Electricity and Magnetism
Magnetic Dipole Moment
Magnetism in Matter

Lana Sheridan

De Anza College

Nov 12, 2015

Last time

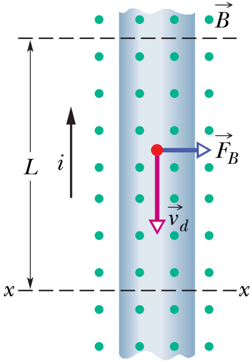
- the Hall effect
- particle accelerators

Overview

- force on a wire with a current in a B-field
- torque on a wire loop in a B-field
- motors
- relating a current loop to a magnet
- magnetic dipole moment
- torque and potential energy of magnetic dipole
- magnetism of matter

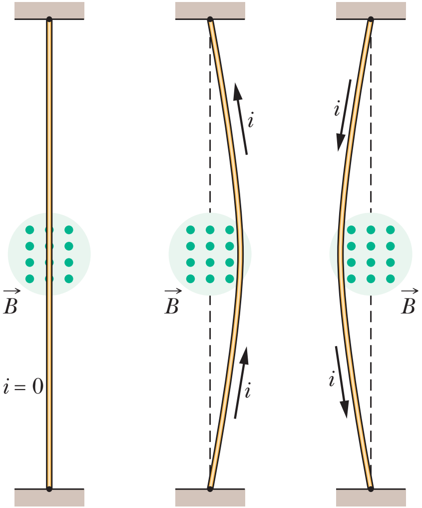
Magnetic Force on a Current Carrying Wire

Charged particles moving in a magnetic field experience a force.



A wire carrying a current also experiences a force, since there is a force on each moving charge confined to the wire.

Magnetic Force on a Current Carrying Wire



The direction of the force depends on the direction of the current.

Magnetic Force on a Current Carrying Wire

The force on the wire in a uniform magnetic field is given by:

$$\mathbf{F} = I\,\mathbf{L} \times \mathbf{B}$$

where $\mathbf{L}$ is a distance vector that points along the length of the wire in the direction of the conventional current I and is as long as the part of the wire inside the field is.

By considering the force on an individual charge, we can motivate this equation.

Magnetic Force on a Current Carrying Wire

The force on an individual conduction electron is
$\mathbf{F}_B = (-e)\,\mathbf{v}_d \times \mathbf{B}$.

The total force will be the sum of the force on all the moving charges together.

How much conduction charge is in the wire?

Magnetic Force on a Current Carrying Wire

The force on an individual conduction electron is
$\mathbf{F}_B = (-e)\,\mathbf{v}_d \times \mathbf{B}$.

The total force will be the sum of the force on all the moving charges together.

How much conduction charge is in the wire?

$$q = -enV$$

where n is the volume density of charge carriers, and V is the volume of the wire.

Also, this charge is negative, since the flowing charges are electrons.

Magnetic Force on a Current Carrying Wire

$$\mathbf{F}_B = -enV\,\mathbf{v}_d \times \mathbf{B}$$

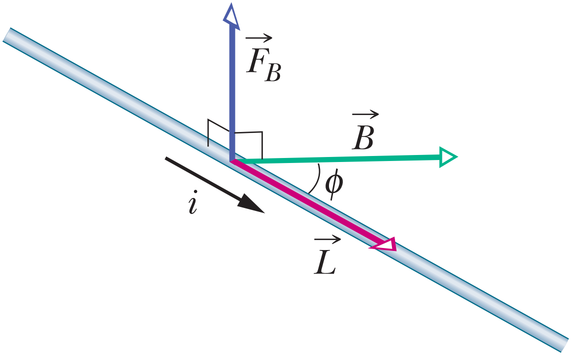
$$v_d = \frac{I}{neA} \;\rightarrow\; I\,L = enVv_d$$

since $V = AL$ where L is the length of the wire.

