Electricity and Magnetism
Coulomb's Law
Electric Field

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De Anza College

Sept 24, 2015

Last time

- charge
- charge interactions
- charge induction

Warm Up: Worksheet

3. Do both balloons A and B have a charge?

(A) yes

(B) no, neither is charged

(C) at least 1 is charged.

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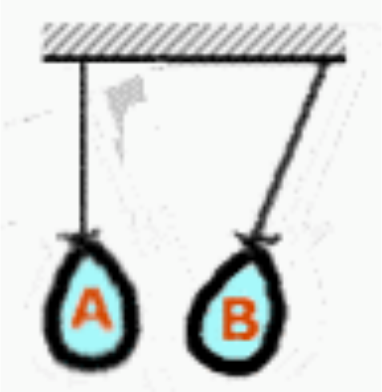

(A) yes

(B) no

Warm Up: Worksheet

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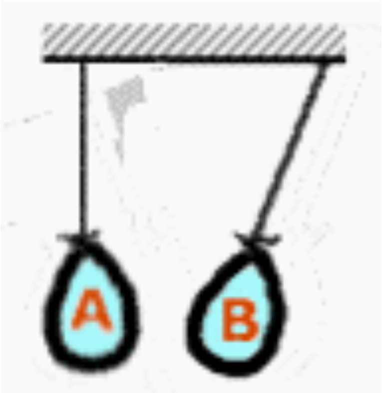

(A) yes

(B) no ← consider Newton's 3rd law

Overview

- Coulomb's Law
- The net force of several charges
- Vector review
- Charge quantization
- Charge conservation
- Current
- Forces at a fundamental level
- Electric field
- Conductors and electric fields

Electrostatic Forces

Charged objects interact via the electrostatic force.

The force that one charge exerts on another can be attractive or repulsive, depending on the signs of the charges.

- Charges with the **same** electrical sign **repel** each other.
- Charges with **opposite** electrical signs **attract** each other.

Charge is written with the symbol q or Q.

Electrostatic Forces

For a pair of point-particles with charges q_1 and q_2, the magnitude of the force on each particle is given by **Coulomb's Law**:

$$F_{1,2} = \frac{k\,|q_1 q_2|}{r^2}$$

k is the electrostatic constant and r is the distance between the two charged particles.

$k = \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 \text{ N m}^2/\text{C}^2$

Electrostatic Forces: Coulomb's Law

$$F_{1,2} = \frac{k\,|q_1 q_2|}{r^2}$$

Remember however, forces are vectors. The vector version of the law is:

$$\mathbf{F}_{1\to 2} = \frac{k\,q_1 q_2}{r^2}\,\hat{\mathbf{r}}_{1\to 2}$$

where $\mathbf{F}_{1\to 2}$ is the force that particle 1 exerts on particle 2, and $\hat{\mathbf{r}}_{1\to 2}$ is a unit vector pointing from particle 1 to particle 2.

Coulomb's Law

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Does this look a bit familiar?

Coulomb's Law

Coulomb's Law:

$$\mathbf{F}_{1\to 2} = \frac{k\, q_1 q_2}{r^2}\, \hat{\mathbf{r}}_{1\to 2}$$

Does this look a bit familiar?

Similar to this?

$$\mathbf{F}_{1\to 2} = -\frac{G\, m_1 m_2}{r^2}\, \hat{\mathbf{r}}_{1\to 2}$$

Coulomb's Law

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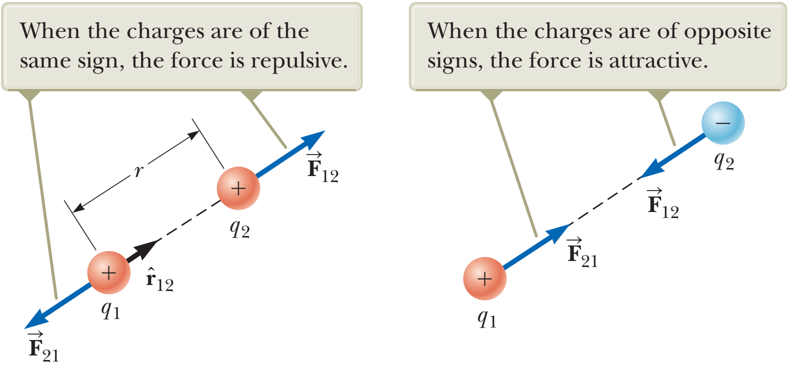


[1]Figure from Serway & Jewett, Physics for Scientists and Engineers, 9th ed.

