Electricity and Magnetism
Current and Resistance

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Oct 22, 2015

Last time

- energy stored in a capacitor
- uses of capacitors

Overview

- current
- current density
- drift speed
- resistance
- resistivity
- conductance
- Ohm's Law
- power

Motion of Charge

Up until now, we have mostly considered charges in fixed positions.

We will now look at moving charges, particularly in circuits.

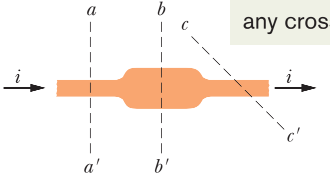
Electric Current

Electric current, I, is the rate of flow of charge through some defined plane:

$$I = \frac{\Delta q}{\Delta t}$$

Δq is an amount of charge and Δt is a time interval.

The defined plane might be aa'. However, since charge is conserved if an amount of charge Δq flows through aa', then the same amount of charge Δq must flow through bb' and cc' in the same time interval.

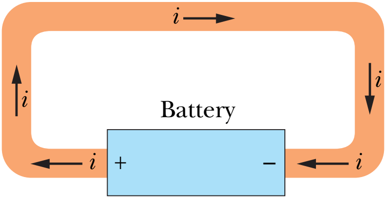


The current is the same in any cross section.

Current

Charge will only move when there is a net force on it. A supplying a potential difference across two points on a wire will do this.

(a)



(b)

Electric Current

The units of current are Amps, A. Formally, amperes.

$1 \text{ A} = 1 \text{ C/s}$

Current is a scalar, however, a negative sign can be used to indicate a current flowing backwards through a loop.

Conventional Current

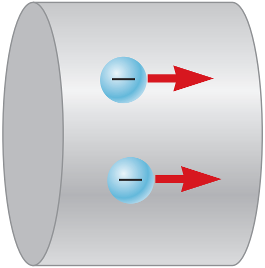
By convention, current is labeled indicating the direction in which **positive charge carriers would move**.

Of course, in very many circumstances, and particularly in conducting metals, electrons, which are negative charge carries, are the moving charges.

This means that a current arrow is drawn opposite to the direction of motion of electrons.

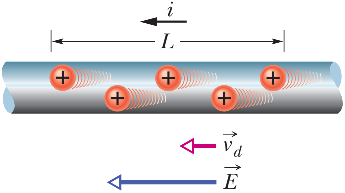
Conventional Current

A conducting wire:



— ← current, i ← +

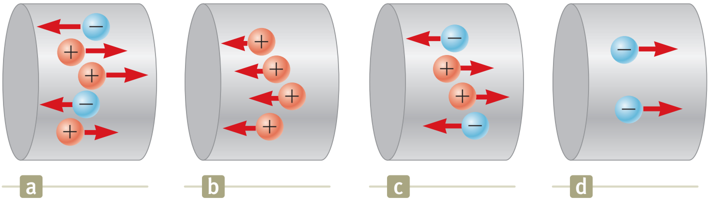
We imagine posititive charges moving:



— +

Current Question

QuickQuiz 27.1: Consider positive and negative charges moving horizontally through the four regions. Rank the current in these four regions from highest to lowest.



(A) b, a, c, d

(B) a, (b and c), d

(C) d, (b and c), a

(D) b, d, a, c

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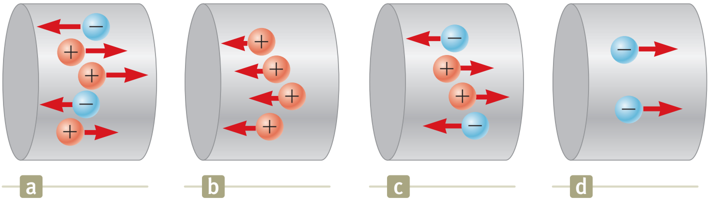

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Current and Junctions

Since charge is conserved, all charge that flows into a point, must flow out of it as well.

We can apply this to a junction: a point at which wires join or split.

This gives Kirchhoff's junction rule:

Junction Rule

The sum of the currents entering any junction must be equal to the sum of the currents leaving that junction.