If the wire is straight and in a uniform field and we define $\mathbf{L}$ to be a vector of length L pointed in the direction of the conventional current, then:

$$\mathbf{F}_B = I\,\mathbf{L} \times \mathbf{B}$$

Magnetic Force on a Current Carrying Wire

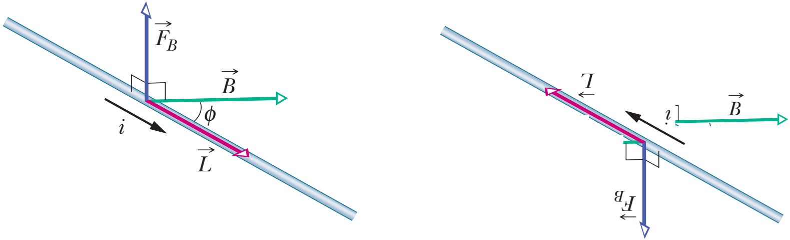


$$\mathbf{F} = I\,\mathbf{L} \times \mathbf{B}$$

Torque on a Loop of Wire with a Current

Or, how to turn electricity into motion.

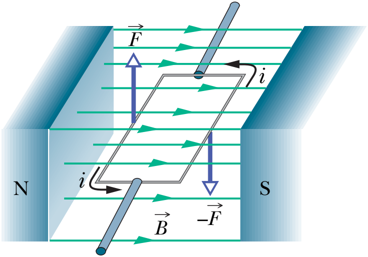
Consider two wires in a magnetic field with currents flowing in opposite directions.



They will experience forces in opposite directions.

Torque on a Loop of Wire with a Current

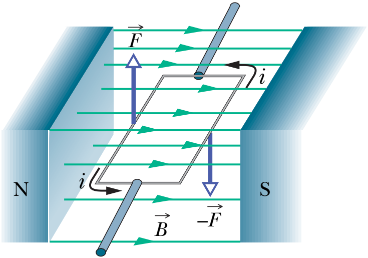
This is the situation that occurs when a loop of wire is placed in a B-field.



These opposing forces on opposite sides of the loop creates a torque on the loop.

Torque on a Loop of Wire with a Current

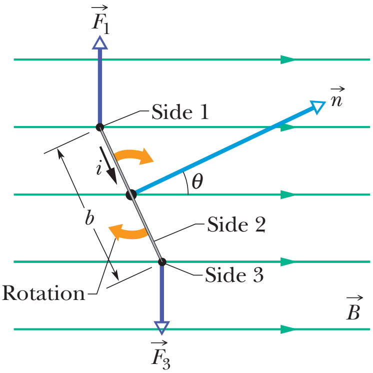
The current on the two sides away from the axle gives an upward force on the left and downward on the right.



On the two ends that connect to the axle, the force is zero when the loop lays flat parallel to the B-field.

When the loop rotates, the forces on those two ends are equal and opposite.

Torque on a Loop of Wire with a Current

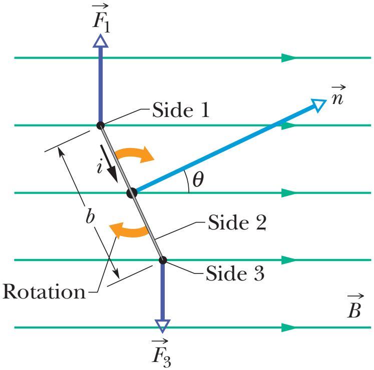


$$\boldsymbol{\tau}_F = \mathbf{r} \times \mathbf{F} \quad ; \qquad \boldsymbol{\tau}_{\text{net}} = \sum_i \boldsymbol{\tau}_i$$

$$\boldsymbol{\tau}_{\text{net}} = \mathbf{r}_1 \times \mathbf{F}_1 + \mathbf{r}_2 \times \mathbf{F}_2$$

