Electricity and Magnetism
DC Circuits
EMF and Internal Resistance
Kirchhoff's Laws

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Last time

- current
- current density
- drift speed
- resistance
- resistivity
- conductance
- Ohm's Law
- power

Overview

- emf
- internal resistance of batteries
- potential drops
- Kirchhoff loop rule
- resistors in series and parallel

Power

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The units for power are Watts, W.

$1\ \mathrm{W} = 1\ \mathrm{J/s}$.

Does this agree with the new equation?

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1 A V = (1 C/s) (1 J/C) = 1 J/s . Yes.

Power Dissipated

$$P = I\,\Delta V$$

We can use this expression along with $R = \frac{\Delta V}{I}$ to find the power dissipated as heat in a resistor.

Power dissipated as heat in a resistor:

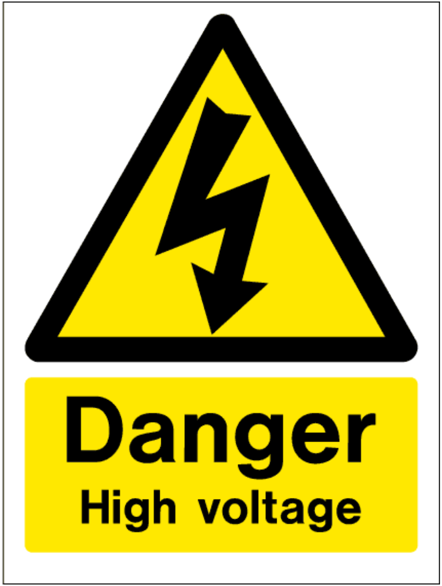
$$P = I^2 R$$

or equivalently,

$$P = \frac{(\Delta V)^2}{R}$$

where I in the first equation is the current **in the resistor** and ΔV in the second equation is the potential difference **across the resistor**.

Example: Why High Voltage?



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$$R = \frac{\rho L}{A} = 13.4\ \Omega$$

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$$P = I^2 R = (500 \text{ A})^2 (13.4 \ \Omega) = 3.36 \text{ MW}$$

Much more loss!

Example

This is why power stations transmit power at a very high voltage.

The voltage is "stepped down" before being delivered to your house.

Mains electricity in the US is distributed throughout a house at 120 V. (The "line voltage".)

Circuits (Ch 27)

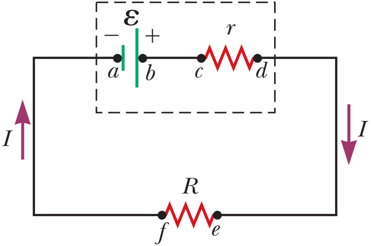
Circuits consist of a collection of electrical components connected by conducting wires through which charge is driven by an energy source.

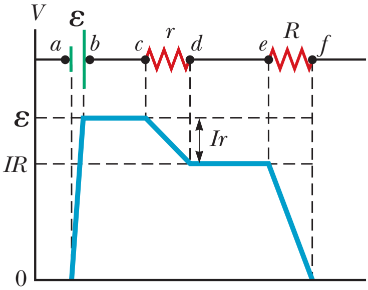
Right now we focus on **direct-current (DC)** circuits.

In a direct-current circuit current flows in one direction only.

This is the only type of situation we have been considering so far. However, in the coming labs you may look at some situations with **alternating-current (AC)**, in which the current flows forward, then backward, through the circuit.

Potential in a Circuit





The potential drops across each resistor in the circuit as each transforms electrical power to heat.

Equivalently, the potential energy of a charge q decreases as it moves through a resistor.

Batteries increase the potential energy of a charge / raise the potential.

A closer look at batteries and power supplies

Batteries and power supplies fill a critical role in circuits.

They supply the energy to drive the charges around the circuit.

They do this by creating a charge imbalance and causing each charge to experience a force.

Electomotive Force

We say that a battery or power supply contributes an electromotive force (emf) and we can call batteries and power supplies emf devices.

These devices act as “charge pumps” in a circuit.

emf device

A device that maintains a potential difference between two points (terminals) in the circuit.

