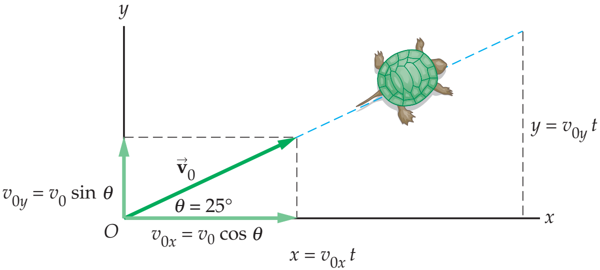
$$\mathbf{v}_{\mathrm{avg}} = \frac{\boldsymbol{\Delta r}}{\Delta t}$$

$$\mathbf{a} = \frac{\mathrm{d}\mathbf{v}}{\mathrm{dt}} \quad \text{(same expression as 1 dim)}$$

where $\mathbf{a} = a_x\mathbf{i} + a_y\mathbf{j}$

$$\mathbf{a}_{\mathrm{avg}} = \frac{\boldsymbol{\Delta v}}{\Delta t} \quad \text{(same expression as 1 dim)}$$

Constant Velocity in 2 Dimensions



Or, we can find the distance it travels in the x-direction by considering what is its rate of change of x-position with time!

$$v_{0x} = \frac{\Delta x}{\Delta t} = v_0 \cos\theta \quad \Rightarrow \quad x = (v_0 \cos\theta)\, t$$

And in the y-direction:

$$v_{0y} = \frac{\Delta y}{\Delta t} = v_0 \sin\theta \quad \Rightarrow \quad y = (v_0 \sin\theta)\, t$$

[1] Figure from Walker, "Physics".

Relative Motion

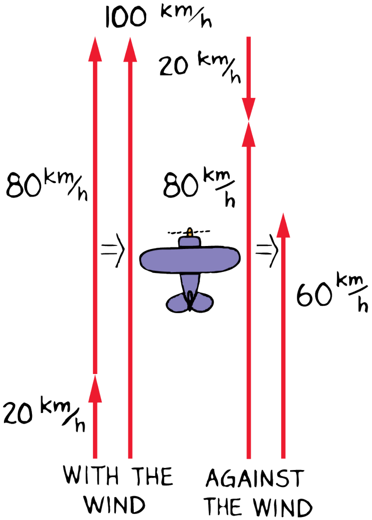
We can use the notion of motion in 2 dimensions to consider how one object moves **relative** to something else.

All motion is relative.

Our **reference frame** tells us what is a fixed position.

An example of a reference for time and space might be picking an object, declaring that it is at rest, and describing the motion of all objects relative to that.

Intuitive Example for Relative Velocities



[1]Figure by Paul Hewitt.

Intuitive Example

Now, imagine an airplane that is flying North at 80 km/h but is blown off course by a cross wind going East at 60 km/h.

How fast is the airplane moving? In which direction?

Sketch:

[1] Figure by Paul Hewitt.

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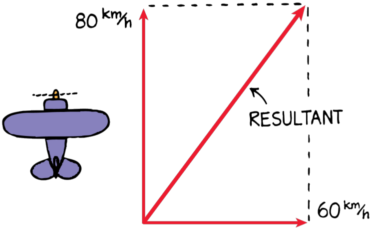
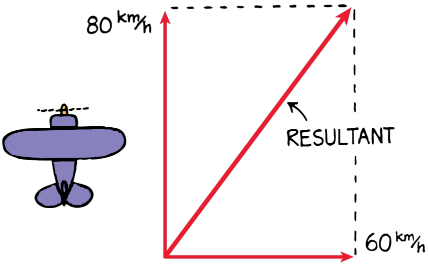


[1]Figure by Paul Hewitt.

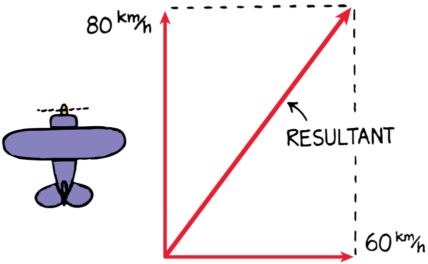
Intuitive Example



Strategy: vector addition!

In this case, the two vectors are at right-angles. We can use the Pythagorean theorem.

Intuitive Example



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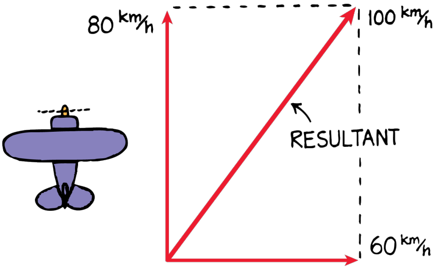

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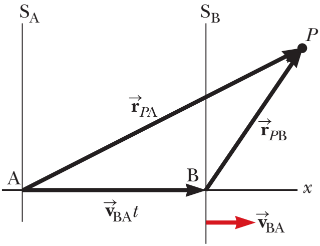
Relative Motion

One *very* useful technique for physical reasoning is considering other *frames of reference*.

