Electricity and Magnetism
Electric Fields

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De Anza College

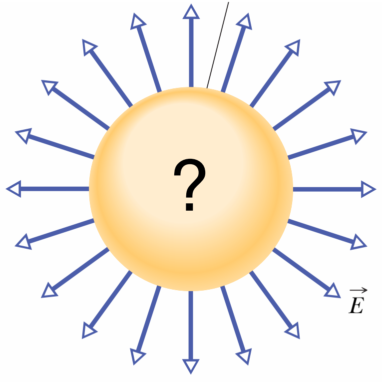
Sept 29, 2015

Last time

- Coulomb's law
- force from many charges
- current
- electric field
- charges and conductors

Warm Up Questions

Which of the following could be the charge on the particle hidden by the question mark?



(A) 0 C

(B) -1 C

(C) -1.6×10^{-19} C

(D) $+1$ μC

[1]Figure from Halliday, Resnick, Walker

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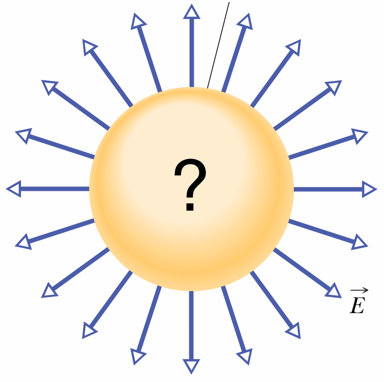

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Which expression relating force to electric field is correct?

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(B) $\mathbf{E} = q_0 \mathbf{F}$

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What are the units of electric field?

(A) Nm

(B) N/C

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Homework Questions

pg 573 42 & 43

42 In Fig. 21-38, two tiny conducting balls of identical mass m and identical charge q hang from nonconducting threads of length L. Assume that θ is so small that $\tan\theta$ can be replaced by its approximate equal, $\sin\theta$. (a) Show that

$$x = \left(\frac{q^2 L}{2\pi\varepsilon_0 mg}\right)^{1/3}$$

gives the equilibrium separation x of the balls. (b) If $L = 120$ cm, $m = 10$ g, and $x = 5.0$ cm, what is $|q|$?

43 (a) Explain what happens to the balls of Problem 42 if one of them is discharged (loses its charge q to, say, the ground). (b) Find the new equilibrium separation x, using the given values of L and m and the computed value of $|q|$.

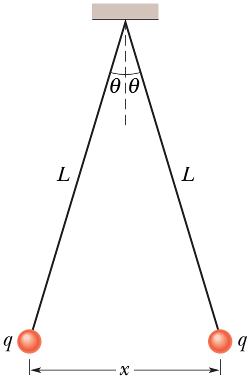


Fig. 21-38
Problems 42 and 43.

Overview

- field due to a point charge
- field from multiple point charges
- electric fields of charge distribution

Field from a Point Charge

We want an expression for the electric field from a point charge, q.

Using **Coulomb's Law** the force on the test particle is
$\mathbf{F}_{\rightarrow 0} = \frac{k\,qq_0}{r^2}\hat{\mathbf{r}}$.

$$\mathbf{E} = \frac{\mathbf{F}}{q_0} = \left(\frac{1}{\cancel{q_0}}\right)\frac{k\,q\,\cancel{q_0}}{r^2}\hat{\mathbf{r}}$$

The field at a displacement $\mathbf{r}$ from a charge q is:

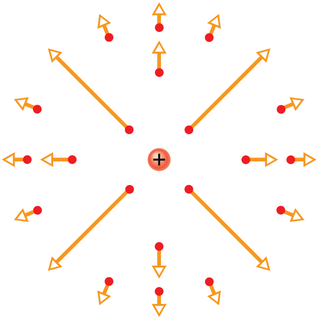
$$\mathbf{E} = \frac{k\,q}{r^2}\hat{\mathbf{r}}$$

Field from a Point Charge

The field at a displacement **r** from a charge q is:

$$\mathbf{E} = \frac{k\,q}{r^2}\hat{\mathbf{r}}$$

This is a vector field:



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$$\mathbf{F}_{\text{net},0} = \mathbf{F}_{1\rightarrow 0} + \mathbf{F}_{2\rightarrow 0} + ... + \mathbf{F}_{n\rightarrow 0}$$

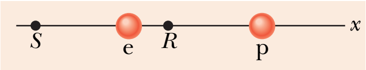
$\mathbf{E}_{\text{net}} = \frac{\mathbf{F}_{\text{net}}}{q_0}$

Total electric field:

$$\mathbf{E}_{\text{net}} = \mathbf{E}_1 + \mathbf{E}_2 + ... + \mathbf{E}_n$$

Question about field from point charges

Consider a proton p and an electron e on an x axis.