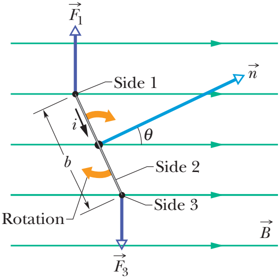
Torque on a Loop of Wire with a Current

$$\mathbf{F}_1 = I\mathbf{a} \times \mathbf{B} = iaB\,\mathbf{j} = -\mathbf{F}_3$$



$$\boldsymbol{\tau}_{net} = \mathbf{r}_1 \times \mathbf{F}_1 + \mathbf{r}_3 \times \mathbf{F}_3$$

Torque on a Loop of Wire with a Current

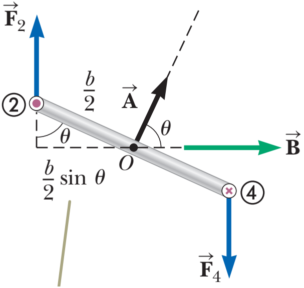


$$\begin{aligned} \boldsymbol{\tau}_{\text{net}} &= \mathbf{r}_1 \times \mathbf{F}_1 + \mathbf{r}_2 \times \mathbf{F}_2 \\ &= \left(\frac{b}{2}\right)(IaB)\sin\theta + \left(\frac{b}{2}\right)(IaB)\sin\theta \quad \text{[cw in diag.]} \end{aligned}$$

Noting that the area of the loop $A = ab$:

$$\boldsymbol{\tau} = IAB\sin\theta$$

Torque on a Loop of Wire with a Current



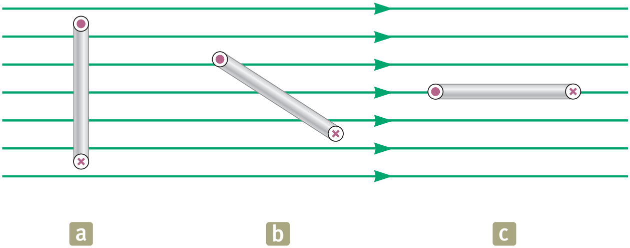
$$\boldsymbol{\tau} = IAB\sin\theta$$

We can make this expression more compact by defining $\mathbf{A} = A\hat{\mathbf{n}}$ where $\hat{\mathbf{n}}$ is normal to the loop plane.

$$\boldsymbol{\tau} = I\mathbf{A} \times \mathbf{B}$$

Torque on a Loop of Wire Question

Which of the rectangular loops has the largest magnitude of the net force acting on it?



(A) a

(B) b

(C) c

(D) all the same

[1]Serway & Jewett, 9th ed.

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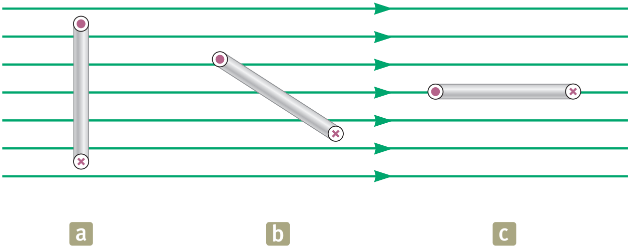

(A) a

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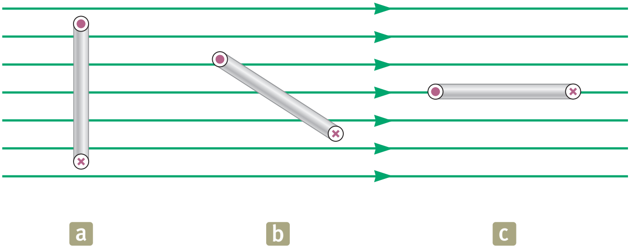
Torque on a Loop of Wire Question

Rank the magnitudes of the torques acting on the rectangular loops from highest to lowest.



