Electricity and Magnetism
Magnetic Fields and Charges

Lana Sheridan

De Anza College

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Last time

- magnets
- magnetic field

Overview

- Earth's magnetic field
- magnetic force on a charge
- motion of a charge in a B-field
- charge in electric and magnetic fields

Compasses and the Earth's Magnetic field

The Earth has a magnetic field.

This is how compasses detect which way North is.

Compasses have a small needle magnet inside, that rotates to align with the Earth's B-field.

The strength of the Earth's field at Earth's surface is ~ 1 Gauss.

Compasses and the Earth's Magnetic field

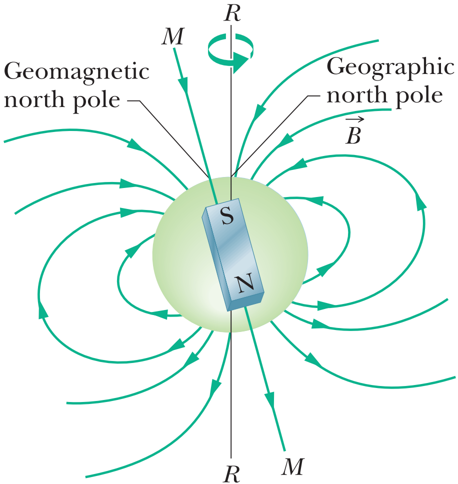

Figure from Halliday, Resnick, Walker, 9th ed, pg 870.[1]

Compasses and the Earth's Magnetic field

North poles of magnets point northward, so the magnetic pole that points (roughly) North is a south pole

The poles of magnets are perhaps more accurately called:

- north-*seeking* pole
- south-*seeking* pole

but almost always they are just called "north" and "south" poles.

Magnetic Fields and Force

If there were magnetic monopoles, deriving a relation between the force on one and the magnetic field it experiences would be easy: just the same as the case for electric fields.

Deriving the force on a magnetic dipole from another dipole is more difficult.

The easiest place to start: the force on a moving electric charge in a magnetic field.

Electric charges can also be affected by magnetic fields.

Force on a Moving Charge

The force on a moving electric charge in a magnetic field:

$$\mathbf{F}_B = q\,\mathbf{v} \times \mathbf{B}$$

where **B is the magnetic field**, $\mathbf{v}$ is the velocity of the charge, and q is the electric charge.

Notice this is similar to the relation between electrostatic force and electric field.

$$\mathbf{F}_E = q\mathbf{E} \quad \longrightarrow \quad \mathbf{F}_B = q\mathbf{v} \times \mathbf{B}$$

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The magnitude of the force is given by:

$$F_B = q\,vB\sin\theta$$

if θ is the angle between the $\mathbf{v}$ and $\mathbf{B}$ vectors.

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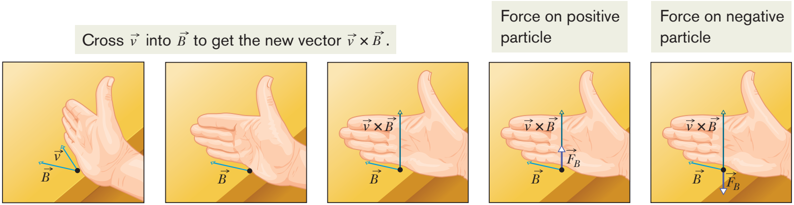
For the direction, right hand rule:



[1]Figure from Halliday, Resnick, Walker, 9th ed.

Force on a Moving Charge

Because this expression involves 3-dimensions, it can be difficult to draw all the vectors involved.

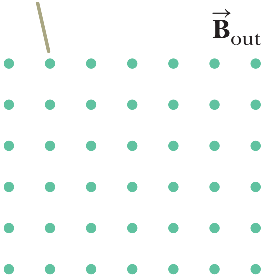
To make diagrams simpler, the magnetic field is often drawn going directly down in to the page or straight up out of it. ($\perp$ to the plane of the diagram).

Dots ($\cdot \quad \cdot \quad \cdot$) or circle-dots ($\odot \quad \odot \quad \odot$) represent field coming up **out** of the page.

