Electricity and Magnetism
Applying Faraday's Law

Lana Sheridan

De Anza College

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Last time

- Ampere's law
- Faraday's law
- Lenz's law

Overview

- induction and energy transfer
- induced electric fields
- inductance
- self-induction
- RL Circuits

Faraday's Law

Faraday's Law

If a conducting loop experiences a changing magnetic flux through the area of the loop, an emf $\mathcal{E}_F$ is induced in the loop that is directly proportional to the rate of change of the flux, Φ_B with time.

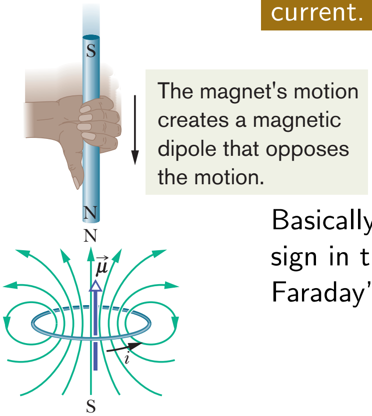
Faraday's Law for a conducting loop:

$$\mathcal{E} = -\frac{\Delta\Phi_B}{\Delta t}$$

Lenz's Law

Lenz's Law

An induced current has a direction such that the magnetic field due to the current opposes the change in the magnetic flux that induces the current.



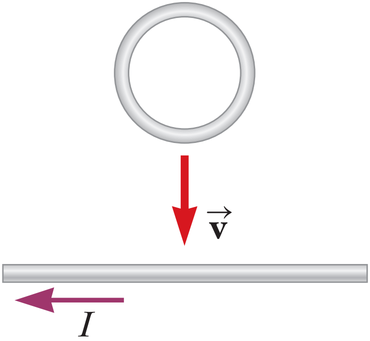
Basically, Lenz's law let's us interpret the minus sign in the equation we write to represent Faraday's Law.

$$\mathcal{E} = -\frac{\Delta\Phi_B}{\Delta t}$$

[1]Figure from Halliday, Resnick, Walker, 9th ed.

Faraday's and Lenz's Laws

What about this case? We found the current should flow counterclockwise.



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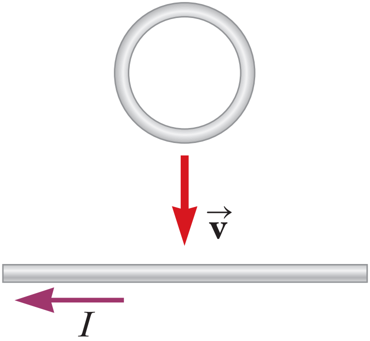


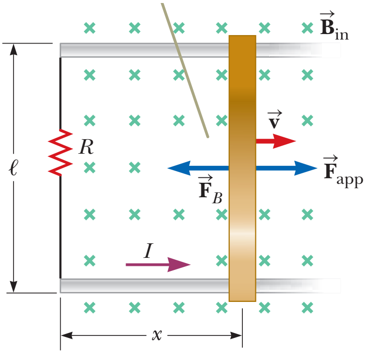
The flux from the wire is into the page and increasing.

The field from the current is out of the page.

There is an upward resistive force on the ring.

Faraday's and Lenz's Laws

Consider a conducting bar placed on conducting rails in a magnetic field, with a resistor (outside the field) completing the circuit.



Using the motional emf approach, what is the induced emf across the bar?

Using Faraday's law, what is the induced emf across the bar?

Faraday's and Lenz's Laws

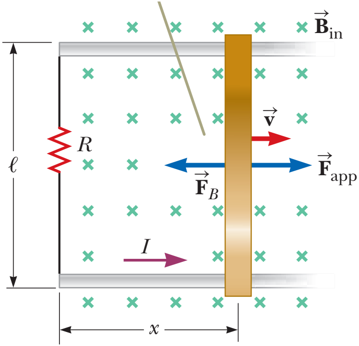
Motional emf:

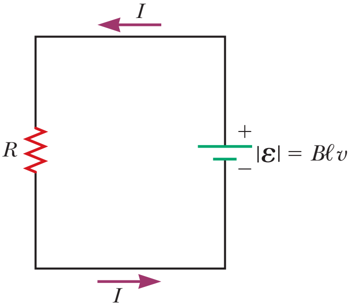
$$\mathcal{E} = Bv\ell$$

upwards

Faraday's Law:

$$\mathcal{E} = -\frac{\Delta\Phi_B}{\Delta t} = -B\ell\frac{\Delta x}{\Delta t} = -Bv\ell$$



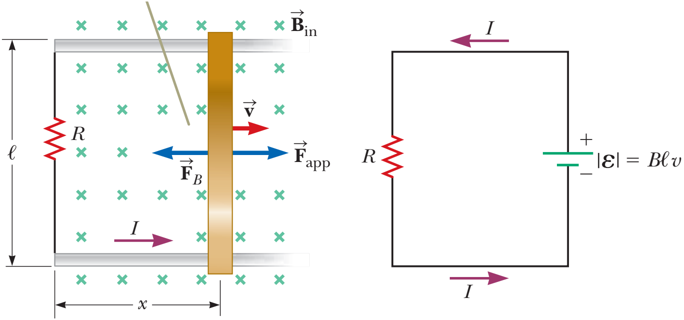


A counterclockwise current begins to flow as the rod moves. (Opposes the field.)

Faraday's and Lenz's Laws

Power is delivered to the resistor as current flows.

That power must come from the force needed to keep the rod in motion.



Prove they are equal.

Faraday's and Lenz's Laws

Power delivered to resistor:

$$P = \frac{\mathcal{E}^2}{R} = \frac{B^2 v^2 \ell^2}{R}$$

Power supplied by applied force needed to keep rod moving with constant velocity v:

$$\mathbf{F}_{\text{net}} = 0 \;\Rightarrow\; \mathbf{F}_{\text{app}} = \mathbf{F}_B$$

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$$\begin{aligned} P &= \mathbf{F}_{\text{app}} \cdot \mathbf{v} \\ &= (I\ell B)v \\ &= \frac{\mathcal{E}}{R} B v \ell \\ &= \frac{B^2 v^2 \ell^2}{R} \end{aligned}$$

Faraday's and Lenz's Laws

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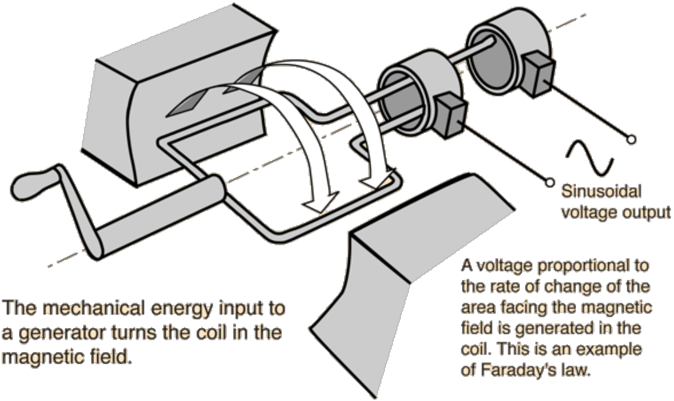
Faraday's and Lenz's Laws

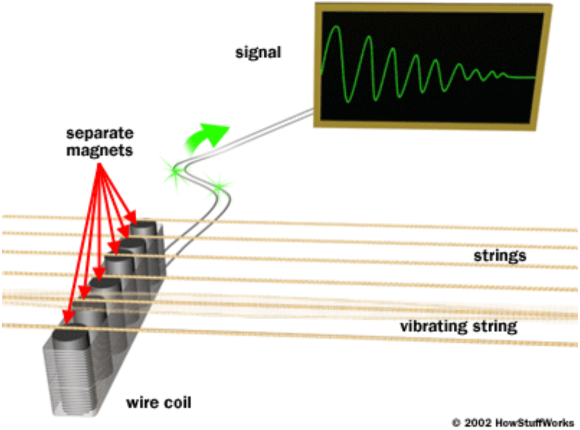
Implication: it is possible to turn mechanical power into electrical power.