(A) a, b, c

(B) b, a, c

(C) c, b, a

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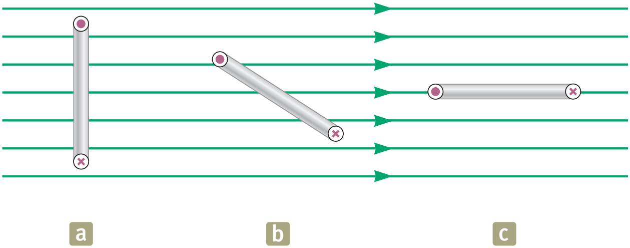

(A) a, b, c

(B) b, a, c

(C) c, b, a ←

(D) c, a, b

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Torque on a Coil of Wire with a Current

$$\boldsymbol{\tau} = I\mathbf{A} \times \mathbf{B}$$

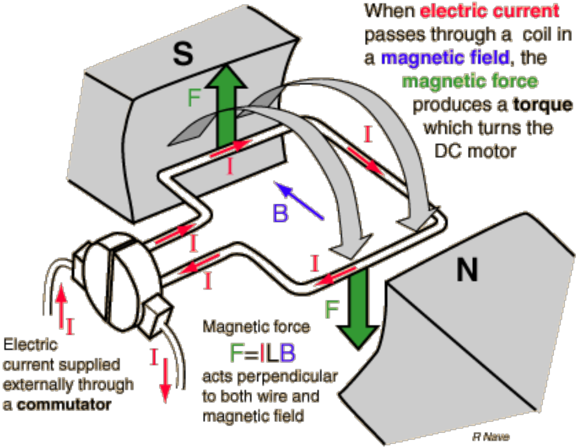
Remarkably, that equation also holds for other shapes of loop as long as they are flat (in one plane). A is the area of the loop.

For a *coil* of N loops stacked together, the effect of each loop adds up:

$$\boldsymbol{\tau} = NI\mathbf{A} \times \mathbf{B}$$

Electric Motors

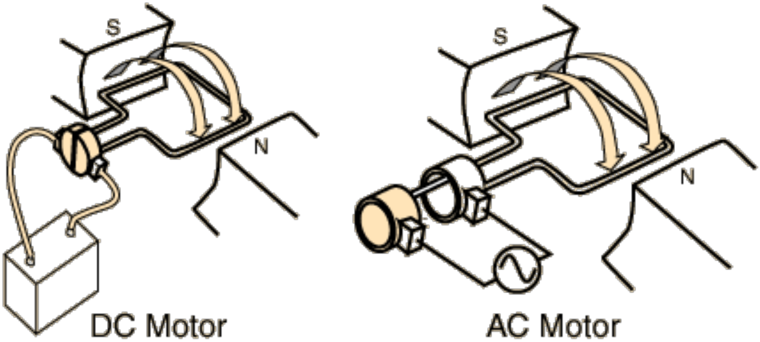
This effect can be used to turn electricity into mechanical work.



[1]Figure from hyperphysics.phys-arstr.gsu.edu

Electric Motors

Either direct current (DC) or alternating current (AC) can be used for a motor.



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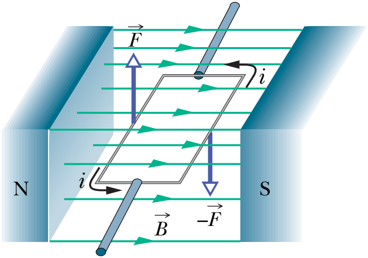
Torque on a Loop of Wire with a Current



$$\boldsymbol{\tau} = IAB\sin\theta$$

We can make this expression more compact by defining $\mathbf{A} = A\hat{\mathbf{n}}$ where $\hat{\mathbf{n}}$ is normal to the loop plane.

$$\boldsymbol{\tau} = I\mathbf{A} \times \mathbf{B}$$

Magnetic Moment for a Current Loop

For a current loop, we can define the **magnetic moment** of the loop as

$$\boldsymbol{\mu} = I\mathbf{A}$$

And for a coil of wire carrying a current:

$$\boldsymbol{\mu} = I\mathbf{A}$$

Then the expression for the torque can be written

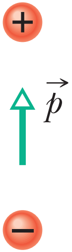
$$\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$$

Magnetic Dipole Moment

Recall our definition for the Electric dipole moment:
dipole moment:

$$\mathbf{p} = q\,\mathbf{d}$$

where **d** is a vector pointing from the negative charge to the positive charge, and its magnitude d is the separation of the charges and each charge in the dipole has magnitude q.