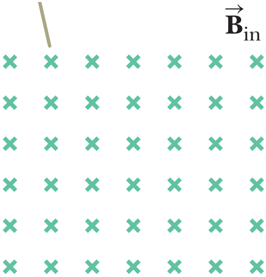
Crosses ($\times \quad \times \quad \times$) or circle-crosses ($\otimes \quad \otimes \quad \otimes$) represent field going down **into** the page.

Magnetic field direction

B points **out of** the page.

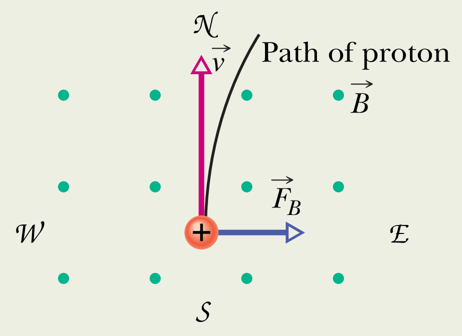


B points **into** the page.



Force on a Moving Charge

For example: here the dots indicate the field is directed upward out of the slide.

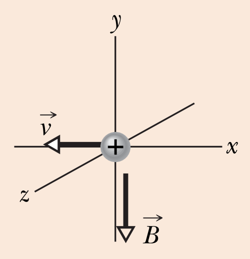


The force on the particle is $\perp$ to its velocity and the field.

Questions: Applying the Force equation

$$\mathbf{F}_B = q\,\mathbf{v} \times \mathbf{B}$$

A **positively** charged particle with velocity $\mathbf{v}$ travels through a uniform magnetic field $\mathbf{B}$. What is the direction of the magnetic force $\mathbf{F}_B$ on the particle?



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(B) $-z$

(C) $-x$

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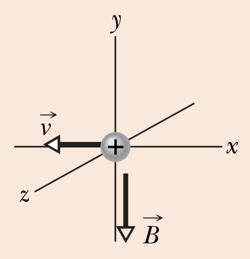

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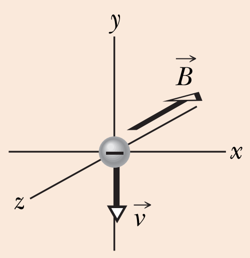

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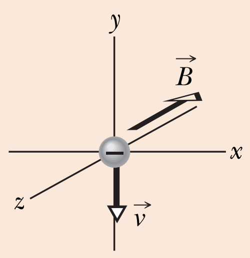

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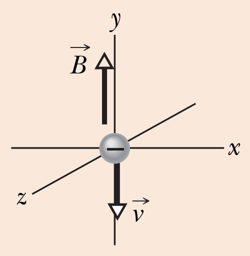

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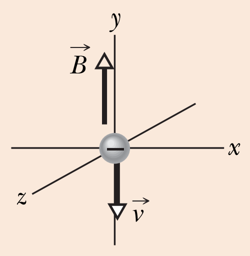

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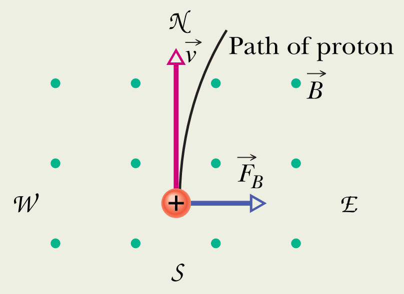


The force on the particle is $\perp$ to its velocity and the field.

[1]Figure from Halliday, Resnick, Walker, 9th ed.