Electomotive “Force” (emf)

There is a force on each free charge in the system because there is an electric field.

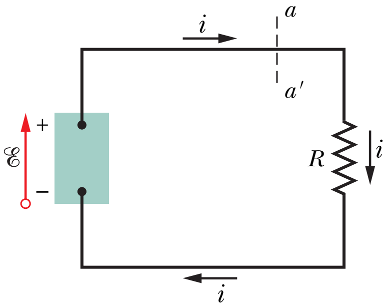
$$\mathbf{F} = q\mathbf{E}$$

The electric field exists because of the potential difference supplied to the circuit by the battery.

But this is not what we mean by emf! The emf is not actually a force.

Electomotive "Force"

We write an emf as $\mathcal{E}$, and label the battery with it:



Emf is actually a energy supplied per unit charge!
(Measured in volts.)

This makes calling it a "force" a bit misleading.

Electomotive “Force”

To be clear: the electromotive “force” is not a force.

It is an energy supplied per unit charge! It has the units volts.

The name is an unfortunate choice that stuck.

EMF

We can define emf by the following relation:

$$\mathcal{E} = \frac{\Delta W}{\Delta q}$$

meaning, an emf device does a work ΔW on anamount of charge Δq:

$$\Delta W = \mathcal{E}\,\Delta q$$

while moving the infinitesimal charge dq from the negative terminal to the positive terminal. (Imagining dq to be positive.)

The amount of work that is done "lifting" this charge to the higher potential terminal depends only on the potential difference, so $\mathcal{E}$ is a potential difference measured in volts.

Power Supplied

This definition for emf gives the power supplied by an emf device.

$$\mathcal{E} = \frac{\Delta W}{\Delta q}$$

Power is the rate at which the work is done:

$$P = \frac{\Delta W}{\Delta t} = \mathcal{E}\,\frac{\Delta q}{\Delta t}$$

(assuming that the emf supplied by a source is constant.)
Then notice that $I = \frac{\Delta q}{\Delta t}$, so

$$P = I\mathcal{E}$$

This is the total power supplied by an emf device!

Compare to $P = I\,(\Delta V)$ as the power delivered to any component.

EMF

Why do we suddenly need to call potential difference ΔV of a battery emf $\mathcal{E}$?

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The emf gives the **maximum potential** a battery can supply.

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The emf gives the **maximum potential** a battery can supply.

There is one other important reason, however: we can now start to encounter circumstances where we cannot define electric potential - this will only be important when we come onto magnetic fields.

Internal resistance

How does internal resistance affect the supplied potential difference?

Internal resistance

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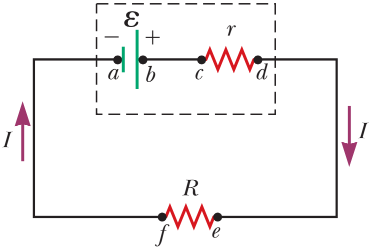
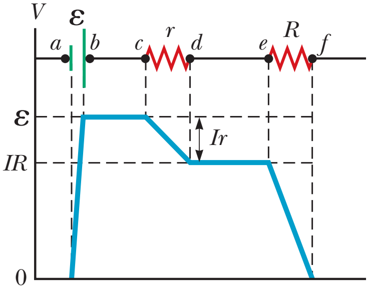
It is another resistance that is in series!



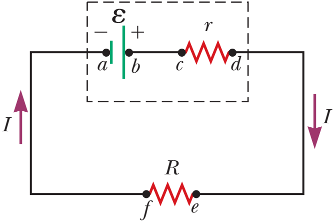


Let r be the internal resistance

$$V_r = I r$$

V_r is the potential drop across the internal resistance.

Internal resistance

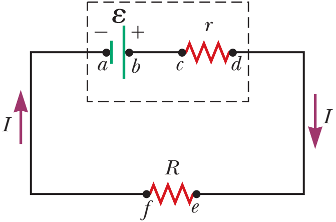


Let V be the potential difference supplied by the battery to the rest of the circuit:

$$V = \mathcal{E} - Ir$$

V is the potential difference between the terminals of the battery at points a and d in the diagram.

