# Electricity and Magnetism
# Hall Effect
# Particle Accelerators
# Magnetic Force on a Wire

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# Last time

- Earth's magnetic field
- magnetic force on a charge
- motion of a charge in a B-field
- charge in electric and magnetic fields

# Overview

- the Hall effect
- particle accelerators
- force on a wire in a magnetic field
- torque on a wire loop in a magnetic field
- motors

# The Lorentz Force

A charged particle can be affected by both electric and magnetic fields.

This means that the total force on a charge is the sum of the electric and magnetic forces:

$$\mathbf{F} = q\,\mathbf{E} + q\,\mathbf{v} \times \mathbf{B}$$

This total force is called the **Lorentz force**.

This can always be used to deduce the electromagnetic force on a charged particle in E- or B-fields.

# Crossed Fields

Both electric and magnetic fields interact with moving charges and produce forces on them.

This can be used to study charged particles.

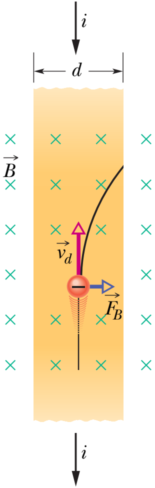
# The Hall effect

Or, how to use a current and a field to create a potential difference.

# The Hall effect

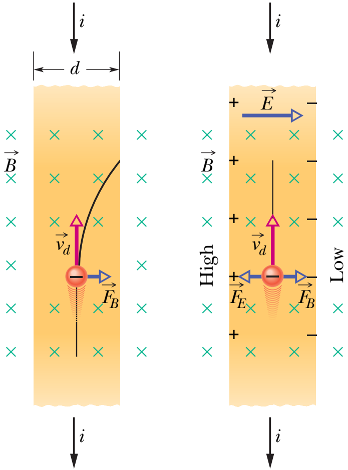
Or, how to use a current and a field to create a potential difference.

Electrons flowing in a conductor can also be deflected by a magnetic field!



# The Hall effect

Electrons are pushed to the right until so much negative charge has built up on the right side that the electrostatic force balances the magnetic force.




At this point we have crossed fields and the potential difference between the left and the right side stabilizes.

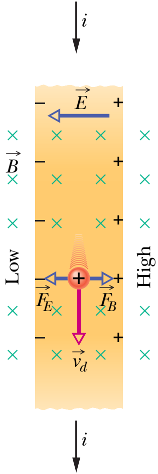
# The Hall effect

The Hall effect allows us to learn many things about the charge carriers in a conductor:

- their charge
- their volume density
- their drift velocity (for a given current)

# The Hall effect

Suppose the charge carriers in a conductor were positively charged:




We would get the opposite polarity for the potential difference!

# The Hall effect

The constant potential difference that appears across the conductor once the current has stabilized is called the *Hall potential difference*.

$$\Delta V = Ed$$

where $d$ is the width of the conductor.

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Since the electric force and magnetic force balance:

$$\begin{aligned} F_E &= F_B \\ eE &= ev_d B \\ v_d &= \frac{E}{B} \end{aligned}$$

We can use our knowledge to estimate $v_d$.

# The Hall effect

Alternatively, we can estimate the density of charge carriers, $n$.

Remember:

$$v_d = \frac{I}{n\,e\,A}$$

Equating this with our expression for $v_d$ on the previous slide:

$$\frac{E}{B} = \frac{I}{n\,e\,A}$$

Rearranging, and using $\Delta V = Ed$ and letting $t = A/d$ be the conductor **thickness**:

$$n = \frac{B\,I}{e(\Delta V)t}$$

# The Hall effect

Remembering $\Delta V = Ed$ and $t = A/d$ is the conductor **thickness**:

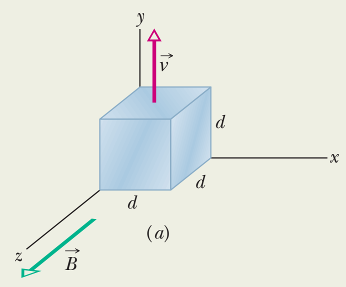
$$n = \frac{B\,I}{e(\Delta V)t}$$

$\Delta V$ is called the Hall Potential Difference:

$$\Delta V = \frac{B\,I}{nte}$$

# The Hall effect - example question

A solid metal cube, of edge length $d = 1.5$ cm, moving in the positive $y$ direction at a constant velocity $\mathbf{v}$ of magnitude 4.0 m/s. The cube moves through a uniform magnetic field $\mathbf{B}$ of magnitude 0.050 T in the positive $z$ direction.