Circular Motion of a Charge

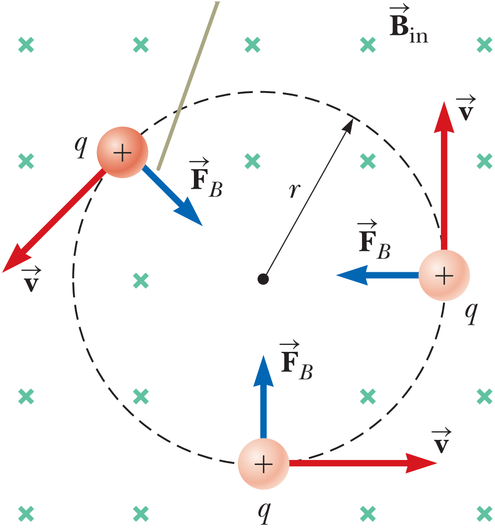
If a charge enters a magnetic field with a velocity at right angles to the field, it will feel a force perpendicular to its velocity.

This will change its trajectory, but not its speed.
$\Rightarrow$ Uniform Circular Motion!

The radius of the circle will depend on 4 things:

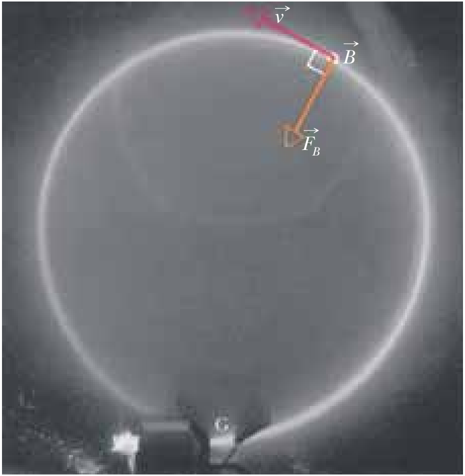
- mass of the particle
- charge of the particle
- initial velocity
- strength of the field

Circular Motion of a Charge



Circular Motion of a Charge

Electrons in a uniform magnetic field:



[1]Photo from Halliday, Resnick, Walker 9th ed, John Le P. Webb, Sussex University.

Circular Motion of a Charge

To find the radius:

$$F_{\text{net}} = F_c = F_B$$

Since $\mathbf{v}$ and $\mathbf{B}$ are perpendicular $F_B = qvB$:

$$\begin{aligned} \frac{mv^2}{r} &= |q|vB \\ r &= \frac{mv}{|q|B} \end{aligned}$$

The sign of q will determine whether the charge circulates clockwise or counter-clockwise.

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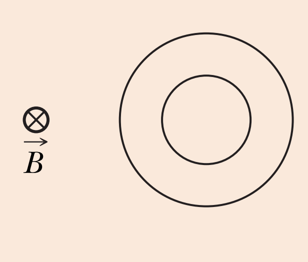
$$\frac{mv^2}{r} = |q|vB$$

Question

The figure here shows the circular paths of two particles that travel at the same speed in a uniform magnetic field **B**, which is directed into the page. One particle is a proton; the other is an electron (which is less massive).

Which particle follows the smaller circle?



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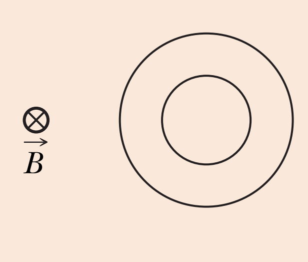

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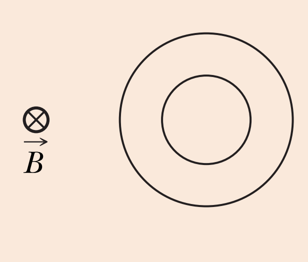

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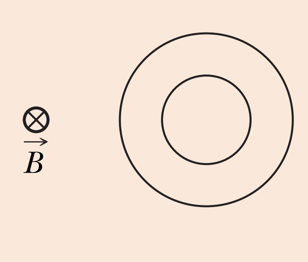

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$$T = \frac{2\pi r}{v}$$

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Also, angular frequency

$$\omega = \frac{|q|B}{m}$$

More general case

What if the velocity vector of a charge particle is not perpendicular to the magnetic field?

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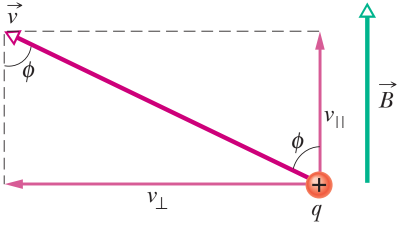
There will be some component of the velocity in the direction of the magnetic field.



