Electricity and Magnetism
Ampère's Law
Motional EMF
Faraday's Law

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Last time

- magnetic fields from moving charges
- magnetic fields around current-carrying wires
- forces between parallel wires
- Gauss's law

Overview

- Ampère's law
- motional emf
- induction
- Faraday's law
- Lenz's law

Gauss's Law for Magnetic Fields

Gauss's Law for magnetic fields.:

$$\oint \mathbf{B} \cdot d\mathbf{A} = 0$$

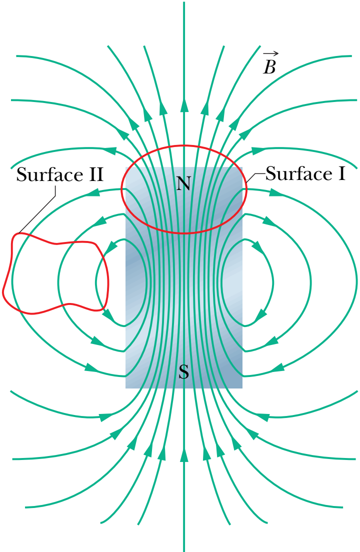
Where the integral is taken over a closed surface A. (This is like a sum over the flux through many small areas.)

We can interpret it as an assertion that magnetic monopoles do not exist.

The magnetic field has no sources or sinks.

Gauss's Law for Magnetic Fields

$$\oint \mathbf{B} \cdot d\mathbf{A} = 0$$



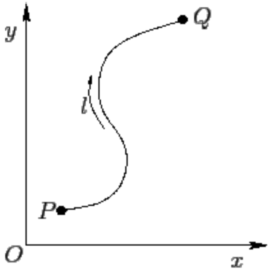
B-Field around a wire revisited

Gauss's law will not help us find the strength of the B-field around a wire, or wires carrying current: the flux through any closed surface will be zero.

Another law can: Ampère's Law.

Line Integrals

To understand Ampère's Law, we first need to understand the basic idea of what a line integral represents.



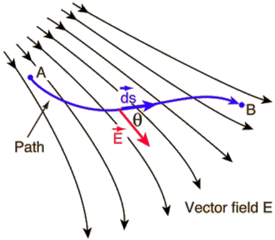
The most basic line integral is just:

$$\ell = \sum_k \Delta s_k = \int_P^Q \mathrm{ds}$$

This is just summing up the length of the line from P to Q.
ℓ is the line length.
Each Δs_k is a little line segment.

Line Integrals

Line integrals involving fields are a little more complicated. Suppose we want to evaluate the dot product between the field vector at each point along the line withe the line segment at that point.



This is a measure of how much the line points along the field.

$$\sum_k \mathbf{E} \cdot \Delta \mathbf{s}_k = \int_A^B \mathbf{E} \cdot d\mathbf{s}$$

Line Integrals

There are two cases that are particularly easy to calculate:

1. The field always points perpendicularly to the path:

$$\int_a^b \mathbf{B} \cdot d\mathbf{s} = 0$$

2. The field always points parallel to the path:

$$\int_a^b \mathbf{B} \cdot d\mathbf{s} = B\ell$$

where ℓ is the path length.

Line Integrals

There is one other special piece of notation used with some line integrals:

$$\oint$$

This symbol means that the integral starts and ends at the same point.

The path is a loop.

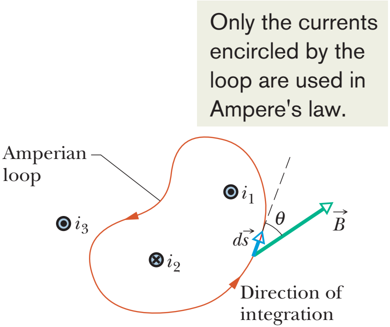
$$\oint \mathbf{B} \cdot \mathrm{d}\mathbf{s}$$

Ampère's Law

For constant currents (magnetostatics):

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{\text{enc}}$$

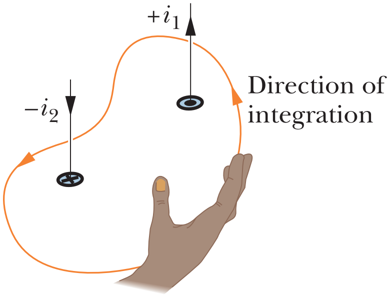
The line integral of the magnetic field around a closed loop is proportional to the current that flows *through* the loop.[1]



[1] That is, the current that flows through any surface bounded by the loop.

