Electricity and Magnetism
Magnetic Field from Moving Charges

Lana Sheridan

De Anza College

Nov 17, 2015

Last time

- force on a wire with a current in a B-field
- torque on a wire loop in a B-field
- motors
- relating a current loop to a magnet
- magnetic dipole moment
- torque and potential energy of magnetic dipole
- magnetism of matter

Overview

- magnetic fields from moving charges
- magnetic fields around current-carrying wires
- forces between parallel wires
- Gauss's law

Magnetic fields from moving charges and currents

We are now moving into chapter 29.

Anything with a magnet moment creates a magnetic field.

This is a logical consequence of Newton's third law.

Magnetic fields from moving charges

A moving charge will interact with other magnetic poles.

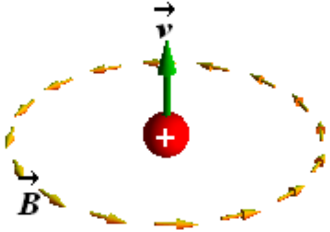
This is because it has a magnetic field of its own.

The field for a moving charge is given by the Biot-Savart Law:

$$\mathbf{B} = \frac{\mu_0}{4\pi}\frac{q\,\mathbf{v}\times\hat{\mathbf{r}}}{r^2}$$

Magnetic fields from moving charges

$$\mathbf{B} = \frac{\mu_0}{4\pi} \frac{q\,\mathbf{v} \times \hat{\mathbf{r}}}{r^2}$$



[1]Figure from rakeshkapoor.us.

Magnetic fields from currents

$$\mathbf{B} = \frac{\mu_0}{4\pi}\frac{q\,\mathbf{v} \times \hat{\mathbf{r}}}{r^2}$$

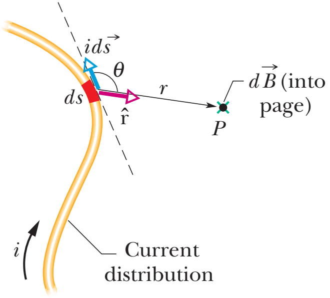
We can deduce from this what the magnetic field do to the current in a small piece of wire is.

Current is made up of moving charges!

$$q\,\mathbf{v} = q\frac{\Delta \mathbf{s}}{\Delta t} = \frac{q}{\Delta t}\Delta \mathbf{s} = I\Delta \mathbf{s}$$

We can replace $q\,\mathbf{v}$ in the equation above.

Magnetic fields from currents



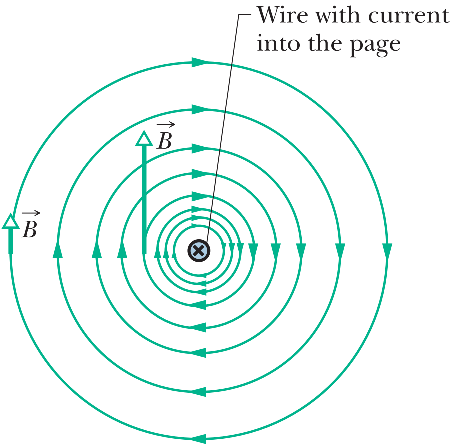
This is another version of the Biot-Savart Law:

$$\mathbf{B}_{\text{seg}} = \frac{\mu_0}{4\pi} \frac{I\,\Delta\mathbf{s} \times \hat{\mathbf{r}}}{r^2}$$

where $\mathbf{B}_{\text{seg}}$ is the magnetic field from a small segment of wire, of length Δs.

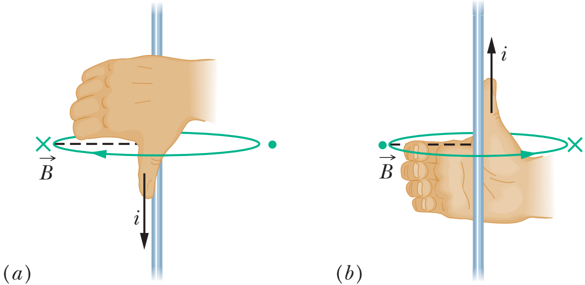
Magnetic fields from currents

Magnetic field around a wire segment, viewed end-on:



Magnetic fields from currents

How to determine the direction of the field lines (right-hand rule):



Magnetic field from a long straight wire

The Biot-Savart Law,

$$\mathbf{B}_{\text{seg}} = \frac{\mu_0}{4\pi} \frac{I \, \Delta\mathbf{s} \times \hat{\mathbf{r}}}{r^2}$$

implies what the magnetic field is at a perpendicular distance R from an **infinitely long straight wire**:

$$\mathbf{B} = \frac{\mu_0 I}{2\pi R}$$

(The proof requires some calculus.)