For the cross product:

$$|\mathbf{v} \times \mathbf{B}| = vB\sin\phi = (v\sin\phi)B = v_{\perp}B$$

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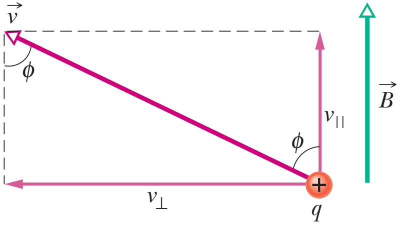

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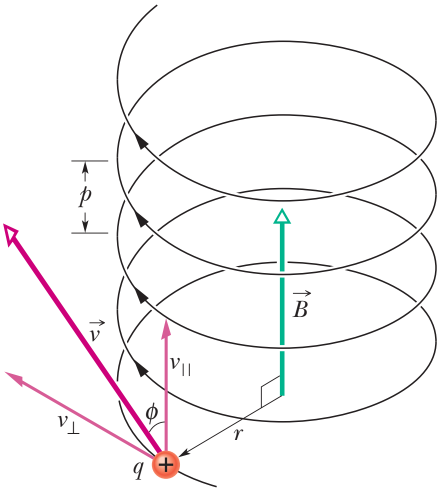
The force will not depend on the $\|$-component and the $\|$-component of velocity will not be changed.

Helical Trajectories



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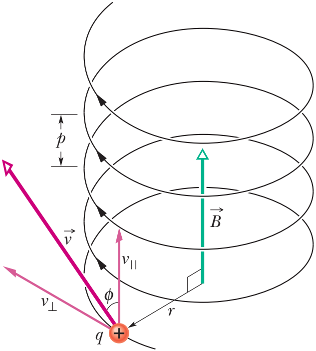


The pitch, p, of the helix is

$$p = v_{\|} T = \frac{2\pi v_{\|} m}{|q|B}$$

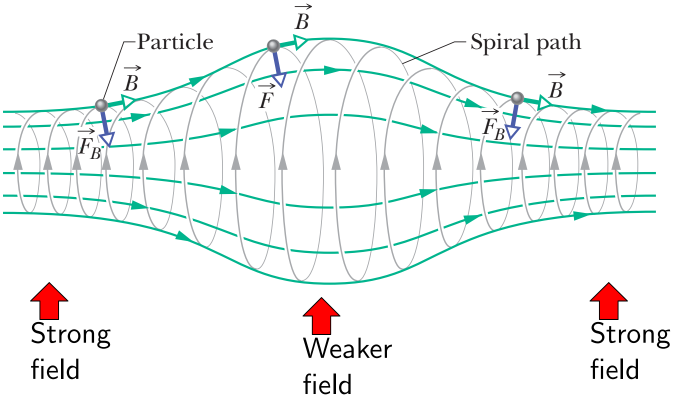
where T is the time period.

The radius is

$$r = \frac{m v_\perp}{|q|B}$$

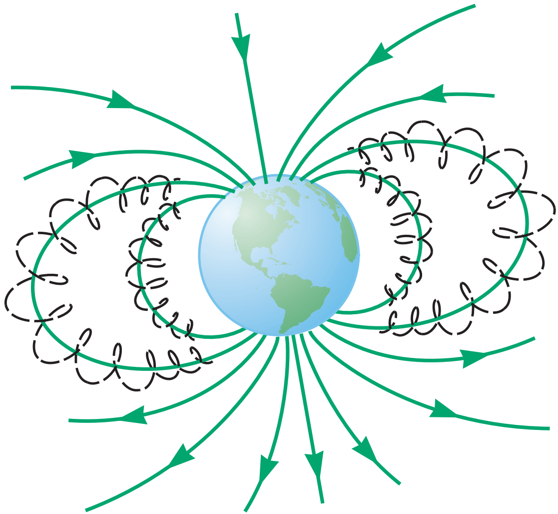
using our equation from earlier.