A reference frame is a coordinate system that an observer adopts.

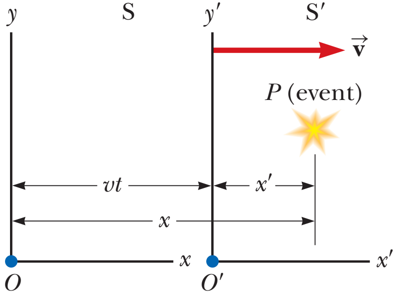
Different observers may have different perspectives: different frames of reference. Consider a pair of observers, one stationary (A), one moving with constant velocity $\mathbf{v}_{BA}$. Both observe a particle P.



Frames of Reference

How do we relate coordinates in different frames of reference?

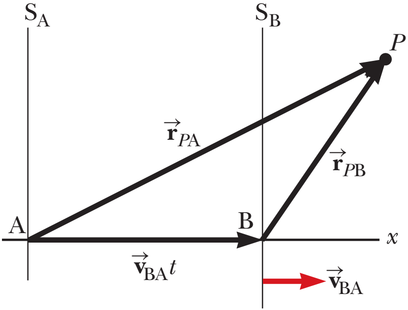
Two frames S and S'



Galilean transformations:

$$x = x' + vt', \quad y = y', \quad z = z', \quad t = t'$$

Relative Motion

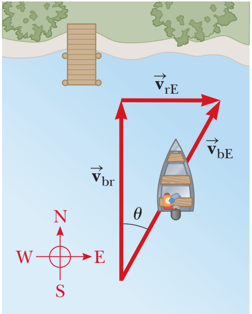


$$\mathbf{r}_{PA} = \mathbf{r}_{PB} + \mathbf{v}_{BA}t$$

$$\mathbf{v}_{PA} = \mathbf{v}_{PB} + \mathbf{v}_{BA}$$

Relative Motion Example

A boat crossing a wide river moves with a speed of 10.0 km/h relative to the water. The water in the river has a uniform speed of 5.00 km/h due east relative to the Earth. If the boat heads due north, determine the velocity of the boat relative to an observer standing on either bank.[1]



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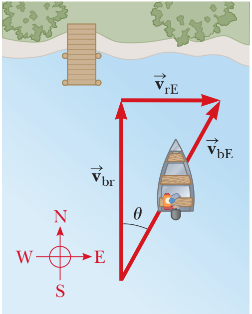

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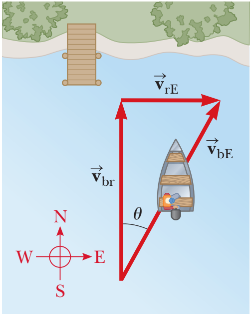

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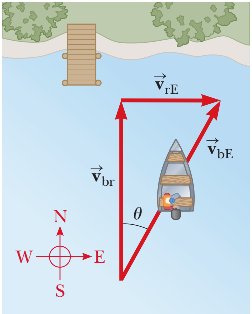

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Simply use vector addition to find v_{bE}.

$$\begin{aligned} v_{bE} &= \sqrt{10^2 + 5^2} \\ &= 11.2 \text{ km/h} \end{aligned}$$

$$\theta = \tan^{-1}\left(\frac{5}{10}\right) = 26.6^\circ$$

[2]Page 97, Serway & Jewett

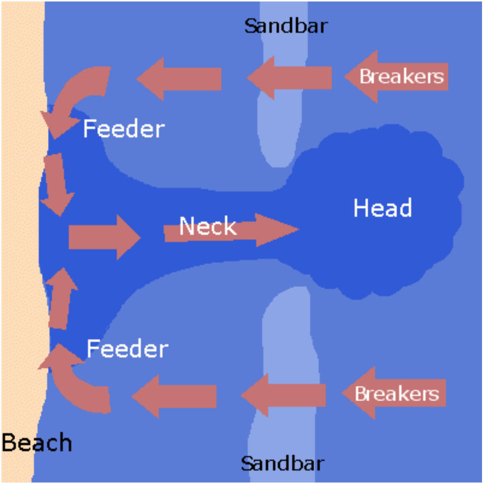
Relative Motion and Rip Currents

Life and death application: rip currents.

In shallow ocean water, a rip current is a strong flow of water away from the shore.

Relative Motion and Rip Currents

If you are caught in one, which way should you swim?