Internal resistance



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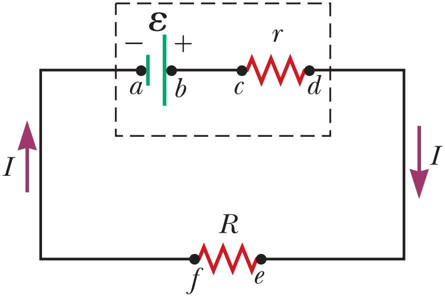
V depends on the current that flows in the circuit!

Internal resistance



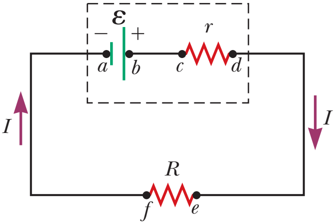
Ideal battery

An ideal battery has no internal resistance. ($r = 0$)

Real batteries do have internal resistance.

Internal resistance and current

The current that flows in the circuit, I, will in turn depend on the **load resistance** R, *ie.* the resistance in the rest of the circuit.



$\Delta V = IR$
and so, $IR = \mathcal{E} - Ir$ and:

$$I = \frac{\mathcal{E}}{r + R}$$

Internal resistance, potential difference, and power

$$I = \frac{\mathcal{E}}{r + R}$$

The potential difference supplied to the circuit ΔV:

$$\Delta V = IR = \frac{\mathcal{E}R}{r + R}$$

It depends on both the internal and external ("load") resistances.

Power:

$$\begin{aligned} \text{power supplied} &= \text{total power delivered} \\ I\mathcal{E} &= I^2 r + I^2 R \end{aligned}$$

Question

Quick Quiz 28.1: To maximize the **percentage** of the power from the emf of a battery that is delivered to a device external to the battery, what should the internal resistance of the battery be?

(A) It should be as low as possible.
(B) It should be as high as possible.
(C) The percentage does not depend on the internal resistance.

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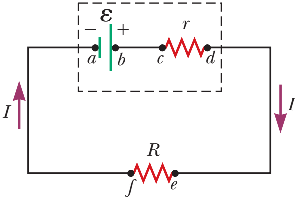
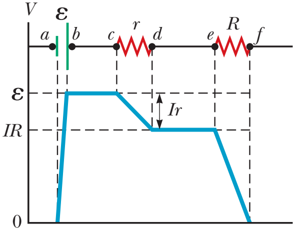
Internal resistance





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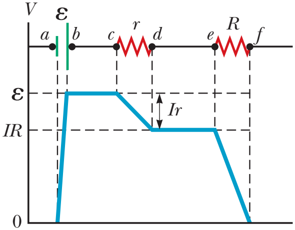
ΔV is the potential difference between the terminals of the battery at points a and d in the diagram.

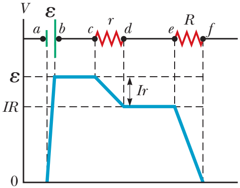
Potential difference between two points



For any circuit we can find the potential difference between points in the circuit by finding the potential drop or jump across the elements between those points.

Two rules can help us track this.

Potential difference between two points



"Voltage Drops":

resistance rule

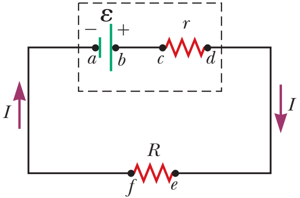
Going through a resistance R in the direction of the current, the change in potential is $-iR$; in the opposite direction it is $+iR$.

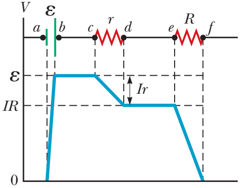
"Voltage jumps":

emf rule

Going through an ideal emf device in the direction of the emf arrow, the change in potential is $+\mathcal{E}$; in the opposite direction it is $-\mathcal{E}$.

The loop rule

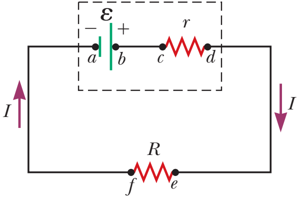


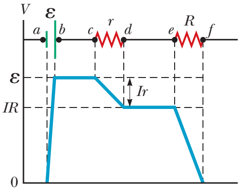


Notice in the lower diagram that we we come back at the right end to the same potential that we started at on the left end.