Force between 2 wires

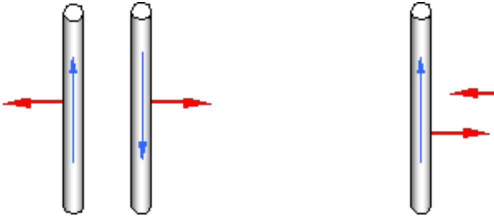
With two current carrying wires, each creates its own magnetic field:

$$\mathbf{B} = \frac{\mu_0 I}{2\pi R}$$

The result is that the wires interact, much like two bar magnets producing magnetic fields would.

Force between 2 wires

Currents in opposite directions repel, currents in the same direction attract.



[1]Figure from salisbury.edu.

Force between 2 wires

It is a bit more intuitive to think about the force per unit length on the wires (since longer wires will experience larger forces).

The force per unit length on a wire due to another parallel wire at a distance d:

$$\frac{F_B}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi d}$$

Force between 2 wires

It is a bit more intuitive to think about the force per unit length on the wires (since longer wires will experience larger forces).

The force per unit length on a wire due to another parallel wire at a distance d:

$$\frac{F_B}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi d}$$

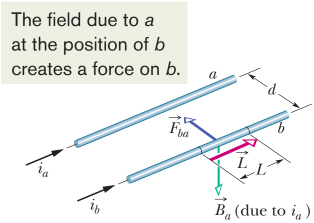
Where does this come from?

The force on a current carrying wire is:

$$\mathbf{F} = I\mathbf{L} \times \mathbf{B}$$

Force between 2 wires

Suppose that wire a produces a field: $B_a = \frac{\mu_0 I_a}{2\pi d}$

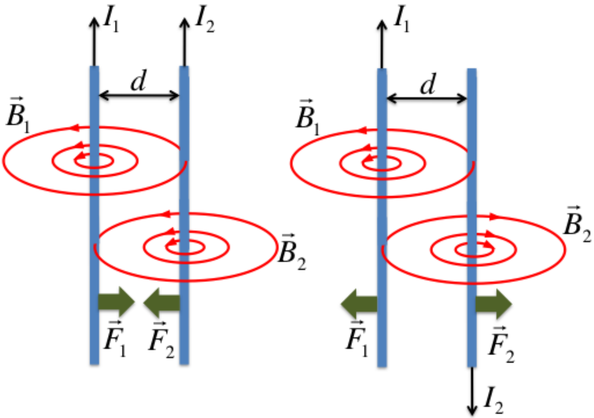


The force on wire b is:

$$F = I_b L \frac{\mu_0 I_a}{2\pi d} \sin(90^\circ)$$

$$\frac{F_B}{L} = \frac{\mu_0 I_a I_b}{2\pi d}$$

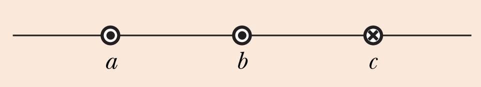
Force between 2 wires



[1]Figure from Stonebrook Physics ic.sunysb.edu.

Question

The figure here shows three long, straight, parallel, equally spaced wires with identical currents either into or out of the page. Rank the wires according to the magnitude of the force on each due to the currents in the other two wires, greatest first.



A a, b, c

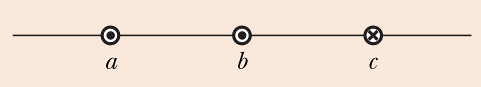
B b, c, a

C c, b, a

[1] Halliday, Resnick, Walker, pg 771.

Question

The figure here shows three long, straight, parallel, equally spaced wires with identical currents either into or out of the page. Rank the wires according to the magnitude of the force on each due to the currents in the other two wires, greatest first.



A a, b, c

B b, c, a ←

C c, b, a

[1] Halliday, Resnick, Walker, pg 771.

Magnetic Permeability

A constant we will need is:

$$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{Tm/A}$$

μ_0 is called the **magnetic permeability of free space**.

It arises when we look at magnetic fields because of our choice of SI units.

Whenever we use μ_0 we assume we are considering the magnetic field to be in a vacuum or air.

μ_0 is not the magnetic dipole moment $\boldsymbol{\mu}$!
Another notation coincidence.

Definition of the Ampère (Amp)

This relation:

$$\frac{F_B}{L} = \frac{\mu_0 I_1 I_2}{2\pi d}$$

gives us the formal definition of the Ampère.

Ampère Unit

Two long parallel wires separated by 1 m are said to each carry 1 A of current when the force per unit length on each wire is 2×10^{-7} N/m.

Gauss's Law for Magnetic Fields

There is more we can say about magnetic fields.