Electrostatic Constant

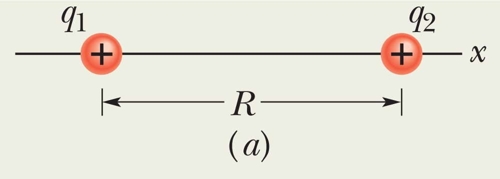
The electrostatic constant is: $k = \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9$ N m^2 C^{-2}

ϵ_0 is called the **permittivity constant** or the **electrical permittivity of free space**.

$\epsilon_0 = 8.85 \times 10^{-12}$ C^2 N^{-1} m^{-2}

Example

(a) Figure 21-8*a* shows two positively charged particles fixed in place on an x axis. The charges are $q_1 = 1.60 \times 10^{-19}$ C and $q_2 = 3.20 \times 10^{-19}$ C, and the particle separation is $R = 0.0200$ m. What are the magnitude and direction of the electrostatic force $\vec{F}_{12}$ on particle 1 from particle 2?



$k = 8.99 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$ or $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$

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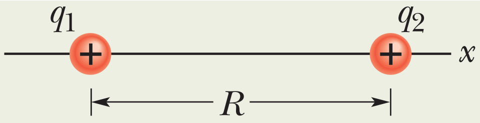


(*a*)

Answer: $\mathbf{F}_{2\to 1} = -1.15 \times 10^{-24}\ \hat{\mathbf{i}}$ N

Force from many charges

Forces from many charges add up to give a net force

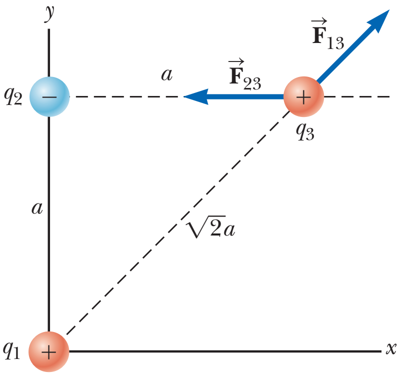
This is (very grandly) called the "principle of superposition".

The net force on particle 1 from particles 2, 3, ... n is:

$$\mathbf{F}_{\text{net},1} = \mathbf{F}_{2\to1} + \mathbf{F}_{3\to1} + ... + \mathbf{F}_{n\to1}$$

Example

Consider three point charges located at the corners of a right triangle as shown, where $q_1 = q_3 = 5.00\ \mu\text{C}$, $q_2 = -2.00\ \mu\text{C}$, and $a = 0.100$ m. Find the resultant force exerted on q_3.



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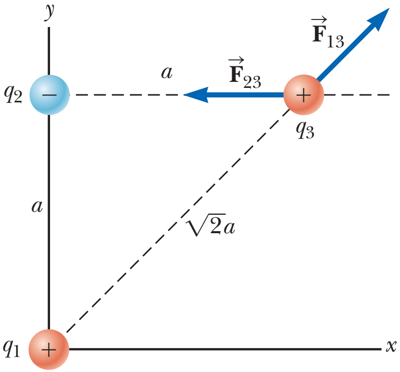

Step 1: What is the force considering ONLY particles 2 and 3?

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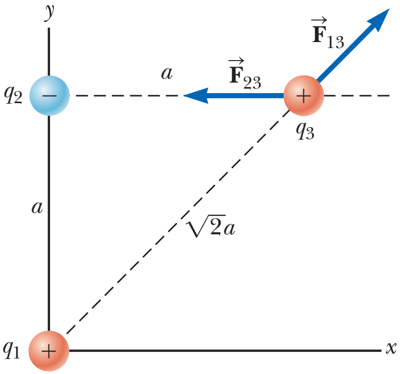

Step 2: What is the force from particle 1 on particle 3? It has 2 components.

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Reminder about Vectors

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A scalar quantity indicates an amount. It is represented by a real number. (Assuming it is a physical quantity.)