(a)

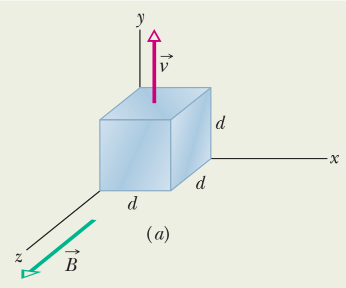
Which cube face is at a lower electric potential and which is at a higher electric potential because of the motion through the field?

[1] Halliday, Resnick, Walker, 9th ed, page 743.

# The Hall effect - example question

Free charges in the conductor will feel a force as they move along with the entire conductor through the field.

The free charges are electrons. We have to find the direction of the force on them.



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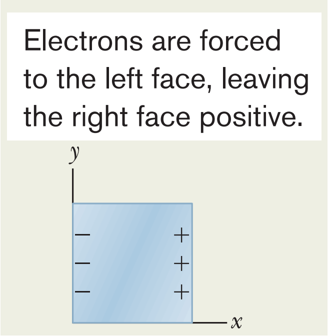

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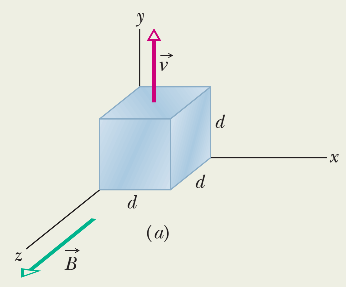

(a)

What is the potential difference between the faces of higher and lower electric potential?

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# The Hall effect - example question

When does the potential difference between the faces stabilize?

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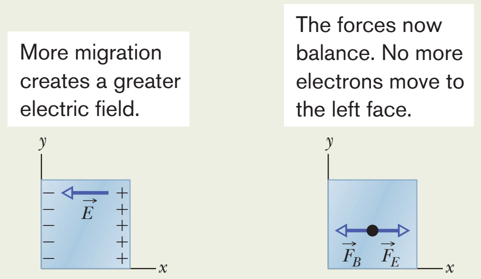


$$F_E = F_B$$

# The Hall effect - example question

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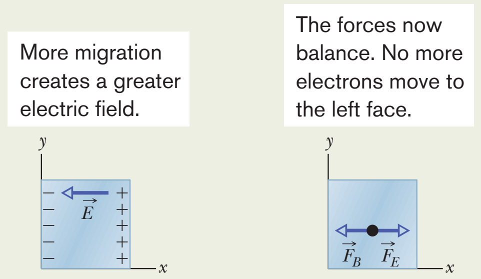


$$
\begin{aligned}
F_E &= F_B \\
eE &= evB \\
\left(\frac{\Delta V}{d}\right) &= vB \\
\Delta V &= vBd
\end{aligned}
$$

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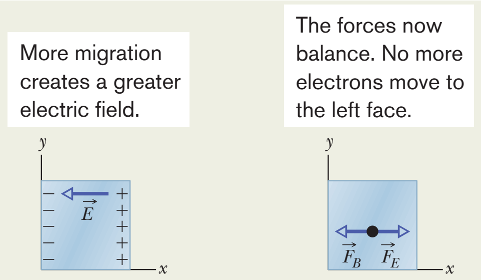



$$
\begin{aligned}
F_E &= F_B \\
eE &= evB \\
\left(\frac{\Delta V}{d}\right) &= vB \\
\Delta V &= vBd \\
\Delta V &= 3.0 \text{ mV}
\end{aligned}
$$

# Related Effects

- the Hall effect in semiconductors - can be more complex! Depends on the material.
- the quantum Hall effect - can observe quantization of the Hall potential difference. Can be used to measure the charge of the electron.

# Accelerating Charged Particles

High speed beams of particles are useful for studying nuclear and particle physics.

They can be tricky to create, however.

Charged particles can be accelerated with a potential difference.

# The electron-Volt

One convenient unit of energy for particles is the electron-Volt, written eV.

