Electricity and Magnetism
Inductance
Transformers
Maxwell's Laws

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Last time

- Ampere's law
- Faraday's law
- Lenz's law

Overview

- induction and energy transfer
- induced electric fields
- inductance
- self-induction
- RL Circuits

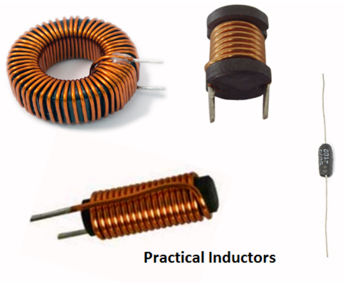
Inductors

A **capacitor** is a device that stores an electric field as a component of a circuit.

inductor

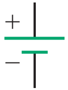
a device that stores a magnetic field in a circuit

It is typically a coil of wire.



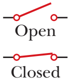
Circuit component symbols

battery V



capacitor C

switch S



resistor R

inductor L

Inductance

Just like capacitors have a capacitance that depends on the geometry of the capacitor, inductors have an inductance that depends on their structure.

For a solenoid inductor:

$$L = \mu_0 n^2 A \ell$$

where n is the number of turns per unit length, A is the cross sectional area, and ℓ is the length of the inductor.

Units: henries, H.

1 henry = 1 H = 1 T m^2 / A

Value of μ_0: New units

The magnetic permeability of free space μ_0 is a constant.

$$\mu_0 = 4\pi \times 10^{-7} \text{ T m / A}$$

It can also be written in terms of henries:

$$\mu_0 = 4\pi \times 10^{-7} \text{ H / m}$$

(Remember, $1 \text{ H} = 1 \text{ T m}^2 / \text{A}$)

Inductance

However, capacitance is defined as being the constant of proportionality relating the charge on the plates to the potential difference across the plates $q = C\,(\Delta V)$. Inductance also is formally defined this way.

inductance

the constant of proportionality relating the *magnetic flux linkage* $(N\Phi_B)$ to the current:

$$N\Phi_B = L\,I \qquad ; \qquad L = \frac{N\Phi_B}{I}$$

Φ_B is the magnetic flux through the coil, and I is the current in the coil.

Inductance of Solenoid Inductors

Suppose now that the only source of magnetic flux in the solenoid is the flux produced by a current in the wire.

Then the field produced within the solenoid is:

$$B = \mu_0 I n$$

where n is the number of turns per unit length.

That means the flux will be:

$$\Phi_B = BA\cos(0^\circ) = BA = \mu_0 I n A$$

where A is the cross sectional area of the solenoid.

Inductance of Solenoid Inductors

$$L = \frac{N\Phi_B}{I}$$

Replacing $N = n\ell$, $\Phi_B = \mu_0 I n A$:

$$L = \frac{n\ell(\mu_0 I n)A}{I}$$

So we confirm our expression for a solenoid inductor:

$$L = \mu_0 n^2 A\ell$$

Induction from an external flux vs Self-Induction

So far we have thought about the effect of a changing magnetic flux on the E-field and emf produced in some region.

We also defined induction by acknowledging that a current in a solenoid will produce a magnetic flux, and relating that to the current in the coil:

$$N\Phi_B = L\,I$$

In general, the magnetic flux $\Phi_B = \mathbf{B} \cdot \mathbf{A}$, could be due not only to the B-field produced by the current in the wire, but also have an additional external source.

If it does not, we say L is the *self-inductance* of the inductor. (This is usually how the symbol L is used.)

Self-Induction

When the current in the solenoid circuit is changing there is a (self-) induced emf in the coil.

From Faraday's Law, we have

$$\mathcal{E} = -\frac{\Delta(N\Phi_B)}{\Delta t}$$

Since L is a constant for a particular inductor,

$$\mathcal{E}_L = -L\frac{\Delta i}{\Delta t}$$

(Derivative form:)

$$\mathcal{E}_L = -L\,\frac{\mathrm{di}}{\mathrm{dt}}$$

The emf opposes the change in current.