In fact, it doesn't matter what point we start at: if we go around a closed loop, when we return to the starting point, we must return to the starting potential also.

The loop rule





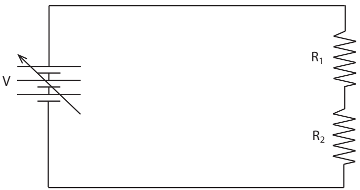
Kirchhoff's loop rule:

Loop Rule

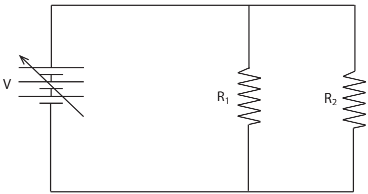
The sum of the changes in potential encountered in a complete traversal of any loop of a circuit must be zero.

Multiloop Circuits

Single loop



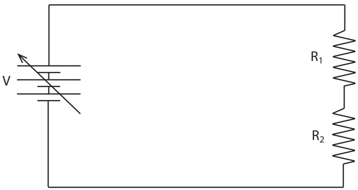
Multiloop



Series and Parallel

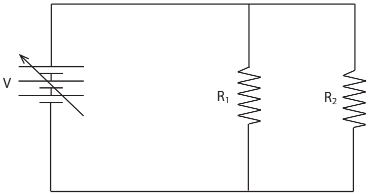
Series

When components are connected one after the other along a single path, they are connected in series.



Parallel

When components are connected side-by-side on different paths, they are connected in parallel.



Resistors in Series

The current though resistors in series in a loop is the same.

Let the total potential difference across two resistors be ΔV, then

$$\Delta V = IR_1 + IR_2 = I(R_1 + R_2)$$

Then the effective equivalent resistance of both together is just the sum

$$R_{\text{eq}} = R_1 + R_2$$

For n resistors in **series**:

$$R_{\text{eq}} = R_1 + R_2 + \ldots + R_n = \sum_{i=1}^{n} R_i$$

Resistors in Parallel

The potential difference across two resistors in parallel is the same. (Loop rule.)

Let i be the total current that flows through both resistors: $I = I_1 + I_2$. (Junction rule.)

$$I = \frac{\Delta V}{R_{\text{eq}}} = \frac{\Delta V}{R_1} + \frac{\Delta V}{R_2}$$

Dividing the equation by V:

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2}$$

For n of resistors in **parallel**:

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n} = \sum_{i=1}^{n} \frac{1}{R_i}$$

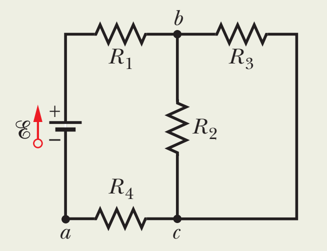
Resistors vs. Capacitors

Table of equivalent capacitances and resistances for series and parallel.

	resistors	capacitors
series	$R_{\text{eq}} = \sum R_i$	$\frac{1}{C_{\text{eq}}} = \sum \frac{1}{C_i}$
parallel	$\frac{1}{R_{\text{eq}}} = \sum \frac{1}{R_i}$	$C_{\text{eq}} = \sum C_i$

Example

Consider the circuit pictured with $\mathcal{E} = 12$ V, and the following resistor values: $R_1 = 20\ \Omega$, $R_2 = 20\ \Omega$, $R_3 = 30\ \Omega$, and $R_4 = 8.0\ \Omega$.



What is the current through the battery?
answer: $I = 0.30$ A

What is the current through resistor R_2?
answer: $I_2 = 0.18$ A

Kirchhoff's Laws

The loop rule for potential difference and the junction rule for current together are called Kirchhoff's laws.

Loop Rule

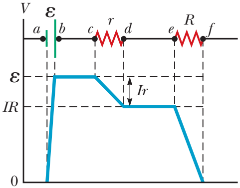
The sum of the changes in potential encountered in a complete traversal of any loop of a circuit must be zero.