Non-Uniform Fields: Magnetic Bottle



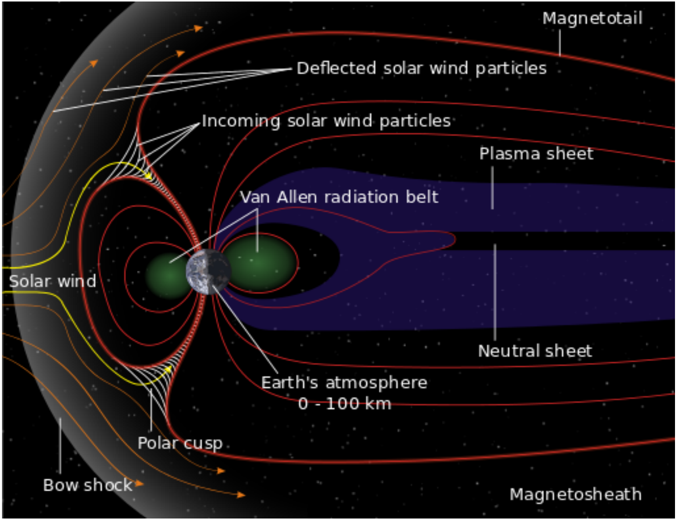
Non-Uniform Fields: Van Allen Belts

Earth's magnetic field acts as a magnetic bottle for cosmic rays.



Non-Uniform Fields: Van Allen Belts

When these charges particles in the belts are disturbed by the solar wind they can drop down into the atmosphere.



[1]Figure by NASA.

Non-Uniform Fields: Van Allen Belts

When these charges particles in the belts are disturbed by the solar wind they can drop down into the atmosphere. The resulting glow is the aurora borealis.

[1] Photo by Donald R. Pettit, Expedition Six NASA ISS science officer, 2013.

The Lorentz Force

A charged particle can be affected by both electric and magnetic fields.

This means that the total force on a charge is the sum of the electric and magnetic forces:

$$\mathbf{F} = q\,\mathbf{E} + q\,\mathbf{v} \times \mathbf{B}$$

This total force is called the **Lorentz force**.

This can always be used to deduce the electromagnetic force on a charged particle in E- or B-fields.

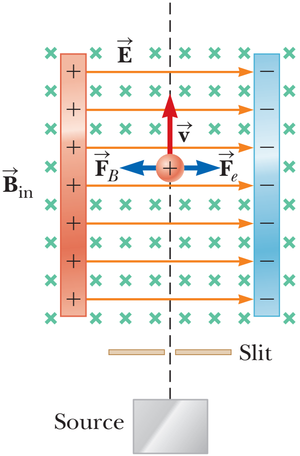
Crossed Fields

Both electric and magnetic fields interact with moving charges and produce forces on them.

This can be used to study charged particles.

Velocity Selector: Using both electric and magnetic fields

Charges are accelerated with and electric field then travel down a channel with uniform electric and magnetic fields.



Velocity Selector: Using both electric and magnetic fields

The particles only reach the end of the channel if $\mathbf{F} = 0$.

$$\mathbf{F} = q\,\mathbf{E} + q\,\mathbf{v} \times \mathbf{B}$$

so that means

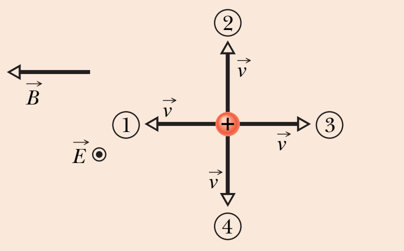
$$q\mathbf{E} = -q\mathbf{v} \times \mathbf{B}$$

supposing $\mathbf{v}$ and $\mathbf{B}$ are perpendicular:

$$v = \frac{E}{B}$$

Crossed Fields Question

The diagram shows four possible directions for the velocity **v** of a positively-charged particle: which direction could possibly result in a net force of zero on the particle?[1]



(A) 1 (left)

(B) 2 (up)

(C) 3 (right)

(D) 4 (down)

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[1]Halliday, Resnick, Walker, 9th ed., page 741.