Inductors vs. Resistors

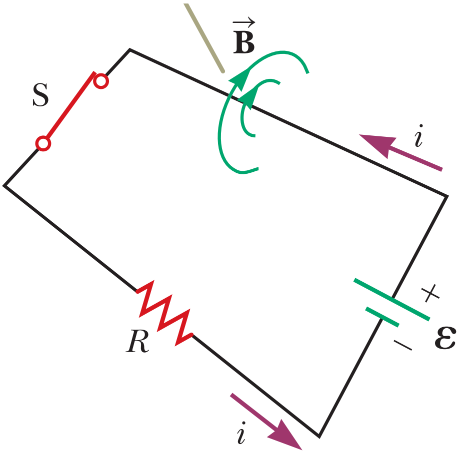
Inductors are a bit similar to resistors.

Resistors resist the flow of current.

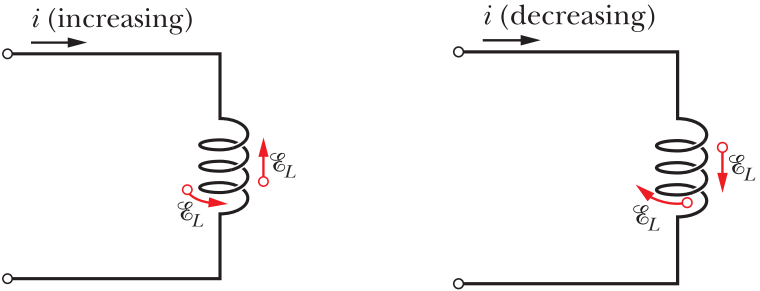
Inductors resist any change in current.

If the current is high and lowered, the emf acts to keep the current flowing. If the current is low and increased, the emf acts to resist the increase.

Self-Induction

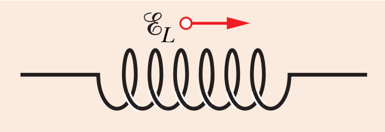


Self-Induction



Self-inductance question

The figure shows an emf $\mathcal{E}_L$ induced in a coil.



Which of the following can describe the current through the coil:

(A) constant and rightward

(B) increasing and rightward

(C) decreasing and rightward

(D) decreasing and leftward

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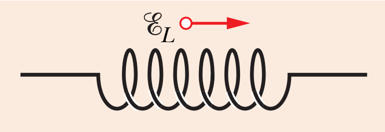

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(C) decreasing and rightward $\leftarrow$

(D) decreasing and leftward

Energy Stored in an Inductor

$$U_B = \frac{1}{2} L i^2$$

Compare with $U_E = \frac{q}{2C}$ (or $U_E = \frac{1}{2} C V^2$) for the energy stored in a capacitor.

Energy Density of a Magnetic Field

Energy is stored in a magnetic field!

$$u_B = \frac{B^2}{2\mu_0}$$

Compare with $u_E = \frac{1}{2}\epsilon_0 E^2$ for electric fields.

Mutual Inductance

An inductor can have an induced emf from its own changing magnetic field.

It also can have an emf from an external changing field.

That external changing field could be **another inductor**.

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For self-inductance on a coil labeled 1:

$$N_1 \Phi_{B,1} = L_1 i_1$$

For mutual inductance:

$$N_1 \Phi_{B,12} = M\, i_2$$

The flux is in coil 1, but the current that causes the flux is in coil 2.

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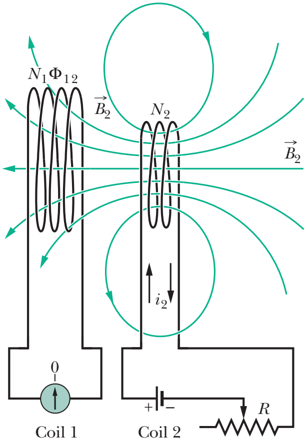
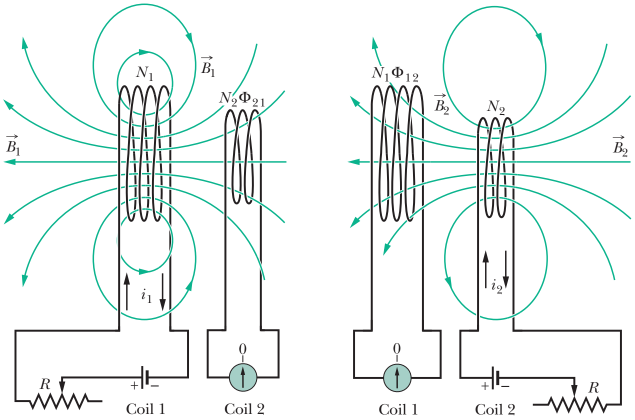