This is the amount of energy that an electron accelerated through a potential difference of 1 Volt has.

$$U = qV$$

$$1 \text{ eV} = (1.6 \times 10^{-19} \text{ C})(1 \text{ V}) = 1.6 \times 10^{-19} \text{ J}.$$

# Accelerating Charged Particles

The acceleration of a particle from a potential difference depends on its mass.

Suppose there is a practical limit on how strong an electric field can be created, $E$.

The force on the particle is:

$$F = qE$$

The acceleration can be deduced from Newton's second law:

$$a = \frac{F}{m} = \frac{qE}{m}$$

The final velocity of a particle with this acceleration will be:

$$v_f^2 = 2ad = \frac{qEd}{m}$$

If $m$ is large than the accelerating distance $d$ must be also. For protons the value of $d$ necessary becomes impractical.

# Cyclotrons

One way around this is to have the particles move in a circle.

The acceleration can take place in a limited space.

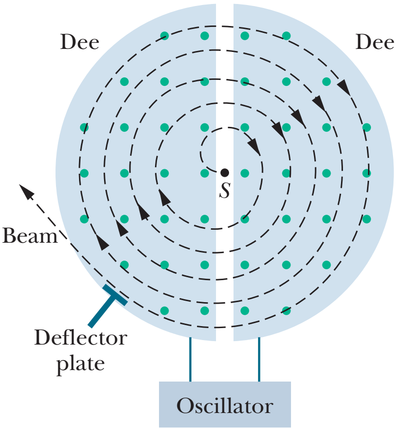
Magnetic fields cause the protons to follow circular arcs.

The protons are directed repeatedly through a potential difference that accelerates them.

The time period of the orbit does not depend on the velocity of the particles!

$$T = \frac{2\pi m}{|q|B}$$

# Cyclotrons




$$f = f_{\text{osc}} = \frac{|q|B}{2\pi m}$$

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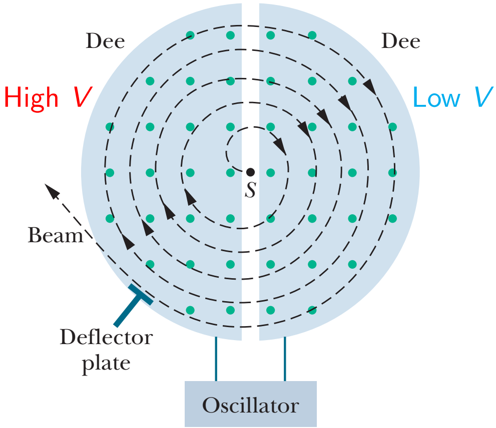


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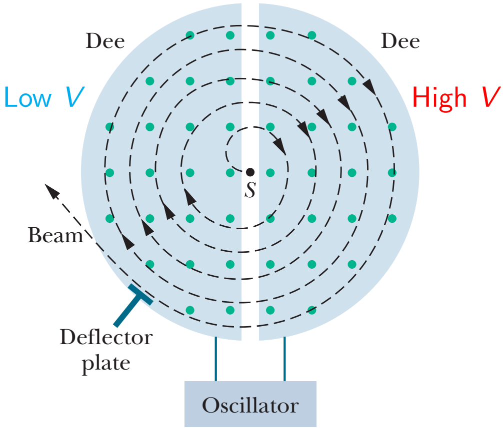


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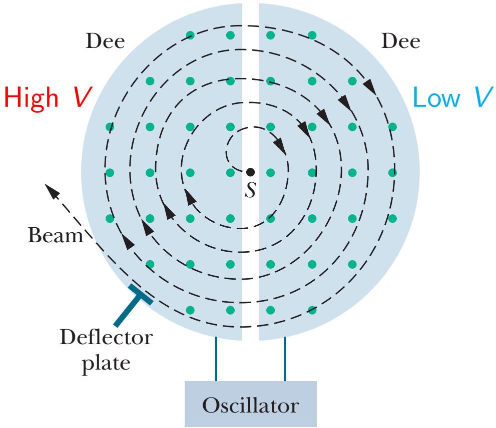
# Cyclotrons




$$f = f_{\mathrm{osc}} = \frac{|q|B}{2\pi m}$$

# Circular Motion of a Charge

To find the radius:

$$F_{\text{net}} = F_c = F_B$$

$$r = \frac{mv}{|q|B}$$

So, $v = qBr/m$.
When the ion exits the cyclotron, it will have kinetic energy:

$$\begin{aligned} K &= \frac{1}{2}mv^2 \\ K &= \frac{(qBr)^2}{2m} \end{aligned}$$

# Cyclotron

The first cyclotron was built in 1934.