Ampère's Law

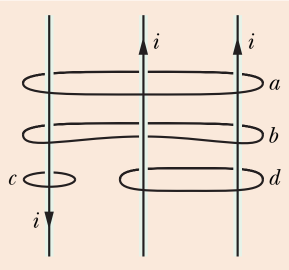
This is how to assign a sign to a current used in Ampere's law.



A current through the loop in the general direction of your outstretched thumb is assigned a plus sign, and a current generally in the opposite direction is assigned a minus sign.

Question

The figure here shows three equal currents i (two parallel and one antiparallel) and four Amperian loops. Rank the loops according to the magnitude of $\oint \mathbf{B} \cdot d\mathbf{s}$ along each, greatest first.



A a, b, c, d

B d, b, c, a

C (a and b), d, c

D d, (a and c), b

[1]Halliday, Resnick, Walker, page 773.

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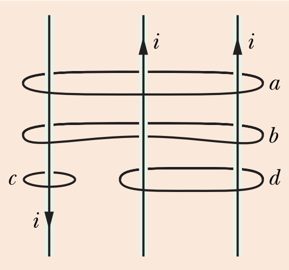

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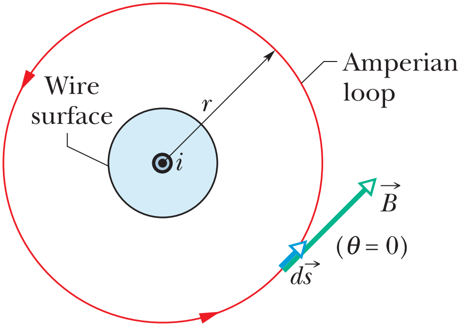
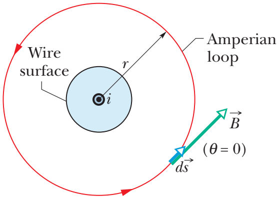
Ampère's Law and the Magnetic Field from a Current Outside a wire

Suppose we want to know the magnitude of the magnetic field at a distance r outside a wire. Using Ampère's Law?



Ampère's Law and the Magnetic Field from a Current Outside a wire



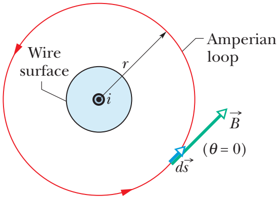
Ampère's Law:

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{enc}$$

To find the B-field at a distance r from the wire's center choose a circular path of radius r.

By cylindrical symmetry, everywhere along the circle $\mathbf{B} \cdot d\mathbf{s}$ is constant.

Ampère's Law and the Magnetic Field from a Current Outside a wire



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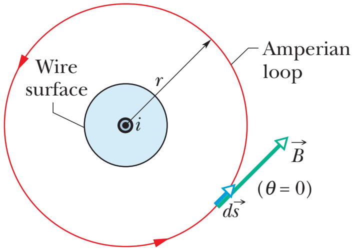
By cylindrical symmetry, everywhere along the circle $\mathbf{B} \cdot \mathrm{d}\mathbf{s}$ is constant.

The magnetic field lines must form a closed loop $\Rightarrow$ $\mathbf{B} \cdot \mathrm{d}\mathbf{s} = B\,\mathrm{d}s$.

Ampère's Law and the Magnetic Field from a Current Outside a wire



$$B \oint ds = \mu_0 I_{enc}$$

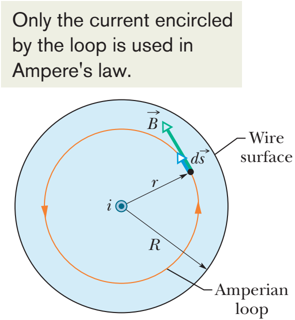
$$B(2\pi r) = \mu_0 I$$

And again we get

$$B = \frac{\mu_0 I}{2\pi r}$$

Ampère's Law and the Magnetic Field from a Current Inside a wire

We can also use Ampère's Law in another context, where using the Biot-Savart Law is harder.



Now we place the Amperian loop *inside* the wire.

We still have $\oint \mathbf{B} \cdot d\mathbf{s} = 2\pi r B$, but now the current that flows through the loop is reduced.