Electric Generators



[1] Figure from hyperphysics.phys-arstr.gsu.edu

Electric Guitar Pickups



Strings are made of ferrous metal: steel (iron) or nickel, which become magnetized by the permanent magnets.

Plucked strings create a changing magnetic field that produces a current in the pickup coil.

[1] Figure from HowStuffWorks.

Induction and Energy Transfer to a Wire Loop

When you move a magnet near a loop of wire or a loop of wire near a magnetic field, a force resists the motion.

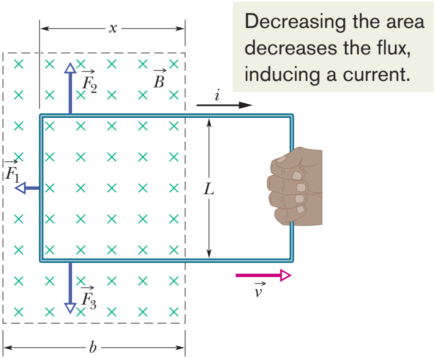
This is due to the induced magnetic dipole in the loop.

Since you must apply a force to overcome the resistance, you do work moving the magnet / loop.

This energy goes to heating the loop of wire.

Energy Transfer

Easy to see for the case of a loop being pulled out of a B-field.

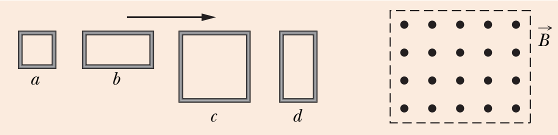


The B-field is uniform within the rectangle and zero outside it.

Loops in B-Fields Question

The figure shows four wire loops, with edge lengths of either L or $2L$. All four loops will move through a region of uniform magnetic field $\mathbf{B}$ (directed out of the page) at the same constant velocity.

Rank the four loops according to the maximum magnitude of the emf induced as they move into the field, greatest first.



A a, b, c, d

B (b and c), (a and d)

C (c and d), (a and b)

D (a and b), (c and d)

---

[1]Halliday, Resnick, Walker, page

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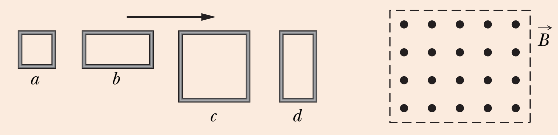


A a, b, c, d

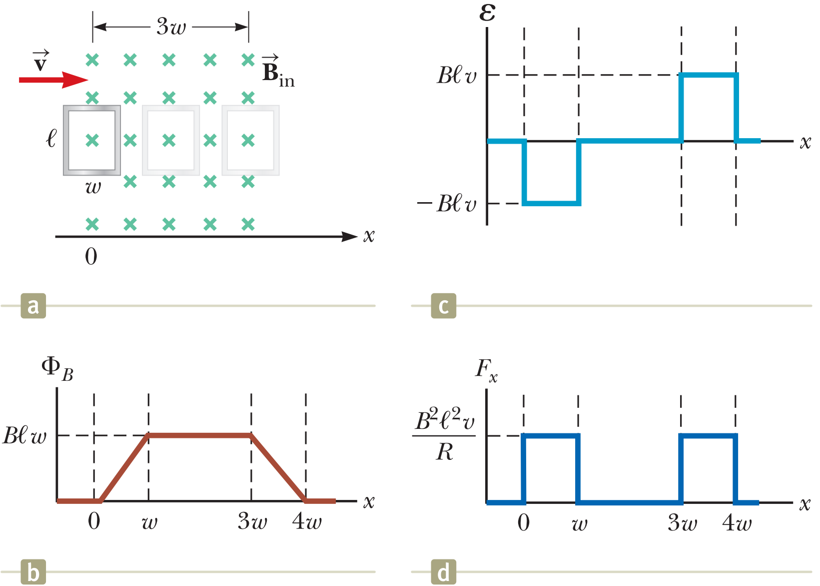
B (b and c), (a and d)

C (c and d), (a and b) $\leftarrow$

D (a and b), (c and d)