Torque on a electric dipole in an electric field:

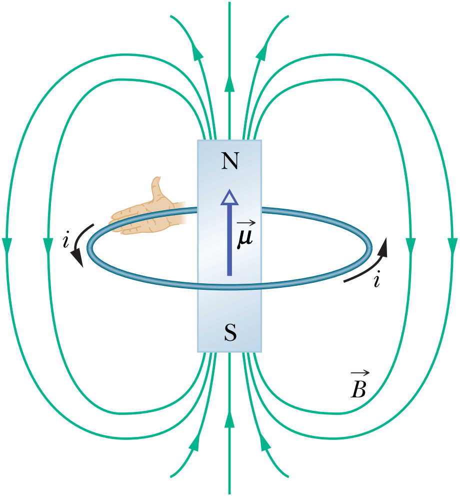
$$\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}$$

Potential energy:

$$U = -\mathbf{p} \cdot \mathbf{E}$$

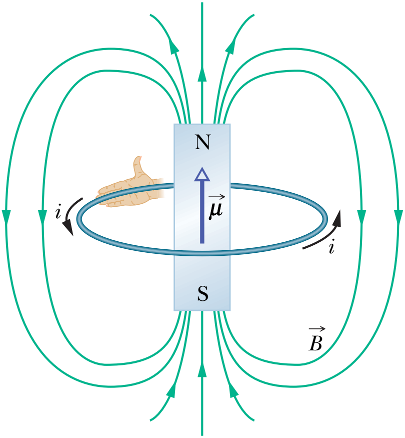
Current Loop vs Bar Magnet

A loop of wire with a current in it produce a similar magnetic field as a bar magnet.



Magnetic Dipole Moment

For a pair of magnetic charges at either end of a thin bar magnet, this would be: $\boldsymbol{\mu} = q_m\,\mathbf{d}$.



Magnetic Dipole Moment

magnetic dipole moment, μ

The quantity relating an external magnetic field that a magnet or coil of wire is in to the torque on the magnet or coil due to that field.

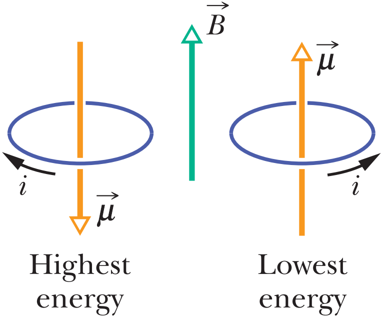
$$\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$$

For a magnet, it is a vector pointing from the south pole of a magnet to the north pole, that is proportional to the strength of the B-field produced by the magnet itself.

For a coil, it is defined according the the right hand rule for current in a wire loop and is proportional to the coil area and current.

Potential Energy of a Dipole in a B-Field

$$\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$$

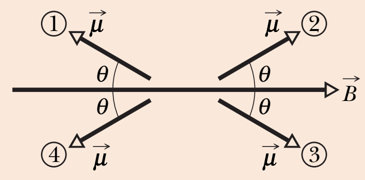


The energy can be found by integrating the torque over the angle of rotation.

$$U = -\boldsymbol{\mu} \cdot \mathbf{B}$$

Question

The figure shows four orientations, at angle θ, of a magnetic dipole moment **μ** in a magnetic field. Rank the orientations according to the magnitude of the torque on the dipole, greatest first.



(A) 1 and 2, 3 and 4

(B) 1 and 4, 2 and 3

(C) 3, 2, 1, 4

(D) all the same

[1] Halliday, Resnick, Walker, 9th ed, page 745.