Ampère’s Law and the Magnetic Field from a Current Inside a wire

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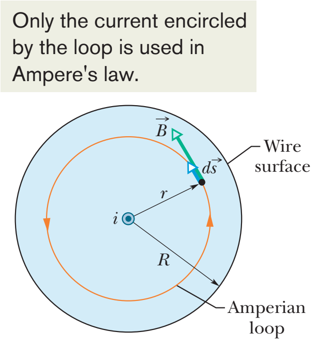
Assuming the wire has uniform resistivity, I_{enc}:



Ampère's Law and the Magnetic Field from a Current Inside a wire

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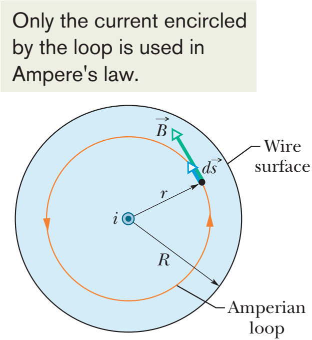


Assuming the wire has uniform resistivity, I_{enc}:

$$I_{\text{enc}} = \frac{\pi r^2}{\pi R^2} I = \frac{r^2}{R^2} I$$

Ampére's Law

$$\oint \mathbf{B} \cdot d\mathbf{s} = 2\pi r B = \mu_0 \frac{r^2}{R^2} I$$

So,

$$B = \frac{\mu_0 I \, r}{2\pi R^2} = \frac{\mu_0 I}{2\pi R} \left(\frac{r}{R} \right)$$

Ampère's Law

For constant currents (magnetostatics):

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{\text{enc}}$$

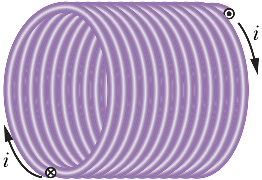
The line integral of the magnetic field around a closed loop is proportional to the current that flows *through* the loop.

Later we will extend this law to deal with the situation where the fields / currents are changing.

Solenoids

solenoid

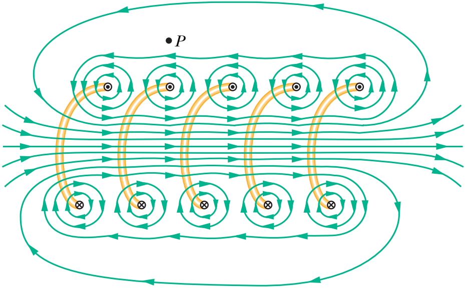
A helical coil of tightly wound wire that can carry a current.



turn

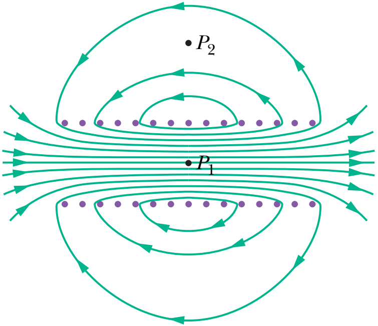
A single complete loop of wire in a solenoid. “This solenoid has 10 turns,” means it has 10 complete loops.

Magnetic Field inside and around a solenoid



Each turn of wire locally has a circular magnetic field around it. The fields from all the wires add together to create very dense field lines inside the solenoid.

Magnetic Field of a solenoid

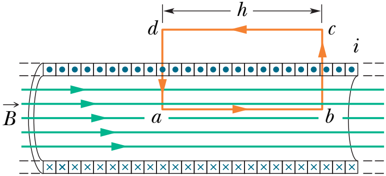


The wires on opposite sides (top and bottom in the picture) have currents in opposite directions. The fields add up between them, but cancel out outside of them.

Magnetic Field of an ideal solenoid

In an ideal solenoid (with infinite length) the field outside is negligible and inside is uniform. (Similar to a capacitor!)

Can use an Amperian loop to find the B-field inside:



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Magnetic Field of an ideal solenoid

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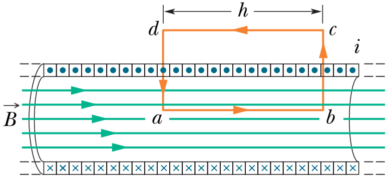


$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{enc}$$

Here, suppose there are n turns per unit length in the solenoid, then $I_{enc} = Inh$

$$Bh = \mu_0 Inh$$

Inside an ideal solenoid:

$$B = \mu_0 In$$

Question

For what current through a solenoid with 50 turns per centimeter will the magnetic field be 20 mT?

Question

For what current through a solenoid with 50 turns per centimeter will the magnetic field be 20 mT?

$$I = 3.18 \text{ A}$$

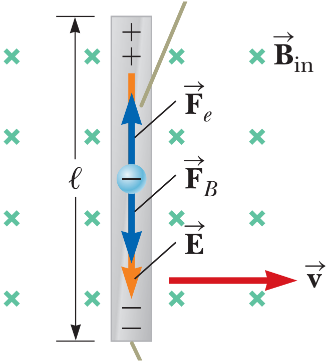
Induction and Inductance

Changing magnetic fields can put forces on charges.