Mutual Inductance

mutual inductance

$$M = \frac{N_1 \Phi_{B,12}}{i_2} = \frac{N_2 \Phi_{B,21}}{i_1}$$



Mutual Inductance

$$N_1 \Phi_{B,12} = M\, i_2$$

Considering the rate of change of both sides with time, and using Faraday's Law $\mathcal{E} = -\frac{\Delta \Phi_B}{\Delta t}$,

$$\mathcal{E}_1 = -M\, \frac{\Delta i_2}{\Delta t}$$

and

$$\mathcal{E}_2 = -M\, \frac{\Delta i_1}{\Delta t}$$

A change of current in one coil causes a

Mutual Inductance Applications

If there is a changing current in one coil, an emf can be induced in the other coil.

The current can be transferred to a whole different circuit that is no directly connected.

This can be used for **wireless charging**.

It is also used in **transformers**: devices that change the voltage and current of a power supply.

For either of those applications to work, there must be a constantly changing current.

Alternating Current (AC)

Alternating current (AC) power supplies are the alternative to direct current (DC) power supplies.

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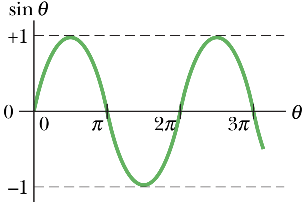
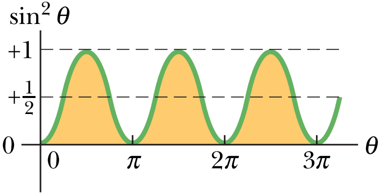
In an alternating current supply, the voltage and current vary sinusoidally with time:





$$i = i_0 \sin(\omega t)$$

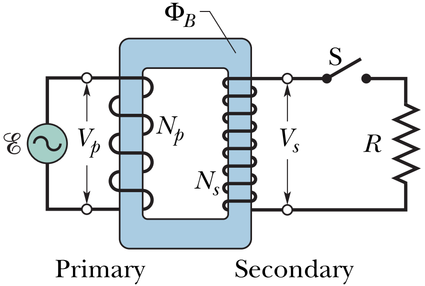
The power delivered to a load fluctuates as $P = P_0 \sin^2(\omega t)$.

Transformers

Transformers change ΔV_{rms} and I_{rms} simultaneously, while keeping the average power:

$$P_{\text{avg}} = I_{\text{rms}}(\Delta V_{\text{rms}})$$

constant (conservation of energy).



This works via mutual inductance. If the current in the first coil did not constantly change (AC) this would not work.

$$\Delta V_s = \Delta V_p \frac{N_s}{N_p}$$

Transformers

The reason for the voltage relation is that the iron core ideally contains all the magnetic flux lines produced.

Then the emf per turn $\mathcal{E}_t = -\frac{\Delta\Phi}{\Delta t}$ is the same in both solenoids.

$$\Delta V_p = -N_p \frac{\Delta\Phi}{\Delta t} \text{ and } \Delta V_s = -N_s \frac{\Delta\Phi}{\Delta t}$$

$$\Delta V_s = \Delta V_p \, \frac{N_s}{N_p}$$

Maxwell's Laws

Amazingly, we can summarize the majority of the relations that we have talked about in this course in a set of just 4 equations.

These are together called Maxwell's equations.