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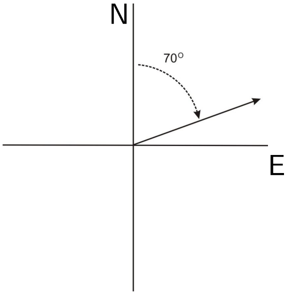
There are many ways to represent a vector.

- a magnitude and (an) angle(s)
- magnitudes in several perpendicular directions

Representing Vectors: Angles

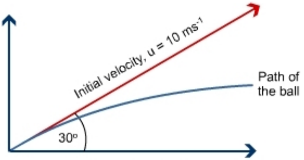
Bearing angles

Example, a plane flies at a bearing of 70°



Reference angles x-axis, CCW

A baseball is thrown at $10\,ms^{-1}$ 30° above the horizontal.



Representing Vectors: Unit Vectors

Another useful way to represent vectors is in terms of *unit vectors.*

Unit vectors have a magnitude of one unit.

In this course, a unit vector $\hat{\mathbf{r}}$ is a one-unit-long vector parallel to the vector $\mathbf{r}$.

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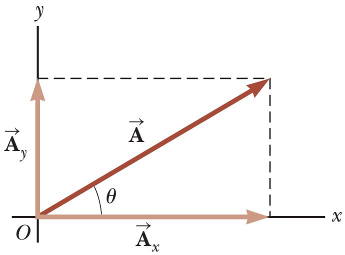
A generic 2 dimensional vector can be written as $\mathbf{v} = a\mathbf{i} + b\mathbf{j}$, where a and b are numbers.

Components

Consider the 2 dimensional vector $\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j}$, where a and b are numbers.

We then say that A_x is the i-component (or x-component) of $\mathbf{A}$ and A_y is the j-component (or y-component) of $\mathbf{A}$.



Notice that $A_x = A\cos\theta$ and $A_y = A\sin\theta$.

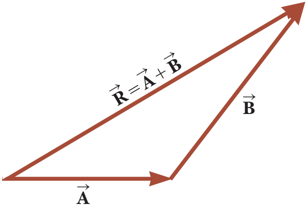
Vectors Properties and Operations

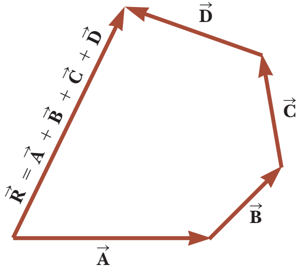
Equality

Vectors **A** = **B** if and only if the magnitudes and directions are the same. (Each component is the same.)

Addition

A + **B**

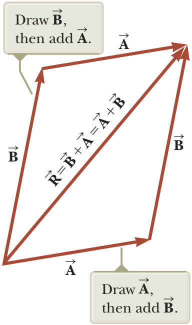




Vectors Properties and Operations

A Property of Addition

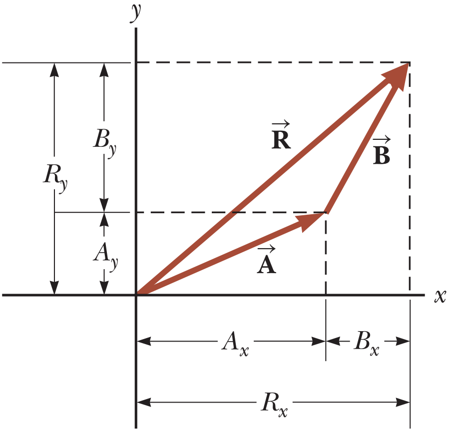
$\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$ (commutative)



Vectors Properties and Operations

Doing addition:

Almost always the right answer is to break each vector into components and sum each component independently.