To see how, we start by looking again at conductors moving in magnetic fields.

Motional EMF

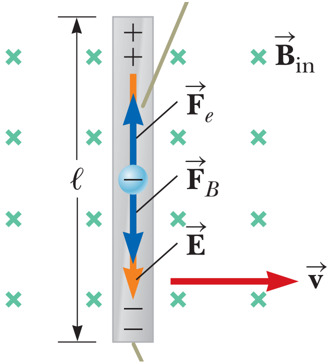
If a conductor moves through a magnetic field at an angle to the field, an emf is induced across the conductor.



There are two ways to see this:

1. force on conduction charges $\mathbf{F} = q\,\mathbf{v} \times \mathbf{B}$
2. in the rest frame of the conductor there is also an electric field

Motional EMF



Once the charge distribution reaches equilibrium, the net force on each charge:

$$\mathbf{F}_{net} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) = 0$$

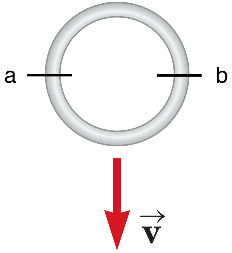
$$E = vB \quad (\mathbf{v} \perp \mathbf{B})$$

$$\frac{\mathcal{E}}{\ell} = vB$$

$$\mathcal{E} = vB\ell$$

Motional emf and loops

Imagine a loop of wire that moves in a **uniform magnetic field**, **B**, directed **into** the page.

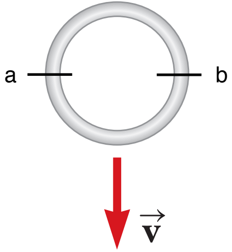


Imagine the loop is composed of a pair of curved rods cut along the lines shown.

Which way (left or right) the emf directed in the top half? In the bottom? How do the magnitudes compare?

Motional emf and loops

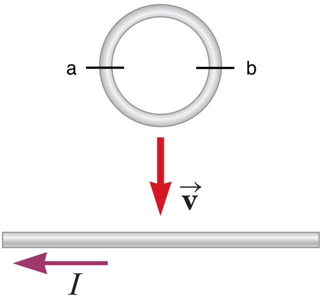
Imagine a loop of wire that moves in a **uniform magnetic field**, **B**, directed **into** the page.



In this case, part of the loop near *a* develops a negative charge and the part near *b* a positive charge, but overall no steady current flows around the loop.

Motional emf and loops

Now imagine a loop of wire that moves in a **non-uniform magnetic field** falling towards a wire.



How do the magnitudes of the emfs in the top and bottom compare?

Motional emf and loops

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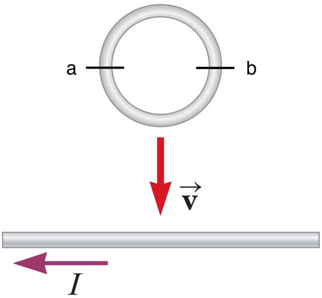
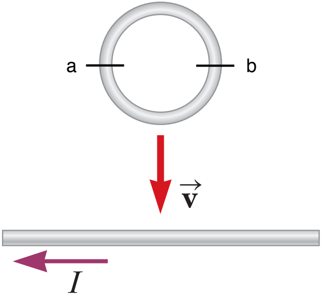


How do the magnitudes of the emfs in the top and bottom compare?

They are not the same! A current can flow.

Motional emf and loops

Now imagine a loop of wire that moves in a **non-uniform magnetic field** falling towards a wire. (Quiz 31.3)



What is the direction of the induced current in the loop of wire?

(A) clockwise

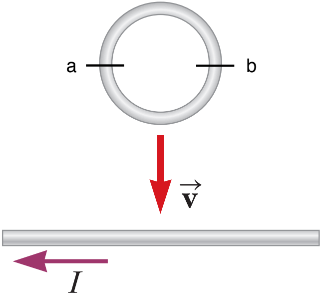
(B) counterclockwise

(C) zero

(D) impossible to determine

Motional emf and loops

Now imagine a loop of wire that moves in a **non-uniform magnetic field** falling towards a wire. (Quiz 31.3)



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Motional emf and loops

What was different in the two cases (uniform vs. non-uniform field)?