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{\text{enc}}}{\epsilon}$$

$$\oint \mathbf{B} \cdot d\mathbf{A} = 0$$

$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{\text{enc}}$$

Gauss's Law for Magnetic Fields

The first of Maxwell's equations is Gauss's Law for E-fields:

$$\oint \mathbf{E} \cdot \mathrm{d}\mathbf{A} = \frac{q_{\text{enc}}}{\epsilon}$$

The second is for Gauss's Law for B-fields:

$$\oint \mathbf{B} \cdot \mathrm{d}\mathbf{A} = 0$$

Maxwell's Law of Induction

Faraday's Law of Induction is the third of Maxwell's laws.

$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$

This tells us that a changing magnetic field will induce an electric field.

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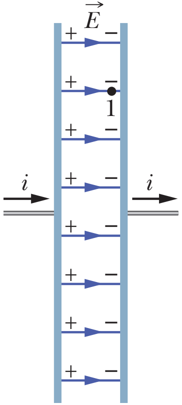
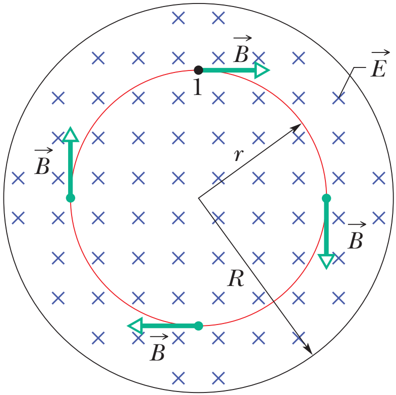
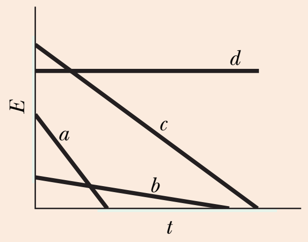




Maxwell's Law of Induction question

The figure shows graphs of the electric field magnitude E versus time t for four uniform electric fields, all contained within identical circular regions as in the circular-plate capacitor. Rank the E-fields according to the magnitudes of the magnetic fields they induce at the edge of the region, greatest first.



A a, b, c, d

B a, c, b, d

C d, b, c, a

D d, c, a, b

[1] Halliday, Resnick, Walker, page 865.

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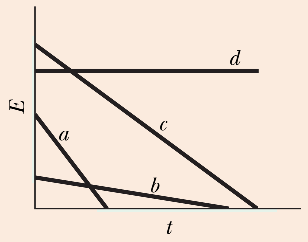

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Ampere-Maxwell Law

However, a changing electric field is not the only cause of a magnetic field.

We know from Ampere's Law:

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 i_{\text{enc}}$$

that a moving charge (current) causes a magnetic field also.

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Since we could have a situation with both a changing E-field *and* a current, we can express it more generally with the Ampere-Maxwell Law:

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{\text{enc}}$$

We add up the contributions from the changing electric flux and the current.

Ampere-Maxwell Law

The Ampere-Maxwell Law is the fourth and last of Maxwell's laws.

Ampere-Maxwell Law

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{enc}$$

Maxwell's Equations

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{enc}}{\epsilon}$$

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[1] Strictly, these are Maxwell's equations in a vacuum.

Ampere-Maxwell Law and Displacement "Current"

Ampere-Maxwell Law:

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{enc}$$

It can be convenient to imagine that even the part of the B-field that is produced by a changing E-field is actually produced by some kind of virtual current.

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This works because the units of $\epsilon_0 \frac{d\Phi_E}{dt}$ are Amps.

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$$i_d = \epsilon_0 \frac{d\Phi_E}{dt}$$

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displacement "current"

$$i_d = \epsilon_0 \frac{d\Phi_E}{dt}$$

Note: The displacement "current" is not a current and has nothing to do with displacement.

Ampere-Maxwell Law and Displacement "Current"

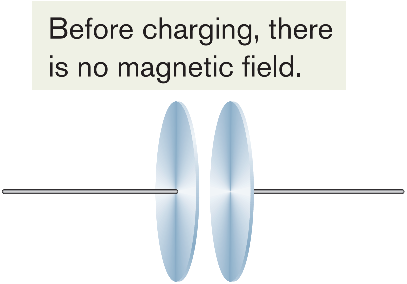
This lets us rewrite the Ampere-Maxwell law as:

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 i_d + \mu_0 i_{\text{enc}}$$

Looking at it this way can give us some insights.