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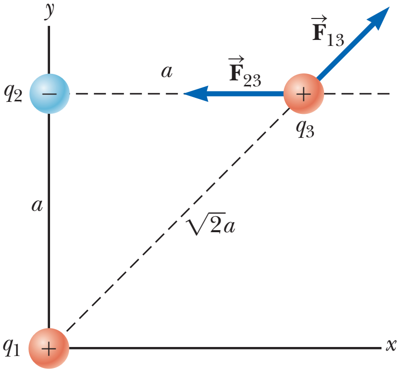

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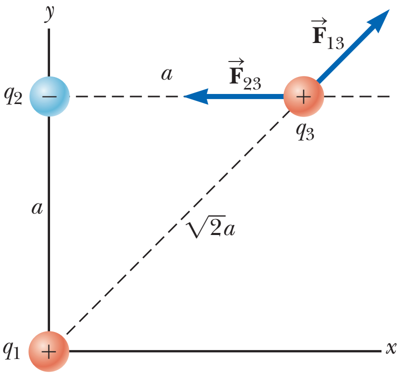

Answer: $\mathbf{F}_{\text{net},3} = (-1.04\ \mathbf{i} + 7.94\ \mathbf{j})$ N

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Charge is Quantized

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A physical quantity is said to be **quantized** if if can only take discrete values.

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It is not.

Just like water has a smallest unit, the H_2O molecule, charge has a smallest unit, written e, the *elementary charge.*

$$e = 1.602 \times 10^{-19}\ \text{C}$$

Basic Unit of Charge

The *elementary charge*.

$$e = 1.602 \times 10^{-19}\ \text{C}$$

Any charge must be

$$q = ne\ , \qquad n \in \mathbb{Z}$$

The charge of an electron is $-e$ and the proton has a charge $+e$.

Question

Initially, sphere A has a charge of $-50e$ and sphere B has a charge of $20e$. The spheres are made of conducting material and are identical in size. If the spheres then touch, what is the resulting charge on sphere A?

(A) $-50e$

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(C) $-15e$

(D) $20e$

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Conservation of Charge

Charge can move from one body to another but the net charge of an isolated system never changes.

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What other quantities are conserved?

Conservation of Charge

One interesting phenomenon that shows the conservation of charge is *pair production*.

A gamma ray (very high energy photon) converts into an electron and a positron (anti-electron):

$$\gamma \rightarrow e^- + e^+$$

New mass is created out of light, but charge is still conserved!

Current

Current is the the rate of flow of charge.

Current is written with the symbol I or i.

$$i = \frac{\Delta q}{\Delta t}$$

(If you like calculus, use $i = \frac{\mathrm{dq}}{\mathrm{dt}}$.)

Coulombs and Ampères

The unit for current is the Ampère, or more commonly, “Amp”.

Using the definition for current, 1 A = 1 C / 1 s.

Therefore, we can formally define the unit of charge in terms of the unit of current:

1 C = (1 A)(1 s)

Question

pg 574, #4

4 Figure 21-15 shows two charged particles on an axis. The charges are free to move. However, a third charged particle can be placed at a certain point such that all three particles are then in equilibrium. (a) Is that point to the left of the first two particles, to their right, or between them? (b) Should the third particle be positively or negatively charged? (c) Is the equilibrium stable or unstable?

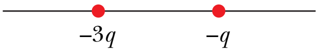
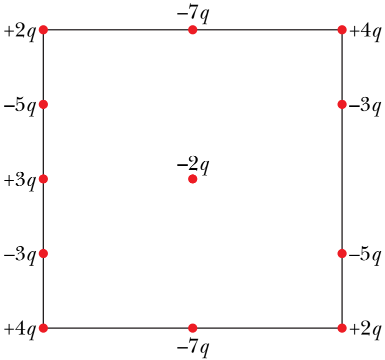


Fig. 21-15 Question 4.

Question

pg 574, #10

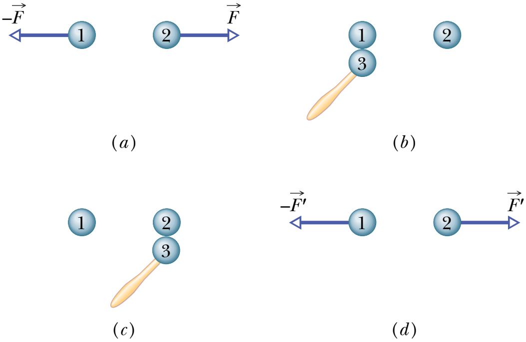
10 In Fig. 21-20, a central particle of charge $-2q$ is surrounded by a square array of charged particles, separated by either distance d or $d/2$ along the perimeter of the square. What are the magnitude and direction of the net electrostatic force on the central particle due to the other particles? (*Hint:* Consideration of symmetry can greatly reduce the amount of work required here.)