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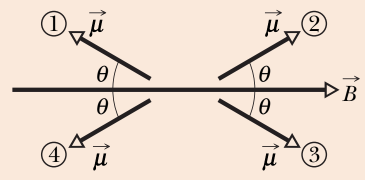

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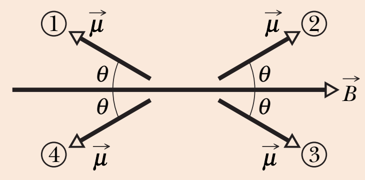

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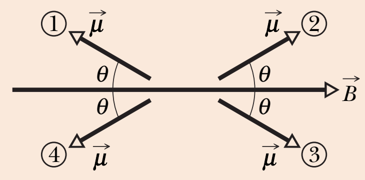

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Electric Dipole and Magnetic Dipole

	electric dipole	magnetic dipole
torque $\boldsymbol{\tau}$	$\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}$	$\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$
potential energy U	$U = -\mathbf{p} \cdot \mathbf{E}$	$U = -\boldsymbol{\mu} \cdot \mathbf{B}$

Magnetism in Matter

Ordinary matter exhibits magnetic properties.

Now we will see why.

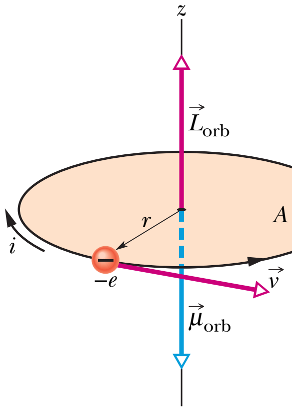
This is covered in Chapter 32 of the book, but we will not go into full detail.

Magnetism in Matter: Magnetic Moment of Atoms

Atoms and subatomic particles also have magnetic moments!

Why? Consider a classical model of a hydrogen atom. One electron orbits the nucleus.

$$\mu = IA$$



Magnetic Moment of Atoms

The current is the rate of charge flow with time:

$$I = \frac{-e}{T} = -e\,\frac{v}{2\pi r}$$

assuming an orbital radius of r, speed v.

$$\begin{aligned}\boldsymbol{\mu} &= I\mathbf{A} \\ &= -e\,\frac{v}{2\pi r}(\pi r^2 \hat{\mathbf{n}}) \\ &= -\frac{evr}{2}\hat{\mathbf{n}}\end{aligned}$$

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Recall that for a particle of mass m orbiting at a radius r, velocity v, the angular momentum is:

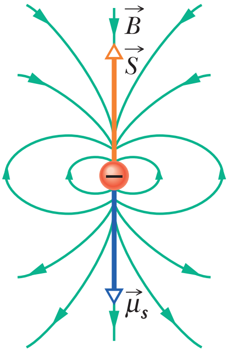
$$L = mvr$$

$$\boldsymbol{\mu} = -\frac{e}{2m_e}\mathbf{L}$$

Electron Spin Angular Momentum

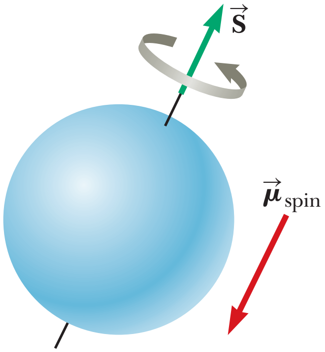
Electrons also have another kind of angular momentum: *intrinsic angular momentum*. This is also called "spin".

Spin is an inherent property of all electrons. It cannot be understood with classical mechanics, but also contributes a magnetic moment.



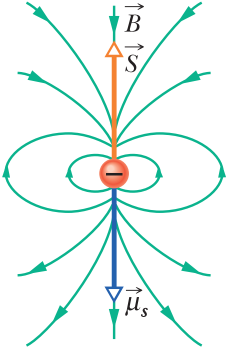
Electron Spin Angular Momentum

You might imagine an electron as a rigid charge sphere spinning on an axis through its center...



...but really, it's not.

Electron Spin Angular Momentum



Electron's spin magnetic dipole moment:

$$\boldsymbol{\mu}_s = -\frac{g\,e}{2m_e}\mathbf{S}$$

where $g \approx 2$.

Magnetic Moment of Atoms

In atoms with many electrons, the electrons tend to cancel out each other's magnetic moments, but outer-shell, unpaired electrons can contribute a significant magnetic moment.