Current and Junctions

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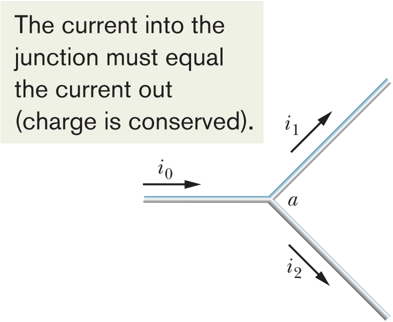
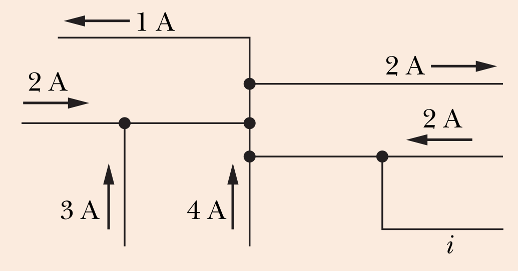


In the diagram, $i_0 = i_1 + i_2$

Question

What are the magnitude and direction of the current i in the lower right-hand wire?



[1]Halliday, Resnick, Walker, 9th ed, page 684.

Current Density

Current Density, J

The current per unit area through a conductor.

$$J = \frac{I}{A}$$

Strictly, this is the **average current density** through the area A, assuming the area A is perpendicular to the direction of the current.

This view of current density will be sufficient for most purposes in this course.

Current Density

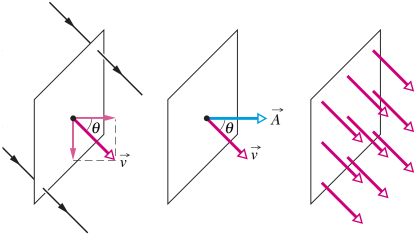
In more detail, current is very similar to flux:

$$I = \sum J\,(\Delta A)\cos\theta$$

Whereas flux:

$$\Phi = \sum E\,(\Delta A)\cos\theta$$

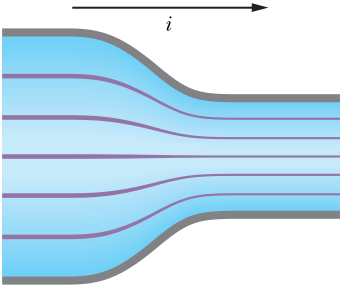
Current density J can be compared with the electric field, E.



Current Density

Current density can be represented with streamlines that are denser where the current density is higher.

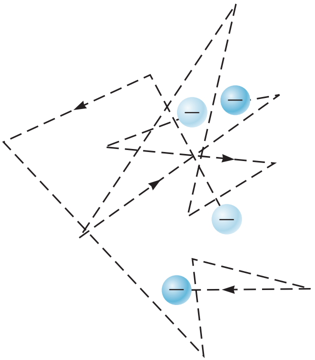
(*cf.* electric field and electric field lines)



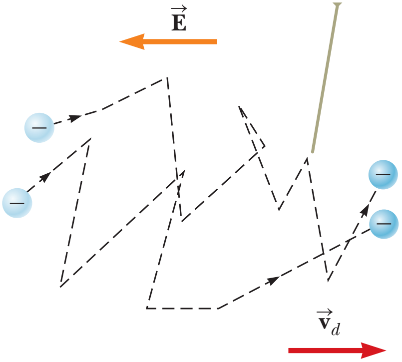
Microscopic Model of Current

Conduction electrons can be though of as moving in a random way, colliding with atoms.

Electrons with $E = 0$:



with an E-field:



With an external field, they tend to drift in the opposite direction to the field lines.

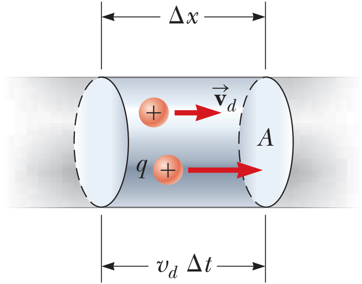
Drift Speed

The drift speed v_d of charge carriers in a conductor is the average speed at which a charge carrier is expected to move through a conductor.

The average speed of a charge carrier through a circuit, by definition is:

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t}$$

How far (Δx) do we expect a charge carrier to move in time Δt?



Drift Speed

Need an expression for Δx in terms of current.

Suppose there are n free conduction electrons per unit volume.