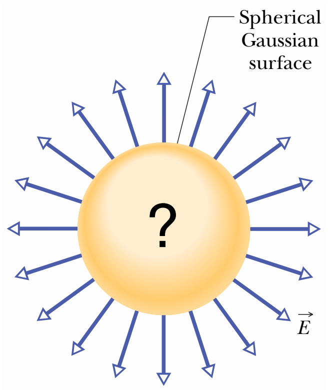
When studying electric fields we used Gauss's law to understand the how the electric field looked around a point charge.

There is also Gauss's law for magnetic fields, but it tells us something different about magnetic fields.

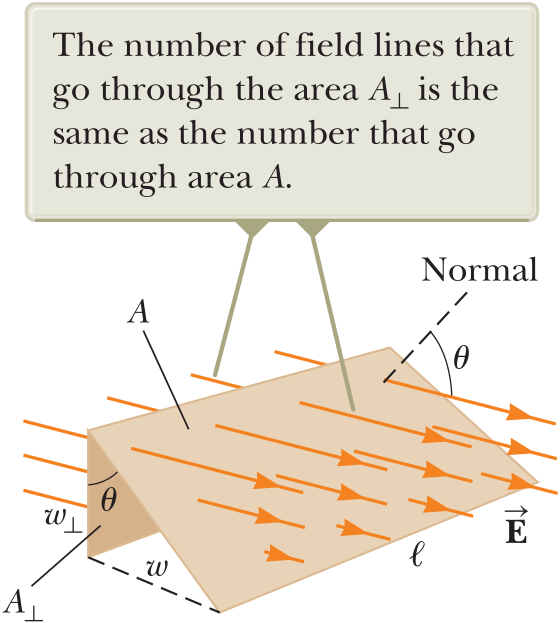
Reminder about Gauss's law for electric fields...

Gauss's Law for Electric Fields basic idea

Gauss's law relates the electric field across a closed surface (*eg.* a sphere) to the amount of net charge enclosed by the surface.



Flux



The number of field lines that go through the area $A_\perp$ is the same as the number that go through area A.
Normal
A
θ
θ
$w_\perp$
$A_\perp$
w
ℓ
$\vec{\mathbf{E}}$

Reminder: Gauss's Law for Electric Fields

Gauss's Law for Electric fields:

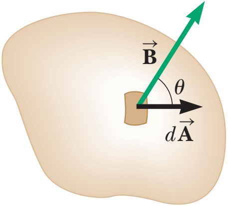
$$\Phi_E = \oint \mathbf{E} \cdot \mathrm{d}\mathbf{A} = \frac{q_{\mathrm{enc}}}{\epsilon_0}$$

The electric flux through a closed surface is equal to the charge enclosed by the surface, divided by ϵ_0.

There is a similar expression for magnetic flux!

First we must define magnetic flux, Φ_B.

Magnetic Flux



Magnetic flux

The magnetic flux of a magnetic field through a surface **A** is

$$\Phi_B = \sum \mathbf{B} \cdot \Delta\mathbf{A}$$

Units: Tm^2

If the surface is a flat plane and **B** is uniform, that just reduces to:

$$\Phi_B = \mathbf{B} \cdot \mathbf{A}$$

Gauss's Law for Magnetic Fields

Gauss's Law for magnetic fields.:

$$\oint \mathbf{B} \cdot \mathrm{d}\mathbf{A} = 0$$

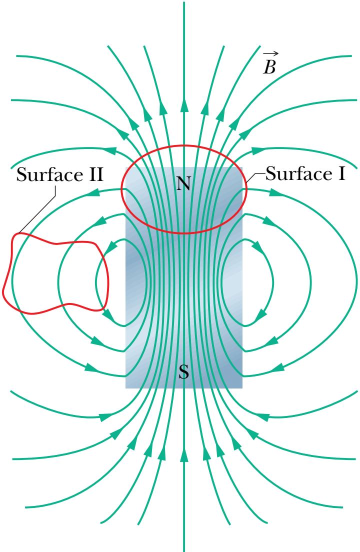
Where the integral is taken over a closed surface A. (This is like a sum over the flux through many small areas.)

We can interpret it as an assertion that magnetic monopoles do not exist.

The magnetic field has no sources or sinks.

Gauss's Law for Magnetic Fields

$$\oint \mathbf{B} \cdot d\mathbf{A} = 0$$



B-Field around a wire revisited

Gauss's law will not help us find the strength of the B-field around a wire, or wires carrying current: the flux through any closed surface will be zero.

Another law can: Ampère's Law.

Summary

- field from a moving charge
- field from a current
- force between two parallel wires
- Guass's law

Homework Halliday, Resnick, Walker:

- PREVIOUS: Ch 28, Problems: 54, 55, 57, 61, 65
- NEW: Ch 29, onward from page 783. Questions: 3; Problems: 1, 11, 21, 23