Junction Rule

The sum of the currents entering any junction must be equal to the sum of the currents leaving that junction.

Using both it is possible to discover many things about how a circuit operates, for example how much power will be dissipated in a particular component.

Potential difference between two points



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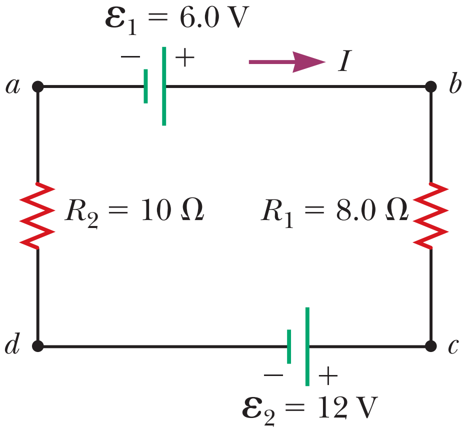
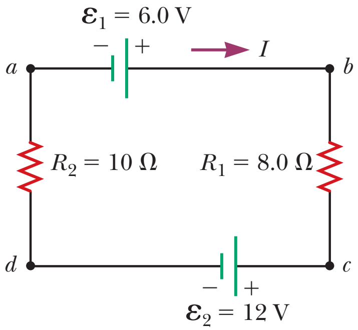
Example with Two Batteries

Find the current in the circuit.



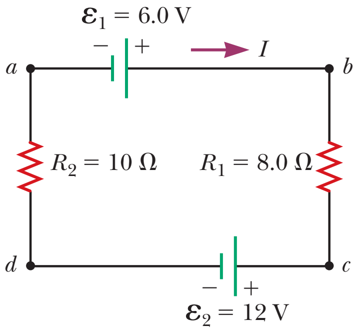
Suppose the current flows in the direction shown.

Example with Two Batteries



$$\sum \Delta V = \mathcal{E}_1 - IR_1 - \mathcal{E}_2 - IR_2 = 0$$

Example with Two Batteries



$$\sum \Delta V = \mathcal{E}_1 - IR_1 - \mathcal{E}_2 - IR_2 = 0$$

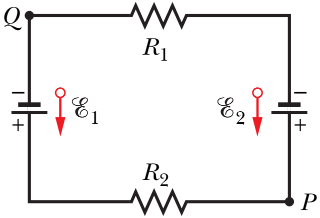
$$\Rightarrow \quad I = \frac{\mathcal{E}_1 - \mathcal{E}_2}{R_1 + R_2} = -0.33\text{ A}$$

Minus sign means that the current flows opposite to the direction shown in the diagram.

Using Kirchhoff's Laws examples

Page 726, #2

•**2** In Fig. 27-26, the ideal batteries have emfs $\mathscr{E}_1 = 150$ V and $\mathscr{E}_2 = 50$ V and the resistances are $R_1 = 3.0\ \Omega$ and $R_2 = 2.0\ \Omega$. If the potential at P is 100 V, what is it at Q?



Using Kirchhoff's Laws examples

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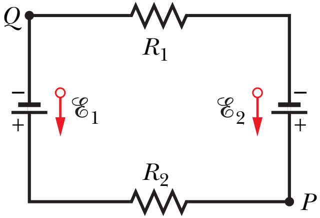
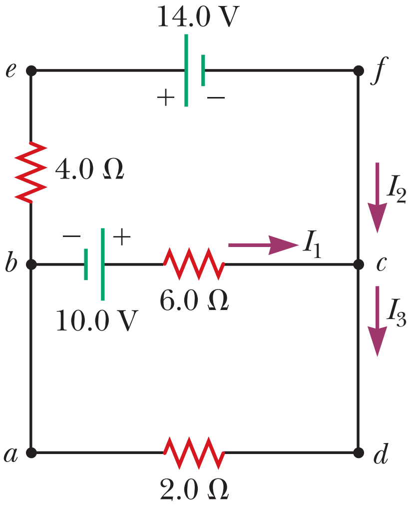


Loop rule: $-\mathcal{E}_2 - IR_2 + \mathcal{E}_1 - IR_1 = 0$, $I = 20$ A.