Then $n\,A\,\Delta x$ electrons move through a cross section A in time Δt.
(Vol $= A\,\Delta x$)

$$I = \frac{Q}{\Delta t} = \frac{(nA\,\Delta x)e}{\Delta t}$$

Drift Speed

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Then we can rearrange for Δx:

$$\Delta x = \frac{I\,\Delta t}{nAe}$$

Drift Speed

Putting this back into the expression for v_d:

$$v_d = \frac{\Delta x}{\Delta t} = \frac{I\,\Delta t}{nAe\,\Delta t}$$

Simplifying,

$$v_d = \frac{I}{nAe} = \frac{J}{ne}$$

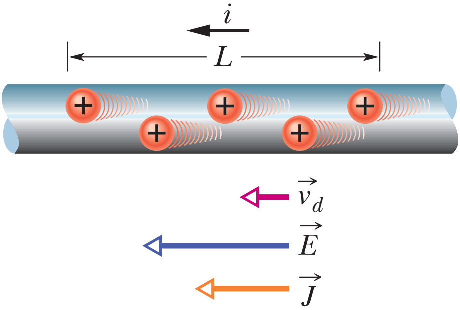
$(J = I/A)$

Drift velocity

We can also express this as a vector relation:

$$\mathbf{J} = n\, q\, \mathbf{v}_d$$

where q is the charge of the charge carrier.



Drift Speed of an Electron in Copper[1]

What is the drift speed of the conduction electrons in a copper wire with radius $r = 900$ μm when it has a uniform current $I = 17$ mA?

Assume that each copper atom contributes one conduction electron to the current and that the current density is uniform across the wire's cross section.

[1] From page 688 in Halliday, Resnick, and Walker, 9th ed.

Drift Speed of an Electron in Copper

How many electrons per unit volume? Same as number of copper atoms:

$$n = \frac{N_A \rho}{M} = \frac{(6.02 \times 10^{23}\ \text{mol}^{-1})(8.96 \times 10^{3}\ \text{kg/m}^3)}{63.54 \times 10^{-3}\ \text{kg/mol}}$$

N_A is Avagadro's number, M is the molar mass (kgs per mole of copper), and ρ is copper's density.

$$n = 8.49 \times 10^{28}\ \text{m}^{-3}$$

This is the number of free conduction electrons in a cubic meter of copper. (A lot.)

Drift Speed of an Electron in Copper

$$
\begin{aligned}
v_d &= \frac{I}{nAe} \\
&= \frac{(17 \times 10^{-3}\ \text{A})}{(8.49 \times 10^{28}\ \text{m}^{-3})(\pi r^2)(1.6 \times 10^{-19}\ \text{C})}
\end{aligned}
$$

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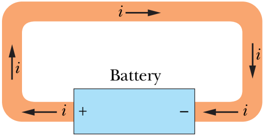
&= \frac{(17 \times 10^{-3}\ \text{A})}{(8.49 \times 10^{28}\ \text{m}^{-3})(\pi r^2)(1.6 \times 10^{-19}\ \text{C})} \\
v_d &= 4.9 \times 10^{-7}\ \text{m/s}
\end{aligned}
$$

Very slow!

Resistance

When a potential difference is applied across a conductor, current begins to flow.

(a)



(b)

However, different amounts of current will flow in different conductors, even when the applied potential difference is the same. What is the characteristic of the conductor which determines the amount of current that will flow?

[1]Figure from Halliday, Resnick, Walker, 9th ed.

Resistance

Resistance

The resistance of a conductor is given by the ratio of the applied potential to the current that flows through the conductor at that potential:

$$R = \frac{V}{I}$$

The units of resistance are Ohms, Ω, symbol is the capital Greek letter "Omega". $1\ \Omega\ = 1$ V/A

We can think of a high resistance as *resisting*, or impeding, the flow of current.

Resistivity

An individual conductor or circuit component has a resistance.

The resistance is based on

- the material it is made of,
- its geometry, and
- the temperature

The material that a component is made from affects the resistance, because different materials have different *resistivities*.

Resistivity

resistivity, ρ

the ratio of the electric field strength in a material to the current density this field causes in the material:

$$\rho = \frac{E}{J}$$

Resistivity is a property of a material. Its symbol is the Greek letter ρ, pronounced "rho".

The units of resistivity are Ω m.

$$1\ \Omega\ \text{m} = 1\ \frac{\text{V}}{\text{A}}\ \text{m} = 1\ \frac{\text{V/m}}{\text{A/m}^2}$$

which agrees with the definition of $\rho = E/J$.