Question

pg 575, #2

•2 Identical isolated conducting spheres 1 and 2 have equal charges and are separated by a distance that is large compared with their diameters (Fig. 21-21*a*). The electrostatic force acting on sphere 2 due to sphere 1 is $\vec{F}$. Suppose now that a third identical sphere 3, having an insulating handle and initially neutral, is touched first to sphere 1 (Fig. 21-21*b*), then to sphere 2 (Fig. 21-21*c*), and finally removed (Fig. 21-21*d*). The electrostatic force that now acts on sphere 2 has magnitude F'. What is the ratio F'/F?



Forces at a Fundamental Level

Often people think about two kinds of forces: contact forces and field forces (*ie.* forces that act at a distance).

In mechanics problems, all forces except gravity are from direct contact.

Gravity is a field force.

The electric and magnetic forces are also field forces.

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And actually, at a fundamental level, *all* forces that we know of are field forces.

Forces at a Fundamental Level

Contact forces are a result of electrostatic repulsion at very small scales.

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Fundamental forces:

Force	$\sim$ Rel. strength	Range (m)	Attract/Repel	Carrier
Gravitational	10^{-38}	∞	attractive	graviton
Electromagnetic	10^{-2}	∞	attr. & rep.	photon
Weak Nuclear	10^{-13}	$< 10^{-18}$	attr. & rep.	W^+, W^-, Z^0
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Gravity is actually quite a weak force, but it is the only one that (typically) matters on large scales - charges cancel out!

Fields

field

A field is any kind of physical quantity that has values specified at every point in space and time.

Fields

In EM we have vector fields. The electrostatic force is mediated by a vector field.

vector field

A field is any kind of physical quantity that has values specified *as vectors* at every point in space and time.

Fields

Fields were first introduced as a calculation tool.

A force-field can be used to identify the force a particular particle will feel at a certain point in space and time based on the other objects in its environment that it will interact with.

Imagine a charge q_0. We want to know the force it would feel if we put it at a specific location.

Fields

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A force-field can be used to identify the force a particular particle will feel at a certain point in space and time based on the other objects in its environment that it will interact with.

Imagine a charge q_0. We want to know the force it would feel if we put it at a specific location.

The electric field $\mathbf{E}$ at that point will tell us that!

$$\mathbf{F} = q_0\mathbf{E}$$

Fields

The source of the field could be another charge.

We do not need a description of the sources of the field to describe what their effect is on our particle. All we need to know if the field!

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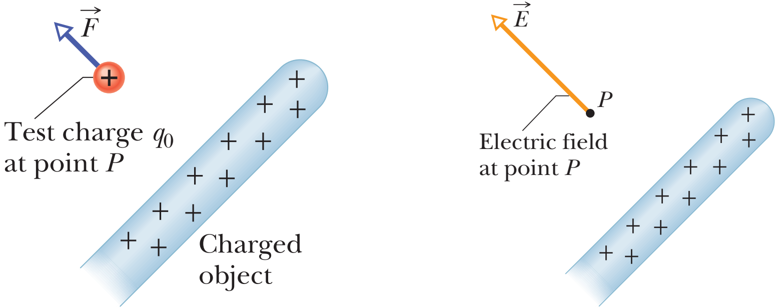
We do not need a description of the sources of the field to describe what their effect is on our particle. All we need to know if the field!

This is also true for gravity. We do not need the mass of the Earth to know something's weight:

$$\mathbf{F}_G = m_0\mathbf{g} \qquad \mathbf{F}_E = q_0\mathbf{E}$$

Force from a Field



$$\mathbf{F} = q_0 \mathbf{E}$$

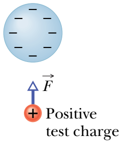
but also:

$$\mathbf{E} = \frac{\mathbf{F}}{q_0}$$

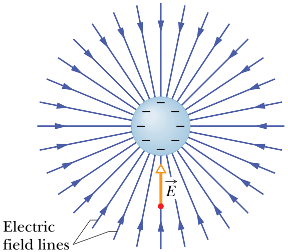
[1]Figure from Halliday, Resnick, Walker.