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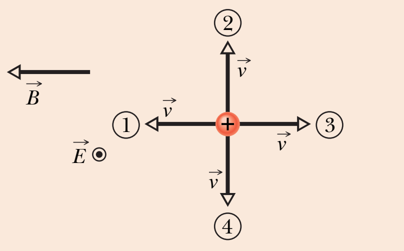

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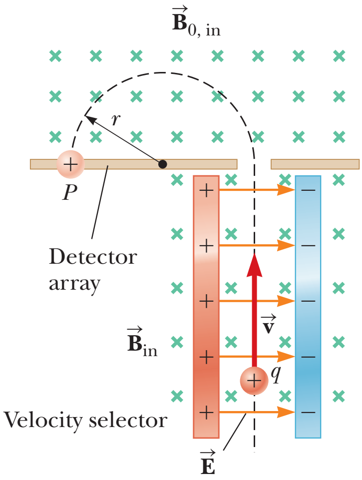
Mass Spectrometer

After selecting particles to have velocity $\mathbf{v} = E/B$ along the channel, they are fed into a magnetic field.



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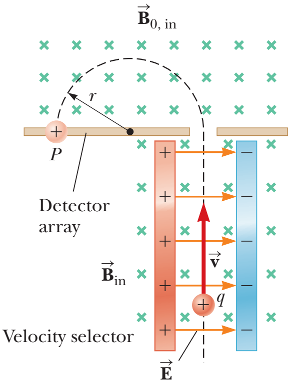


Where they collide with the detector allows us to find the radius of the path, r.

Mass-to-charge ratio:

$$\frac{m}{|q|} = \frac{rB_0}{v}$$

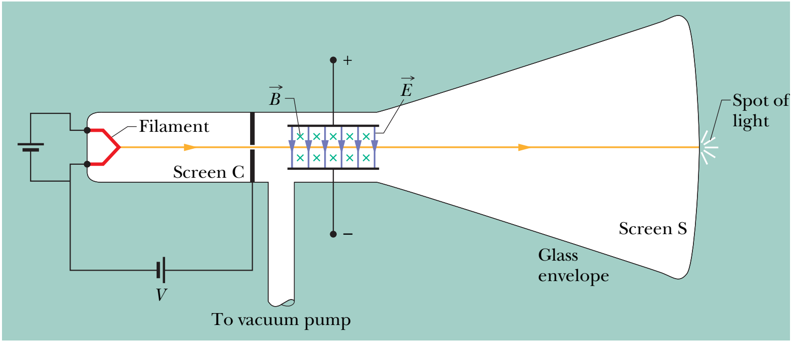
The Discovery of the Electron

Orienting a magnetic field at right angles to an electric field allowed J.J. Thompson in 1897 to determine the ratio of the electron's charge to its mass: $\frac{|q|}{m}$.

This was significant because it showed that the electron was much lighter than other known particles, establishing it as a new kind of particle.

Discovery of the Electron: Main Idea

Electrons are accelerated along the yellow line.



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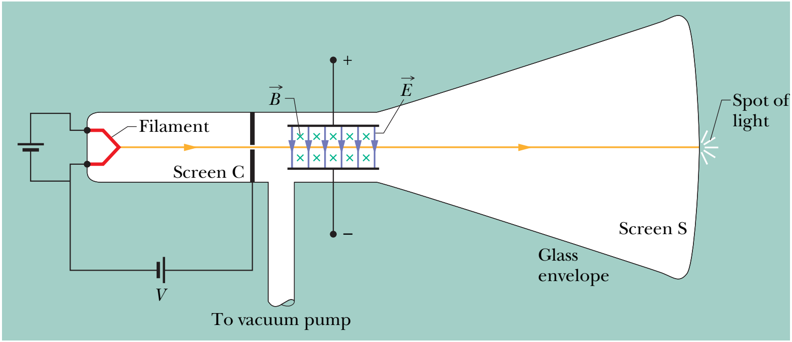