The particles in the nucleus also have magnetic moments, but they are much smaller.

Most of an atom's magnetic moment comes from unpaired electons.

These tiny magnetic moments add up to big effects in bulk materials.

Three Types of Bulk Magnetism

- ferromagnetism
- paramagnetism
- diamagnetism

Ferromagnetism

Atoms of ferromagnetic materials have non-zero magnetic moments.

Interactions between outer electrons in different atoms causes alignment of each atom's magnetic moment.

Magnetic moments reenforce each other and will tend to spontaneously align within domains.

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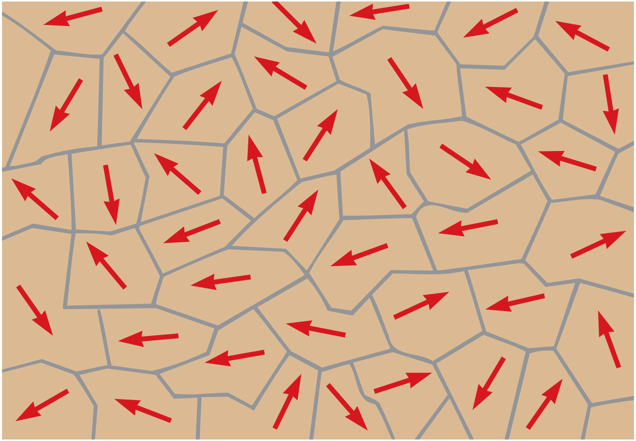
Examples of ferromagnetic materials:

- iron
- nickel
- cobalt
- gadolinium
- dysprosium

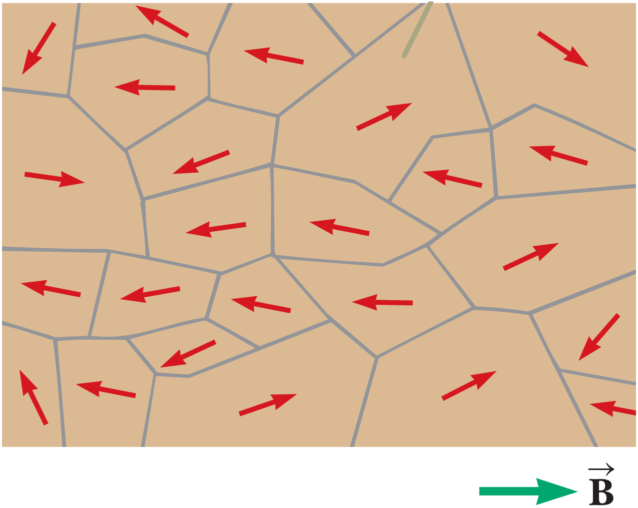
Ferromagnetism

No external B-field



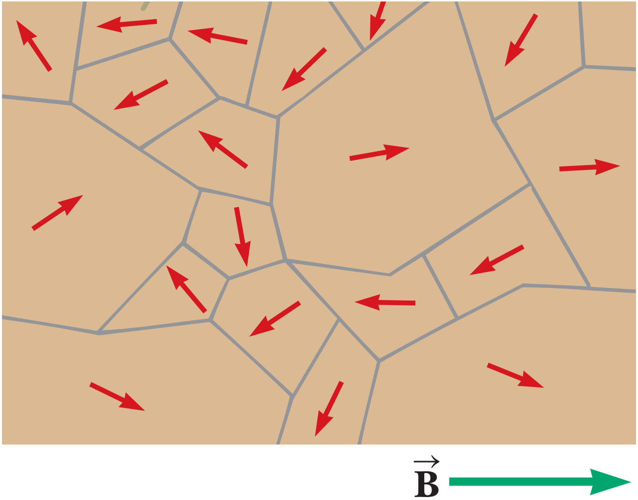
Ferromagnetism

Applied external B-field



Ferromagnetism

Strong external B-field



Paramagnetism

Atoms of paramagnetic materials have non-zero dipole moments, but electrons of different atoms do not interact with each other.