The world's largest cyclotron has a maximum beam radius of 7.9 m.

[1]Photo from Lawrence Berkeley National Laboratory.

# Synchrotron

Once the charged particles reach $\sim 10\%$ of the speed of light this stops working.

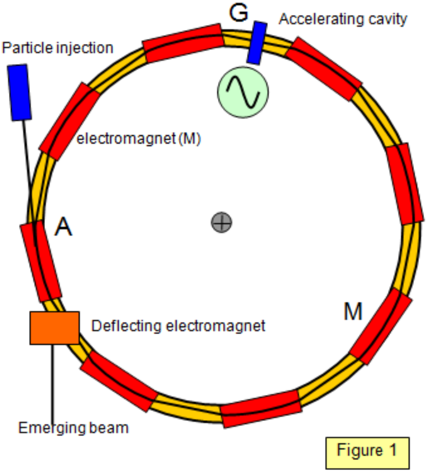
This is because the effective mass of the particles is increasing, so $f_{\text{osc}} = \frac{2\pi m}{|q|B}$ is no longer a constant.

Also, at these speeds the area of the magnetic field for a cyclotron must be quite big as the radius of the path becomes large.

A solution to this is the synchrotron.

# Synchrotron

Synchrotrons operate similarly to cyclotrons, but the frequency of the potential switching can vary.




[1] Figure from schoolphysics.co.uk.

# Synchrotron

This also means that the particles can be kept on a single loop, even as their velocity increases.

The magnetic field only has to cover the ring itself.
(Not the area in the middle of the ring.)

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The LHC (Large Hadron Collider) at CERN is a type of synchrotron.

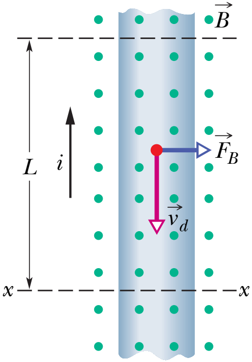
The Tevatron at Fermilab was also one, but it has been shutdown due to Congressional budget cuts.

# Synchrotron

[1]Photo copyright © Synchrotron Soleil, used with permission

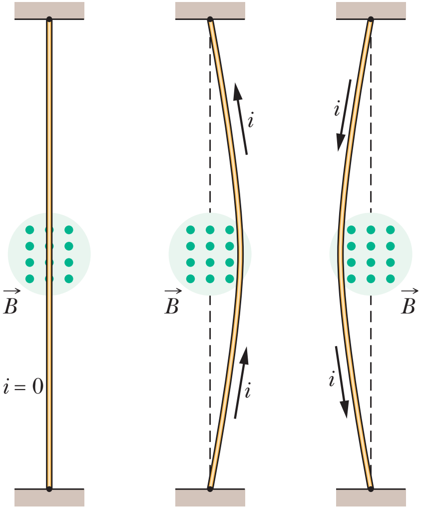
# Magnetic Force on a Current Carrying Wire

Charged particles moving in a magnetic field experience a force.




A wire carrying a current also experiences a force, since there is a force on each moving charge confined to the wire.

# Magnetic Force on a Current Carrying Wire




The direction of the force depends on the direction of the current.

# Magnetic Force on a Current Carrying Wire

The force on the wire in a uniform magnetic field is given by:

$$\mathbf{F} = I\,\mathbf{L} \times \mathbf{B}$$

where $\mathbf{L}$ is a distance vector that points along the length of the wire in the direction of the conventional current $I$ and is as long as the part of the wire inside the field is.

By considering the force on an individual charge, we can motivate this equation.

# Summary

- the Hall effect
- particle accelerators
- motors

**Homework** Halliday, Resnick, Walker:

- PREVIOUS: Ch 28, onward from page 756. Questions: 5, 7; Problems: 7, 9, 11, 17, 21, 23, 25
- NEW: Ch 28, Questions: 3; Problems: 13, 15, 27, 33, 35, 39, 41, (49)