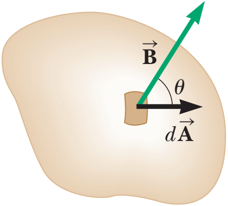
Motional emf and loops

What was different in the two cases (uniform vs. non-uniform field)?

$\rightarrow$ The field at different parts of the loop was different.

$\rightarrow$ The magnetic flux through the loop was changing.

Reminder: Magnetic Flux



Magnetic flux

The magnetic flux of a magnetic field through a surface **A** is

$$\Phi_B = \sum \mathbf{B} \cdot (\Delta \mathbf{A})$$

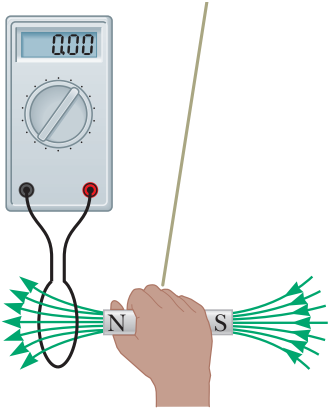
Units: Tm^2

If the surface is a flat plane and **B** is uniform, that just reduces to:

$$\Phi_B = \mathbf{B} \cdot \mathbf{A}$$

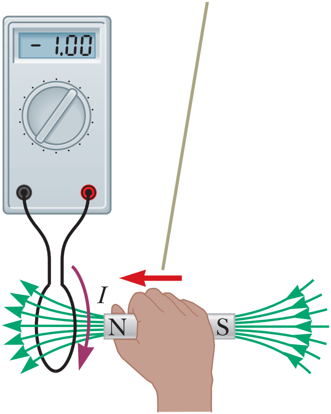
Changing flux and emf

When a magnet is at rest near a loop of wire there is no potential difference across the ends of the wire.



Changing flux and emf

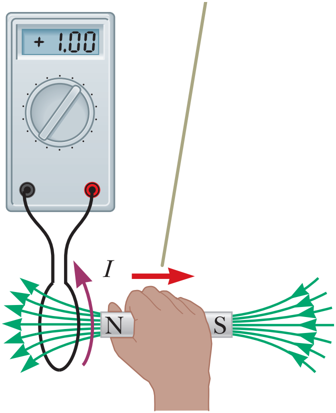
When the north pole of the magnet is moved towards the loop, the magnetic flux increases.



A current flows clockwise in the loop.

Changing flux and emf

When the north pole of the magnet is moved away from the loop, the magnetic flux decreases.



A current flows counterclockwise in the loop.

Faraday's Law

Faraday's Law

If a conducting loop experiences a changing magnetic flux through the area of the loop, an emf $\mathcal{E}_F$ is induced in the loop that is directly proportional to the rate of change of the flux, Φ_B with time.

Faraday's Law for a conducting loop:

$$\mathcal{E} = -\frac{\Delta \Phi_B}{\Delta t}$$

Faraday's Law

Faraday's Law for a coil of N turns:

$$\mathcal{E}_F = -N \frac{\Delta \Phi_B}{\Delta t}$$

if Φ_B is the flux through a single loop.

Changing Magnetic Flux

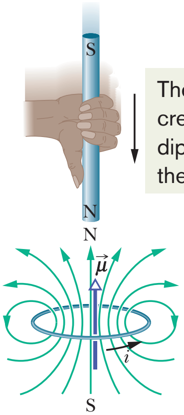
The magnetic flux might change for any of several reasons:

- the magnitude of **B** can change with time,
- the area A enclosed by the loop can change with time, or
- the angle θ between the field and the normal to the loop can change with time.

Lenz's Law

Lenz's Law

An induced current has a direction such that the magnetic field due to the current opposes the change in the magnetic flux that induces the current.



The magnet's motion creates a magnetic dipole that opposes the motion.

Basically, Lenz's law let's us interpret the minus sign in the equation we write to represent Faraday's Law.

$$\mathcal{E} = -\frac{\Delta \Phi_B}{\Delta t}$$

[1]Figure from Halliday, Resnick, Walker, 9th ed.

Summary

- Ampère's law
- motional emf
- Faraday's law
- Lenz's law

Homework

Halliday, Resnick, Walker:

- PREVIOUS: Ch 29, onward from page 783. Questions: 3; Problems: 1, 11, 21, 23
- NEW: Ch 29, Questions: 7; Problems: 35, 43, 45, 51, 53
- NEW: Ch 30, onward from page 816. Questions: 1, 3; Problems: 1, 3, 4.