Potential at $Q = -10$ V.

Example with a Multiloop Circuit

Find the currents I_1, I_2, and I_3 in the circuit.



Suppose the currents flow in the direction shown.

Example with a Multiloop Circuit

Junction rule:

$$I_1 + I_2 = I_3 \tag{1}$$

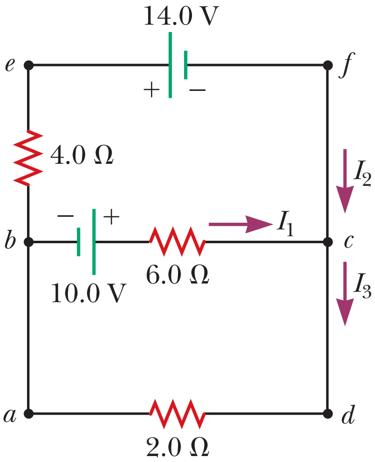
Loops:

$$10\text{V} - (6\Omega)I_1 + (2\Omega)I_3 = 0 \tag{2}$$

$$-14\text{V} + (6\Omega)I_1 - 10V - (4\Omega)I_2 = 0 \tag{3}$$

$$-14\text{V} - (2\Omega)I_3 - (4\Omega)I_2 = 0 \tag{4}$$

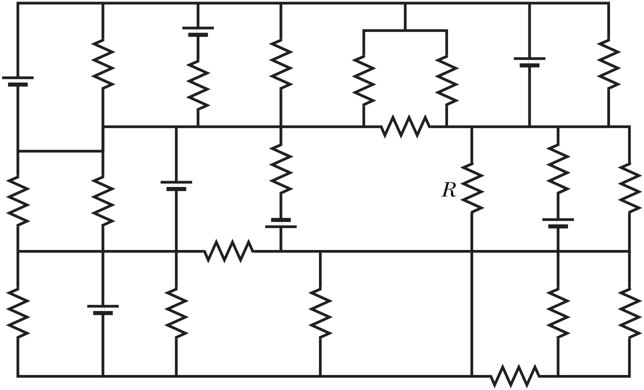
Example with a Multiloop Circuit



$I_1 = +2.0$ A $I_2 = -3.0$ A $I_3 = -1.0$ A

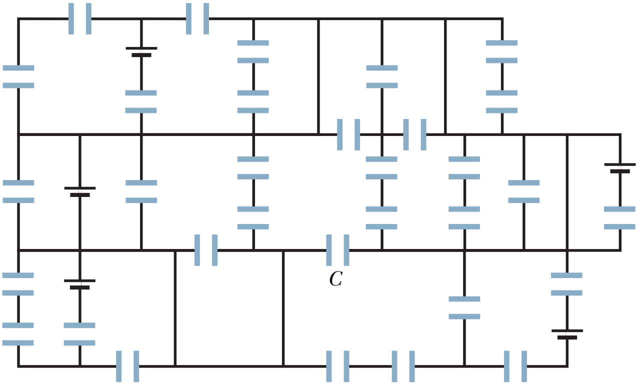
Using Kirchhoff's Laws examples

6 *Res-monster maze.* In Fig. 27-21, all the resistors have a resistance of 4.0 Ω and all the (ideal) batteries have an emf of 4.0 V. What is the current through resistor *R*? (If you can find the proper loop through this maze, you can answer the question with a few seconds of mental calculation.)



Using Kirchhoff's Laws examples

8 *Cap-monster maze.* In Fig. 27-22, all the capacitors have a capacitance of 6.0 μF, and all the batteries have an emf of 10 V. What is the charge on capacitor C? (If you can find the proper loop through this maze, you can answer the question with a few seconds of mental calculation.)



Summary

- Kirchhoff's laws
- resistors in series and parallel

Midterm on Thursday May 14.

Homework Halliday, Resnick, Walker:

- Ch 26, onward from page 699. Problems: 41, 43, 45, 47, 55, 71
- Ch 27, onward from page 725. Questions: 1, 3; Problems: 1, 5, 7, 33