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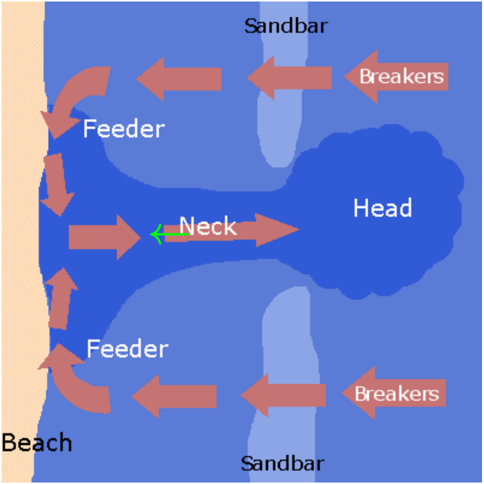

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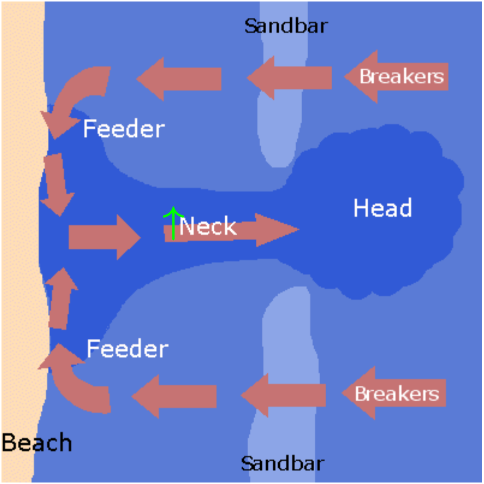

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Example, Ch 4, #72

A rugby player runs with the ball directly toward his opponent's goal, along the positive direction of an x axis. He can legally pass the ball to a teammate as long as the ball's velocity relative to the field does not have a positive x component.

Suppose the player runs at speed 4.00 m/s relative to the field while he passes the ball with velocity $\mathbf{v}_{BP}$ relative to himself. If $\mathbf{v}_{BP}$ has magnitude 6.00 m/s, what is the smallest angle it can have for the pass to be legal?

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$$\mathbf{v}_{PA} = \mathbf{v}_{PB} + \mathbf{v}_{BA}$$

$\mathbf{v}_{PA}$ must have a zero or negative x-component. For the smallest angle, $v_{PA,x} = 0$.

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$$\begin{aligned} v_{PA,x} &= v_{PB,x} + v_{BA,x} \\ v_{PB,x} &= v_{PA,x} - v_{BA,x} \\ v_{PB,x} &= -4 \text{ m/s} \end{aligned}$$

$$\mathbf{v}_{PB} = v_{PB,x}\mathbf{i} + v_{PB,y}\mathbf{j}$$

$$\theta = \cos^{-1}\left(\frac{-4}{6}\right) = \underline{132^\circ} \text{ (counterclockwise from } x \text{ azxis)}$$

Projectiles

projectile

Any object that is thrown. We will use this word specifically to refer to thrown objects that experience a vertical acceleration g.

Assumption

Air resistance is negligible.

Why do we care?

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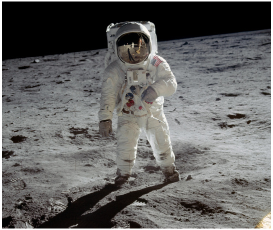

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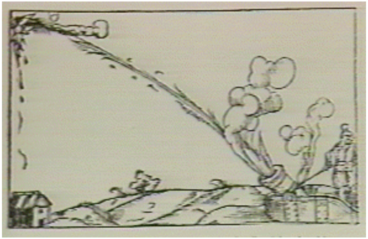
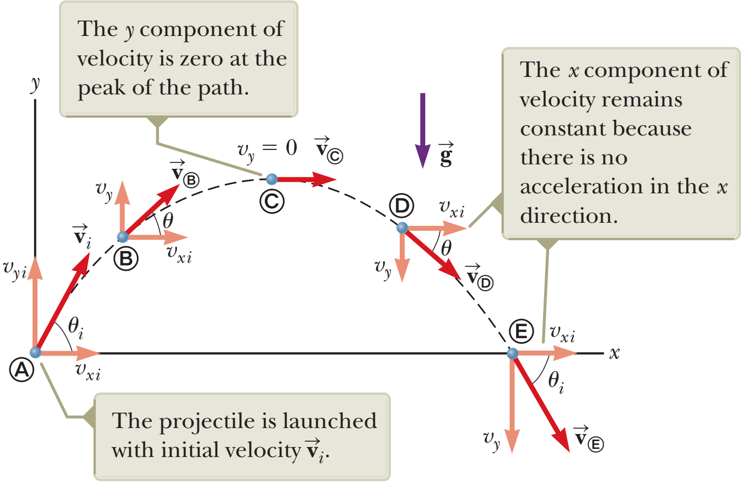