B-field around a charging capacitor

Suppose a capacitor is being charged with a constant current, i.



How does i relate to i_d the "current" from the E-field between the plates?

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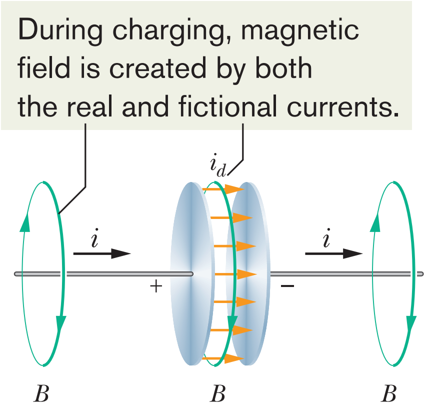

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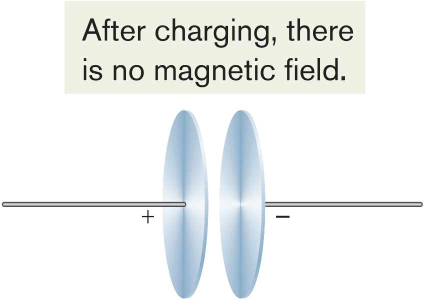


How does i relate to i_d the "current" from the E-field between the plates?

B-field around a charging capacitor

$$i_d = \epsilon_0 \frac{\mathrm{d}\Phi_\mathrm{E}}{\mathrm{dt}} = \epsilon_0 A \frac{\mathrm{dE}}{\mathrm{dt}}$$

Gauss's law allows us to relate q, the charge on the capacitor to the flux:

$$\frac{q}{\epsilon_0} = \oint \mathbf{E} \cdot \mathrm{d}\mathbf{A} = EA$$

The current is the rate of flow of charge:

$$i = \frac{\mathrm{dq}}{\mathrm{dt}} = \epsilon_0 A \frac{\mathrm{dE}}{\mathrm{dt}}$$

So, $i_d = i$!

The B-field outside a circular capacitor looks the same as the B-field around the wire leading up to the capacitor.

Magnetic Field around a circular capacitor

Can be calculated just like the field around a wire!

Magnetic Field around a circular capacitor

Can be calculated just like the field around a wire!

Outside the capacitor at radius r from the center:

$$B = \frac{\mu_0 i_d}{2\pi r}$$

Inside the capacitor (plates have radius R) at radius r from the center:

$$B = \frac{\mu_0 i_d}{2\pi R^2} r$$

But remember: i_d is not a current. No current flows across the gap between the plates.

Another Implication of Maxwell's Equations

Using all 4 equations (in their differential form) it is possible to reach a pair of wave equations for the electric and magnetic fields:

$$\nabla^2 \mathbf{E} = \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2}$$

$$\nabla^2 \mathbf{B} = \frac{1}{c^2}\frac{\partial^2 \mathbf{B}}{\partial t^2}$$

with wave solutions:

$$\mathbf{E} = E_0 \sin(\mathbf{k}\cdot\mathbf{r} - \omega t)$$

$$\mathbf{B} = B_0 \sin(\mathbf{k}\cdot\mathbf{r} - \omega t)$$

Another Implication of Maxwell's Equations

$$\nabla^2 \mathbf{E} = \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2}$$

The constant c appears as the wave speed and

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$c = 3.00 \times 10^8$ m/s, is the speed of light.

The values of ϵ_0 and μ_0 together predict the speed of light!

Relation between Electric and Magnetic Fields

These oscillating electric and magnetic fields make up light.

Faraday's Law of Induction

A changing magnetic field gives rise to an electric field.