Field Lines

Fields are drawn with lines showing the direction of force that a test particle will feel at that point. The density of the lines at that point in the diagram indicates the approximate magnitude of the force at that point.



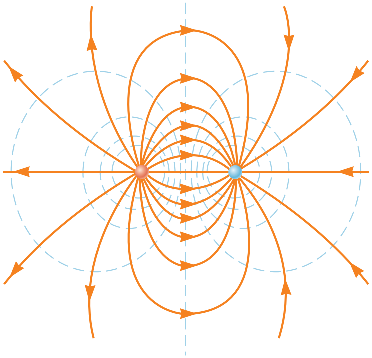
(*a*)



(*b*)

Field Lines

The electrostatic field caused by an electric dipole system looks something like:

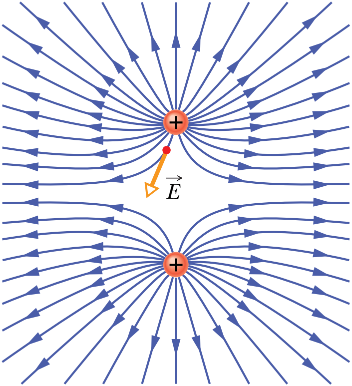


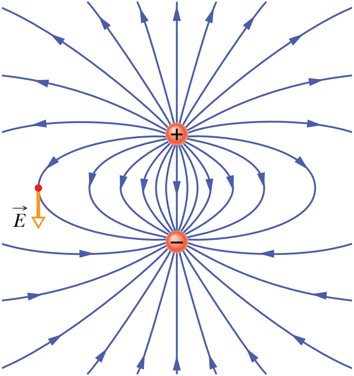
Notice that the lines point **outward** from a positive charge and **inward** toward a negative charge.

[1]Figure from Serway & Jewett

Field Lines

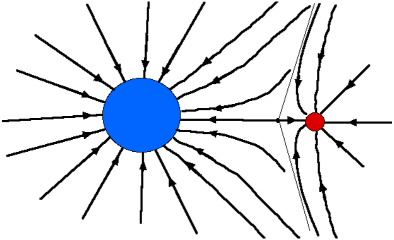
Compare the electrostatic fields for two like charges and two opposite charges:

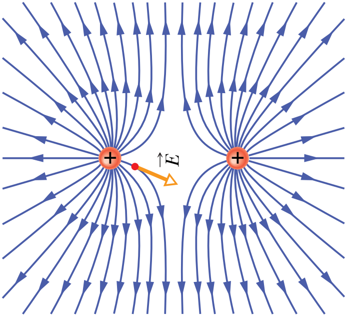




Field Lines

Compare the fields for gravity in an Earth-Sun system and electrostatic repulsion of two charges:

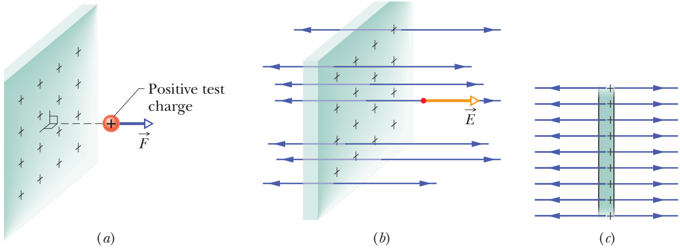




[1]Gravity figure from http://www.launc.tased.edu.au ; Charge from Halliday, Resnick, Walker

Field Lines

Imagine an infinite sheet of charge. The lines point **outward** from the positively charged sheet.



[1]Figure from Halliday, Resnick, Walker.

Field from a Point Charge

We want an expression for the electric field from a point charge, q.