Resistivity

Resistivities of Some Materials at Room Temperature (20°C)

Material	Resistivity, ρ ($\Omega \cdot m$)	Temperature Coefficient of Resistivity, α (K^{-1})
	Typical Metals	
Silver	1.62×10^{-8}	4.1×10^{-3}
Copper	1.69×10^{-8}	4.3×10^{-3}
Gold	2.35×10^{-8}	4.0×10^{-3}
Aluminum	2.75×10^{-8}	4.4×10^{-3}
Manganin[a]	4.82×10^{-8}	0.002×10^{-3}
Tungsten	5.25×10^{-8}	4.5×10^{-3}
Iron	9.68×10^{-8}	6.5×10^{-3}
Platinum	10.6×10^{-8}	3.9×10^{-3}
	Typical Semiconductors	
Silicon, pure	2.5×10^{3}	-70×10^{-3}
Silicon, *n*-type[b]	8.7×10^{-4}	
Silicon, *p*-type[c]	2.8×10^{-3}	
	Typical Insulators	
Glass	$10^{10} - 10^{14}$	
Fused quartz	$\sim 10^{16}$	

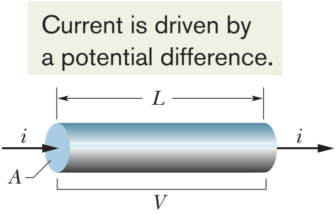
Resistivity

Together with the geometry of the component made of that material, we can predict the resistance of the component.

For a wire or cylinder made of material with resistivity ρ:

$$R = \frac{\rho L}{A}$$

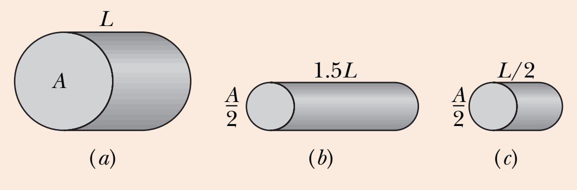
where A is the cross-sectional area of the wire, and L is the length of the wire.



(This follows from the definition of ρ.)

Question

Rank the three cylindrical copper conductors according to the current through them, greatest first, when the same potential difference V is placed across their lengths.



(A) a, b, c

(B) c, b, a

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(D) (a and c), b

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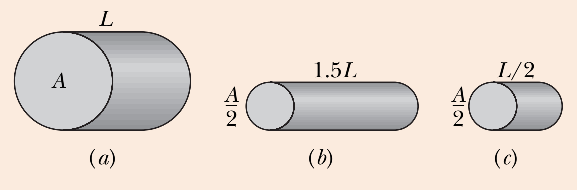

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Conductivity

Sometimes it is useful to represent how conductive a material is: how readily it permits current to flow, as opposed to how much it resists the flow of current.

conductivity, σ

a measure of what the current density is in a material for a particular electric field; the inverse of resistivity:

$$\sigma = \frac{1}{\rho} = \frac{J}{E}$$

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This is different than surface charge density (also written σ). This is just an unfortunate coincidence of notation.

Conductivity

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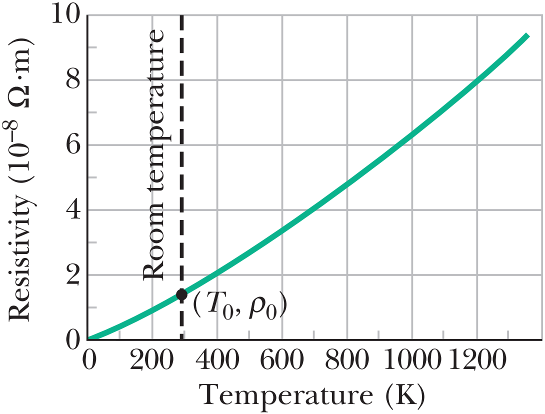
$$\sigma = \frac{1}{\rho} = \frac{J}{E}$$

The units of conductivity are $(\Omega \text{ m})^{-1}$.

We can use conductivity to relate the current density to the electric field in a material:

$$J = \sigma E$$

Resistivity can depend on Temperature



Resistivity can depend on Temperature

The relationship between resistivity and temperature is close to linear.

For most engineering purposes, a linear model is good enough.

The model:

$$\rho - \rho_0 = \rho_0 \alpha (T - T_0)$$

The resistivity varies linearly with the difference in temperature from some reference value T_0.

ρ_0 is the resistivity at T_0.

Resistivity can depend on Temperature

$$\rho - \rho_0 = \rho_0 \alpha (T - T_0)$$

α is just a constant, however it takes different values for different materials.

α is called the **temperature coefficient of resistivity**. It has units K^{-1}.

For example for copper:

$\rho_0 = 1.62 \times 10^{-8}$ Ω m

$\alpha = 4.3 \times 10^{-3}$ K^{-1}

Ohm's Law

Ohm's Law

The current through a device is directly proportional to the potential difference applied across the device.