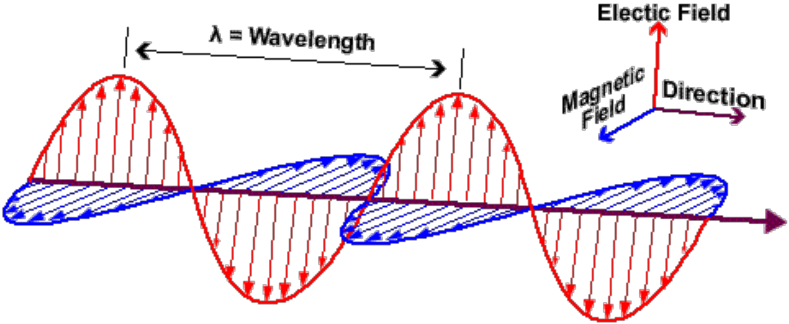
Ampere-Maxwell Law of Induction

A changing electric field gives rise to an magnetic field.

Light

Faraday's Law $\Rightarrow$ a changing magnetic field causes an electric field.

Maxwell's Law $\Rightarrow$ a changing electric field causes a magnetic field.



Light (Electromagnetic Radiation)

All light waves in a vacuum travel at the same speed, the speed of light, $c = 3.00 \times 10^8$ m s^{-1}.

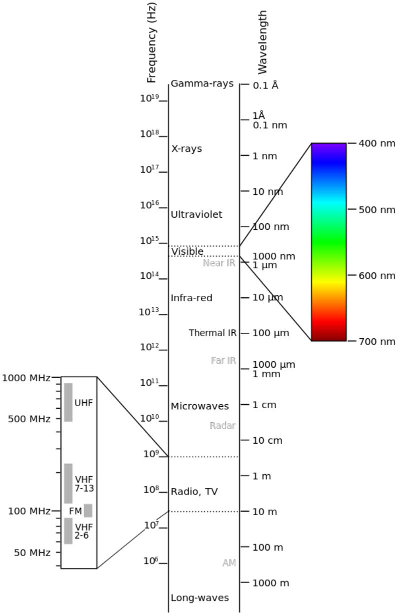
Maxwell's equations possess the 'wrong' symmetry for Gallilean transformations between observers; they are Lorentz-invariant. This gave Einstein an important idea.

All observers, no matter how they move relative to one another all agree that any light wave travels at that same speed.

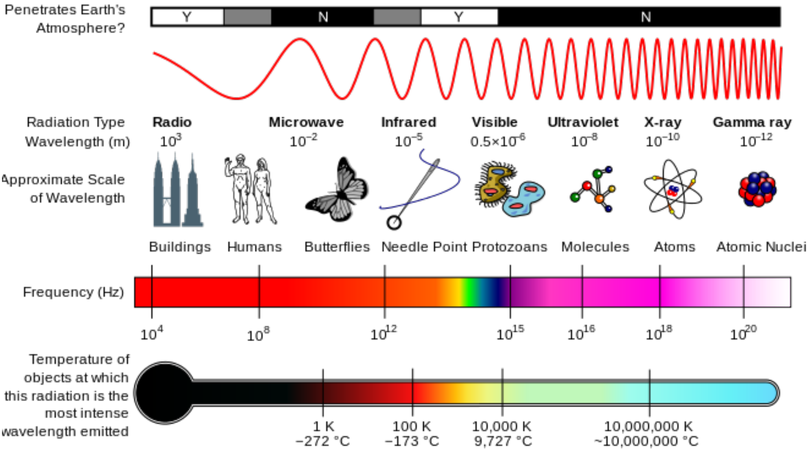
Since light travels at this fixed speed and $c = v = f\lambda$, if the frequency of the light is given, you also know the wavelength, and vice versa.

$$\lambda = \frac{c}{f} \quad ; \quad f = \frac{c}{\lambda}$$

Electromagnetic spectrum



Electromagnetic spectrum



Summary

- applications of Faraday's law
- inductance
- self-induction
- RL Circuits

Homework Halliday, Resnick, Walker:

- NEW: Ch 30, onward from page 816. Problems: 41, 45, 61, 63, 67, 69, 73
- NEW: Ch 31 onward from page 858. Problems: 62, 63
- NEW: Ch 32, onward from page 883. Questions: 1, 3; Problems: 1, 5, 13