They can interact with a strong magnetic field, and will align with the field.

Paramagnetic effects tend to be much smaller than ferromagnetic ones.

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They can interact with a strong magnetic field, and will align with the field.

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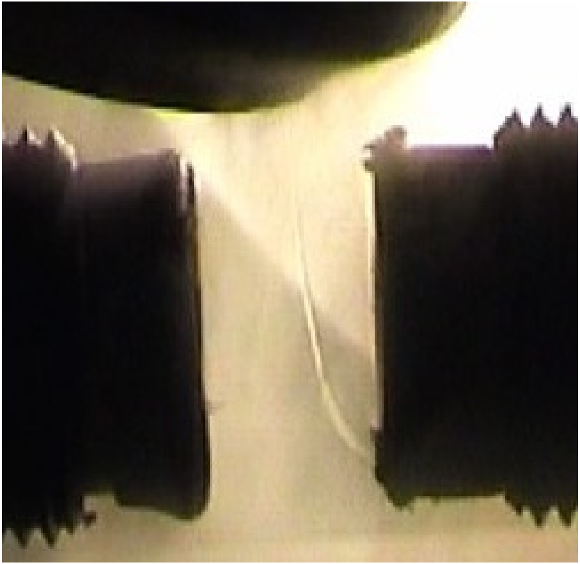
Examples of paramagnetic materials:

- Tungsten
- Cesium
- Aluminium
- Lithium
- Magnesium
- Sodium

Paramagnetism

Liquid oxygen stream deflected in a strong magnetic field. The stream collects in the field.



[1] Image created by Pieter Kuiper.

Diamagnetism

Diamagnetism occurs in all materials, but is a weak effect, so it is "drowned out" if a material is ferro- or paramagnetic.

It is the dominant (but weak) effect when the net magnetic moment of a material's atoms is zero.

The field polarizes the atoms and the resulting magnetic moments **oppose** the external magnetic field.

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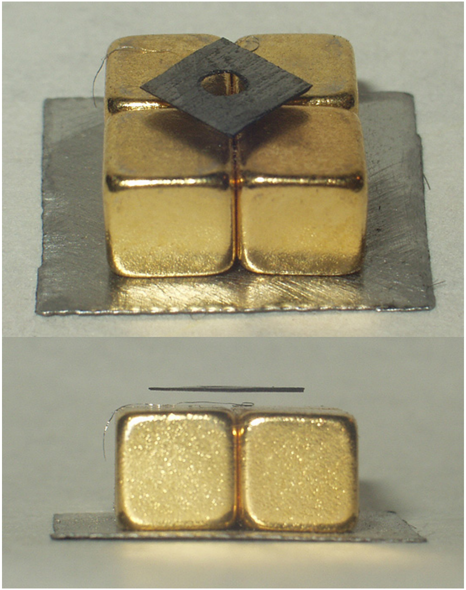
The field polarizes the atoms and the resulting magnetic moments **oppose** the external magnetic field.

Examples of diamagnetic materials:

- Pyrolytic carbon
- Bismuth
- Mercury
- Silver
- diamond (form of Carbon)
- water

Also superconductors can be said to exhibit extreme diamagnetism.

Diamagnetism



[1]Levitating pyrolytic carbon on neodymium magnets. Image by Splarka.

Diamagnetism

[1]Magnet photo by Mai-Linh Doan, Wikipedia; Frog photo by Lijnis Nelemans/High Field Magnet Laboratory/Radboud University Nijmeg.

Summary

- force on a wire in a magnetic field
- torque on a wire loop in a magnetic field
- relating a current loop to a magnet
- magnetic dipole moment
- torque and potential energy of magnetic dipole
- magnetism in matter

Homework Halliday, Resnick, Walker:

- PREVIOUS: Ch 28, Questions: 3; Problems: 13, 15, 27, 33, 35, 39, 41
- NEW: Ch 28, Problems: 49, 54, 55, 57, 61, 65