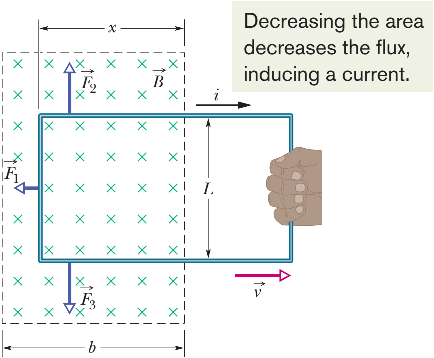
[1]Halliday, Resnick, Walker, page

Loop moving into and out of a B-field



Energy Transfer

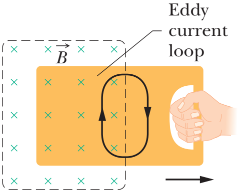
If we pull a loop out of a magnetic field, a current flows in the loop.

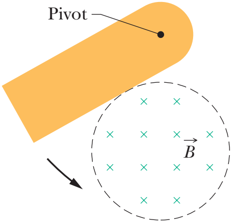


The B-field is uniform within the rectangle and zero outside it.

Eddy Currents

If the wire is replaced by a solid conducting plate, circulations of current form in the plate.





Since the cross section of the plate is larger than that of a similar wire, the resistance will be low, but the current can be high.

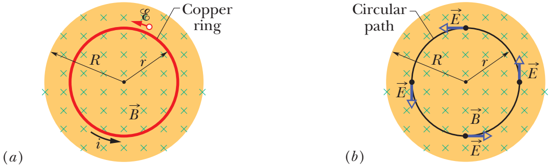
The plate will heat.

Induced Electric Fields

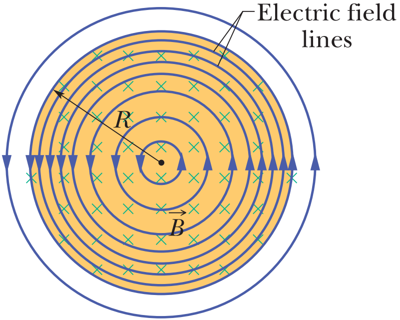
If moving a conductor in a magnetic field causes a current to flow, if must be because the process has created an electric field across the conductor.

The fact that in a conducting plate circulations of current appear tells us that the electric field lines must also makes these circles.

Another way to cause a current and electric field is to change the flux by increasing or decreasing the magnetic field.



Induced Electric Fields



The circulation E-field occurs whether or not a conductor is present: it is the direct result of the changing magnetic flux.

Faraday's Law of Induction (in words)

A changing magnetic field gives rise to an electric field.

Induced emf and the Electric Field

For a closed path, **s**,

$$\mathcal{E} = \oint \mathbf{E} \cdot d\mathbf{s}$$

Notice that by definition $\Delta V = \oint \mathbf{E} \cdot d\mathbf{s} = 0$. Emf does not have this property.

When a charge is moved around a closed path in an electrostatic electric field the work done is zero:

$$W_{\text{app}} = q(\Delta V) = 0$$

Induced emf and the Electric Field

When a charge is moved around a closed path in an electrostatic electric field the work done is zero:

$$W_{\text{app}} = q(\Delta V) = 0$$

For the induced E-field from a changing magnetic flux, the associated force $\mathbf{F} = q\mathbf{E}$ is **not conservative**.

We say the E-field is **nonconservative**.

This is no longer the "electrostatic" case.

Faraday's Law

Faraday's Law for a conducting loop:

$$\mathcal{E} = -\frac{\Delta\Phi_B}{\Delta t}$$

(Differential form:)

$$\mathcal{E} = -\frac{\mathrm{d}\Phi_\mathrm{B}}{\mathrm{dt}}$$

Faraday's Law

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(Differential form:)