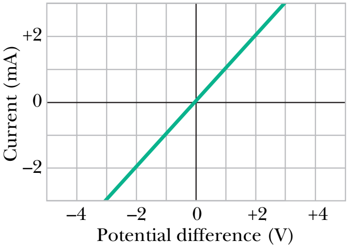
Not all devices obey Ohm's Law!

In fact, for all materials, if ΔV is large enough, Ohm's law fails.

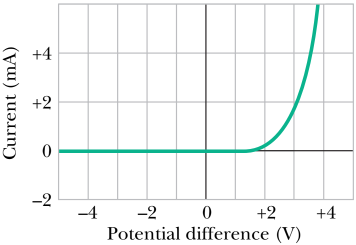
They only obey Ohm's law when the resistance of the device is independent of the applied potential difference and its polarity (that is, which side is the higher potential).

Ohm's Law

Obeys Ohm's law:



Does not obey Ohm's law:



We can write this linear relationship as $\Delta V = IR$ if and only if R is constant and independent of ΔV.

However, notice that we can always define $R(\Delta V) = \frac{\Delta V}{I}$ even when resistance does depend on ΔV.

Ohm's Law Question

The following table gives the current i (in amperes) through two devices for several values of potential difference V (in volts). Which of the devices obeys Ohm's law?

Device 1		Device 2	
V	i	V	i
2.00	4.50	2.00	1.50
3.00	6.75	3.00	2.20
4.00	9.00	4.00	2.80

(A) 1 only

(B) 2 only

(C) both

(D) neither

[1] Halliday, Resnick, Walker, page 692.

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A reason for Ohm's Law?

Electrons in an electric field accelerate.

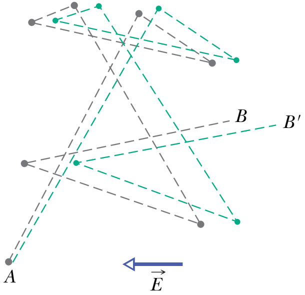
We supply a constant potential difference across a resistor. Why do the electrons not move faster and faster?

A reason for Ohm's Law?

Electrons in an electric field accelerate.

We supply a constant potential difference across a resistor. Why do the electrons not move faster and faster?

The mechanism for resistance that the electrons collide with atoms in the resistive material.



The collisions slow the drift of the electrons.

A reason for Ohm's Law?

We could then model resistance as being inversely proportional to the average time between collisions τ:

$$\rho \sim \frac{1}{\tau}$$

So, does the time between collisions depend on the potential difference across the conductor?

It would, if the electric field cause a large change in the average electron's velocity. We would expect faster moving electrons to collide more frequently (τ would decrease).

(The book has more details on this: it give the Drude model.)

A reason for Ohm's Law?

$$\rho \sim \frac{1}{\tau}$$

However, the average velocity of an electron in room temperature copper is $v \sim 1.6 \times 10^6$ m/s.

The drift velocity is perhaps $v_d \sim 10^{-7}$ m/s : $\frac{v_d}{v} \approx 10^{-13}$!

This means that varying the potential difference will have a negligible affect on τ and therefore also on the resisitivity ρ.

$\Rightarrow$ R is independent of ΔV in many cases.

Power

Power is the rate of energy transfer or the rate at which work is done:

$$P = \frac{W}{\Delta t}$$

For an electrical circuit we can ask about the rate at which a battery or other power supply transfers energy to a device.

This depends on the current and the potential difference:

$$P = I\,(\Delta V)$$

Power

$$P = I\,\Delta V$$

The units for power are Watts, W.

$1\ \text{W} = 1\ \text{J/s}$.

Does this agree with the new equation?

Power

$$P = I\,\Delta V$$

The units for power are Watts, W.

1 W = 1 J/s.

Does this agree with the new equation?

1 A V = (1 C/s) (1 J/C) = 1 J/s . Yes.

Power Dissipated

$$P = I\,\Delta V$$

We can use this expression along with $R = \frac{\Delta V}{I}$ to find the power dissipated as heat in a resistor.

Power dissipated as heat in a resistor:

$$P = I^2 R$$

or equivalently,

$$P = \frac{(\Delta V)^2}{R}$$

where I in the first equation is the current **through the resistor** and ΔV in the second equation is the potential difference **across the resistor**.

Summary

- current
- current density
- drift speed
- resistance
- Ohm's Law
- power

Midterm on Nov 3rd.
Homework Halliday, Resnick, Walker:

- Ch 26, onward from page 699. Questions: 3, 5; Problems: 1, 5, 13, 17, 19, 21, 27