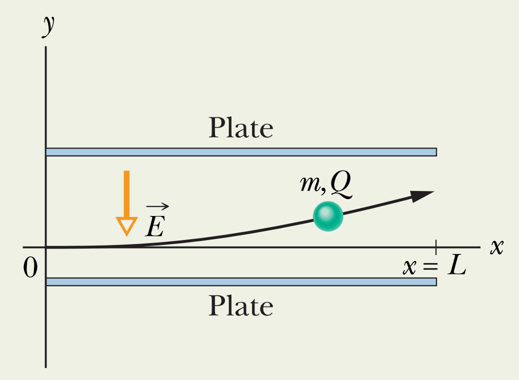
The electric field deflects them upward.

The magnetic field deflects them downward.

Adjust the magnetic field until the deflections cancel out and the spot returns to the center.

Why does that tell us about q/m?

Consider only the E-field from 2 parallel charged plates:



A charge particle follows a parabola, because the field is **uniform**.

This is exactly like projectile motion.

[1] Figure from Halliday, Resnick, Walker, 9th ed, page 593.

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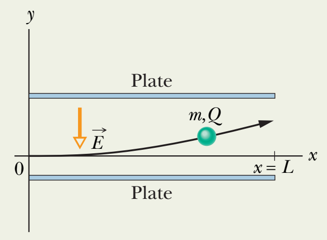

The displacement in the vertical (y) direction (same dir. as field lines)

$$y = v_{i,y}t + \frac{1}{2}at^2$$

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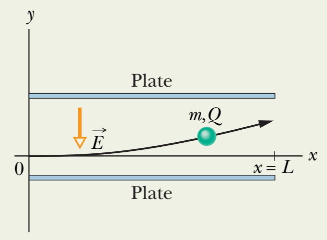

The displacement in the vertical (y) direction (same dir. as field lines)

$$y = v_{i,y} t + \frac{1}{2} a t^2$$

If the particle is moving horizontally only on entry into the field, $v_{i,y} = 0$.

Also $a = F_E / m$, giving:

$$y = \frac{1}{2} \frac{F_E}{m} t^2$$

Why does that tell us about m/q?

There is no acceleration in the x direction:

$$x = L = v_x t \qquad \Rightarrow \qquad t = \frac{L}{v}$$

Therefore the deflection in the y direction due to the electric field by the end of the plates (length L):

$$y = \frac{(qE)L^2}{2mv^2}$$

This gives an expression for q/m:

$$\frac{|q|}{m} = \frac{2\,y\,v^2}{E\,L^2}$$

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But what is the speed v?

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$$x = L = v_x t \qquad \Rightarrow \qquad t = \frac{L}{v}$$

Therefore the deflection in the y direction due to the electric field by the end of the plates (length L):

$$y = \frac{(qE)L^2}{2mv^2}$$

This gives an expression for q/m:

$$\frac{|q|}{m} = \frac{2\,y\,v^2}{E\,L^2}$$

But what is the speed v? $v = E/B$

How to determine v with Crossed Fields

The deflection of a charged particle moving through the fields is 0, only if $\mathbf{F}_{\text{net}} = 0$.

Assuming $\mathbf{v} \perp \mathbf{B}$:

$$\begin{aligned} F_E &= F_B \\ qE &= qvB \\ v &= \frac{E}{B} \end{aligned}$$

Switch on both fields to get a measurement of v. Then switch off the magnetic field and measure the deflection y (E-field only):

$$\frac{|q|}{m} = \frac{2\,y\,E}{B^2\,L^2}$$

Discovery of the Electron

For an electron, $|q| = e$:

$$\frac{e}{m_e} = 1.759 \times 10^{11} \text{ C/kg}$$

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$$m_e = 9.11 \times 10^{-31} \text{ kg}$$

Summary

- magnetic force on a charged particle
- motion of a charged particle in a magnetic field
- crossed fields

Homework Halliday, Resnick, Walker:

- PREVIOUS: Ch 28, onward from page 756. Questions: 1; Problems: 1, 3, 5
- NEW: Ch 28, Questions: 5, 7; Problems: 7, 9, 11, 17, 21, 23, 25