Projectile Velocity



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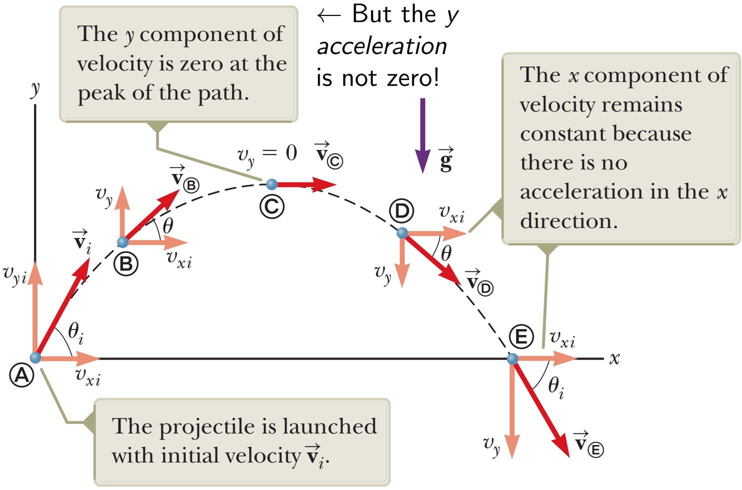
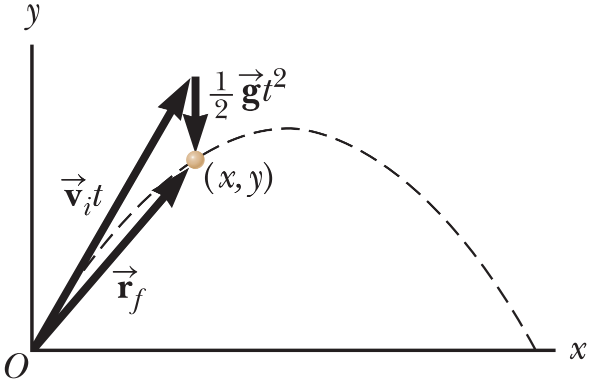


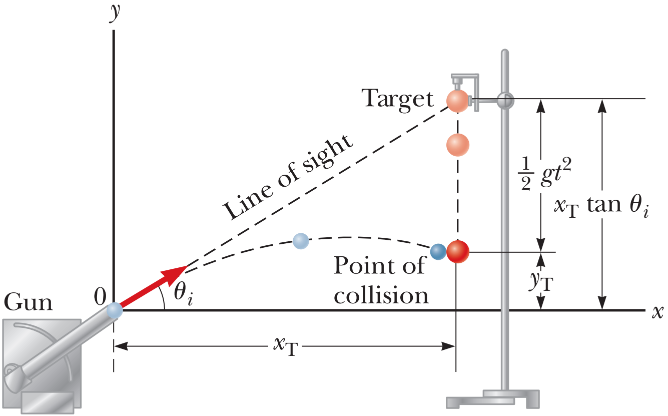
Vector Addition can give a Projectile's Trajectory



$$\Delta \mathbf{r} = \mathbf{r}_f - 0 = \mathbf{v}_i t + \frac{1}{2}\mathbf{a}t^2$$

Motion in 2 Dimensions

A method of testing that the vectors add as asserted!



$$\mathbf{r}_f = \mathbf{v}_i t - \frac{1}{2}gt^2\mathbf{j}$$

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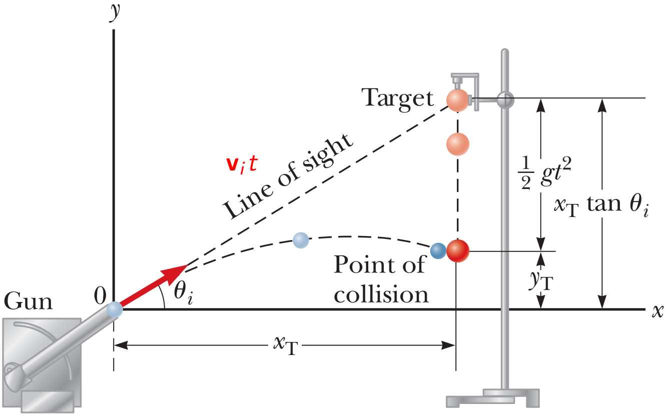

$$\mathbf{r}_f = \mathbf{v}_i t - \frac{1}{2}gt^2\mathbf{j}$$

Summary

- motion in 2 dimensions
- relative motion
- introducing projectiles

Homework

- prev: Ch 4 Problem 76, 83 (relative motion).
- new: Ch 4 Problem 73, 75 (relative motion).
- new: Ch 4 (projectiles)