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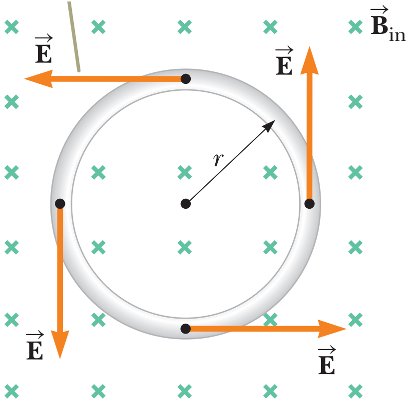
Faraday's law reformulated:

$$\oint \mathbf{E} \cdot \mathrm{d}\mathbf{s} = -\frac{\mathrm{d}\Phi_\mathrm{B}}{\mathrm{d}t}$$

Induced emf and the Electric Field



We can also write Faraday's Law as:

$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$

Reformulated Faraday's Law

We can use this value of $\mathcal{E}$ together with $\mathcal{E} = -\frac{d\Phi_B}{dt}$, to reformulate Faraday's Law.

Faraday's law

$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{\Delta \Phi_B}{\Delta t}$$

(Differential form:)

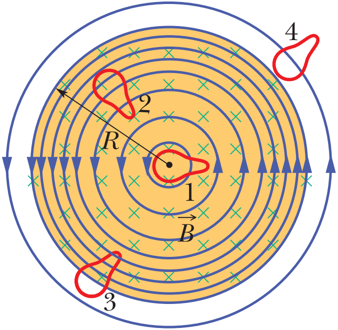
$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$

Faraday's Law Examples

Faraday's law:

$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{\Delta\Phi_B}{\Delta t}$$

(Differential form:) $\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$



Electric Potential

Electric potential has meaning only for electric fields that are the result of static charges; it has no meaning for electric fields that are produced by induction.

For E-fields produced by static charges $\oint \mathbf{E} \cdot d\mathbf{s} = 0$

For induced E-fields, the integral may not be zero.

$$\mathcal{E} = \oint \mathbf{E} \cdot d\mathbf{s}$$

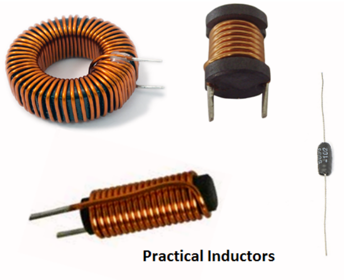
Inductors

A **capacitor** is a device that stores an electric field as a component of a circuit.

inductor

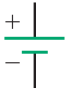
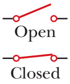
a device that stores a magnetic field in a circuit

It is typically a coil of wire.



Practical Inductors

Circuit component symbols

battery	V	
capacitor	C	
switch	S	
resistor	R	
inductor	L	

Inductance

Just like capacitors have a capacitance that depends on the geometry of the capacitor, inductors have an inductance that depends on their structure.

For a solenoid inductor:

$$L = \mu_0 n^2 A \ell$$

where n is the number of turns per unit length, A is the cross sectional area, and ℓ is the length of the inductor.

Units: henries, H.

1 henry = 1 H = 1 T m^2 / A

Value of μ_0: New units

The magnetic permeability of free space μ_0 is a constant.

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m / A}$$

It can also be written in terms of henries:

$$\mu_0 = 4\pi \times 10^{-7} \text{ H / m}$$

(Remember, $1 \text{ H} = 1 \text{ T m}^2 \text{ / A}$)

Inductance

However, capacitance is defined as being the constant of proportionality relating the charge on the plates to the potential difference across the plates $q = C\,(\Delta V)$. Inductance also is formally defined this way.

inductance

the constant of proportionality relating the *magnetic flux linkage* $(N\Phi_B)$ to the current:

$$N\Phi_B = L\,I \qquad ; \qquad L = \frac{N\Phi_B}{I}$$

Φ_B is the magnetic flux through the coil, and I is the current in the coil.

Summary

- applications of Faraday's law
- inductance
- self-induction
- RL Circuits

Homework Halliday, Resnick, Walker:

- PREV: Ch 29, Questions: 7; Problems: 35, 43, 45, 51, 53
- PREV: Ch 30, Questions: 1, 3; Problems: 1, 3, 4.
- NEW: Ch 30, onward from page 816. Problems: 29, 31, 35, 37