Using **Coulomb's Law** the force on the test particle is $\mathbf{F}_{\to 0} = \frac{k\, q q_0}{r^2}\hat{\mathbf{r}}$.

$$\mathbf{E} = \frac{\mathbf{F}}{q_0} = \left(\frac{1}{\cancel{q_0}}\right)\frac{k\, q\, \cancel{q_0}}{r^2}\hat{\mathbf{r}}$$

The field at a displacement $\mathbf{r}$ from a charge q is:

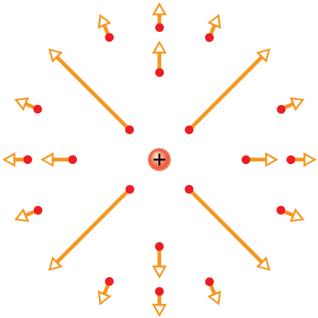
$$\mathbf{E} = \frac{k\, q}{r^2}\hat{\mathbf{r}}$$

Field from a Point Charge

The field at a displacement **r** from a charge q is:

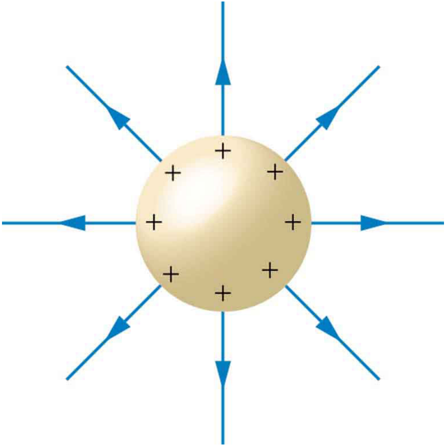
$$\mathbf{E} = \frac{k\,q}{r^2}\hat{\mathbf{r}}$$

This is a vector field:



Charges and Conductors

Excess charge sits on the outside surface of a conductor.

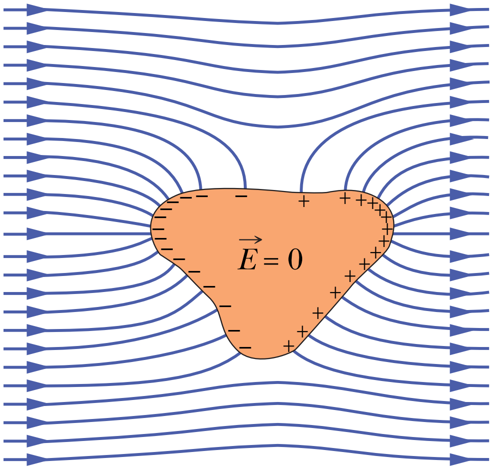


The electric field lines are perpendicular to the surface.

[1]Figure from OpenStax College Physics.

Conductors and Electric fields

Consider a **neutral conductor** placed in an electric field:



Conductors and Electric fields

Electric fields exert forces on free charges in conductors.

Each charge keeps moving until:

1. the charges reaches the edge of the conductor and can move no further OR
2. the field is cancelled out!

Inside a conducting object, the electric field is zero!

Faraday Cages

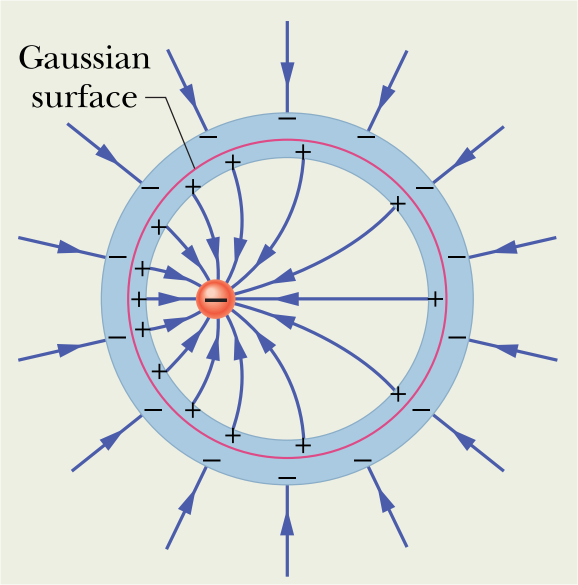
A conducting shell can shield the interior from even very strong electric fields.

[1]Photo from Halliday, Resnick, Walker

Faraday Cages

[1]Photo found on TheDailySheeple, credits unknown.

Charges Inside Conductors: The Faraday Ice Pail



Summary

- Coulomb's law
- Quantization of charge
- Charge conservation
- Current
- electric field
- field of a point charge

Homework

worksheets:
physicsclassroom.com/getattachment/curriculum/estatics/...

- ...static5.pdf
- ...static7.pdf

Halliday, Resnick, Walker:

- Ch 21, onward from page 573. Questions: 1; Sec Qs: 3, 9, 27, 42 & 43