What is the direction of the electric field due to the electron at point S and point R?

(A) leftward at S, leftward at R

(B) leftward at S, rightward at R

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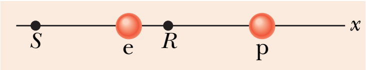

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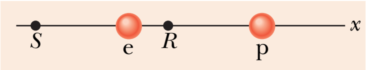

What is the direction of the net electric field at point S and point R?

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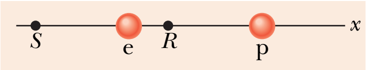

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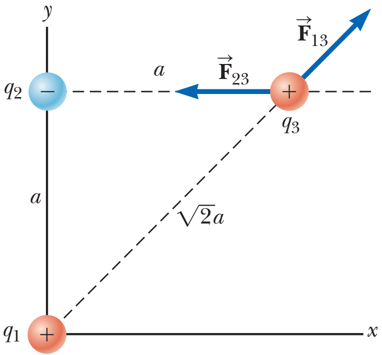
Electric Field Question

$q_1 = q_3 = 5.00\ \mu\text{C}$, $q_2 = -2.00\ \mu\text{C}$, and $a = 0.100$ m.
The resultant force exerted on q_3 is $\mathbf{F}_{\text{net},3} = (-1.04\ \mathbf{i} + 7.94\ \mathbf{j})$ N.

What is the electric field at the location of q_3 due to the other two charges?



(A) $(-1.04\ \mathbf{i} + 7.94\ \mathbf{j})$ N

(B) $(-1.04\ \mathbf{i} + 7.94\ \mathbf{j})$ N/C

(C) $(-0.208\ \mathbf{i} + 1.59\ \mathbf{j})$ MN/C

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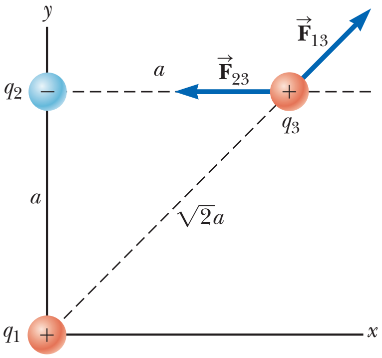

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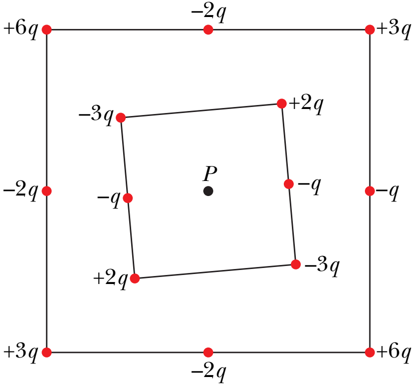
Question about net field

2 Figure 22-21 shows two square arrays of charged particles. The squares, which are centered on point P, are misaligned. The particles are separated by either d or $d/2$ along the perimeters of the squares. What are the magnitude and direction of the net electric field at P?



[1]Figure from Halliday, Resnick, Walker, page 597, problem 2.

Electric Dipole

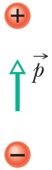
electric dipole

A pair of charges of equal magnitude q but opposite sign, separated by a distance, d.

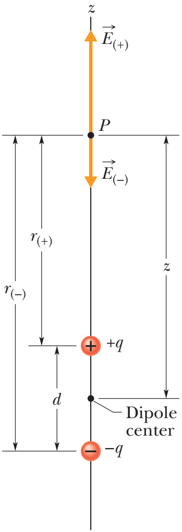
dipole moment:

$$\mathbf{p} = qd\,\hat{\mathbf{r}}$$

where $\hat{\mathbf{r}}$ is a unit vector pointing from the negative charge to the positive charge.



Electric Field from an Electric Dipole



We will find an expression for the magnitude of the field *along the dipole axis*

$$
\begin{aligned}
E &= E_{(+)} - E_{(-)} \\
&= \frac{kq}{r_{(+)}^2} - \frac{kq}{r_{(-)}^2} \\
&= \frac{kq}{z^2}\left(\frac{1}{(1-d/2z)^2} - \frac{1}{(1+d/2z)^2}\right)
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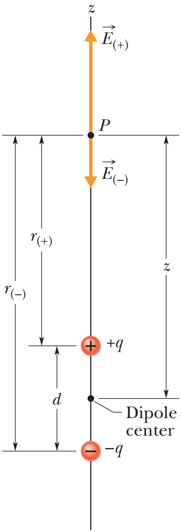

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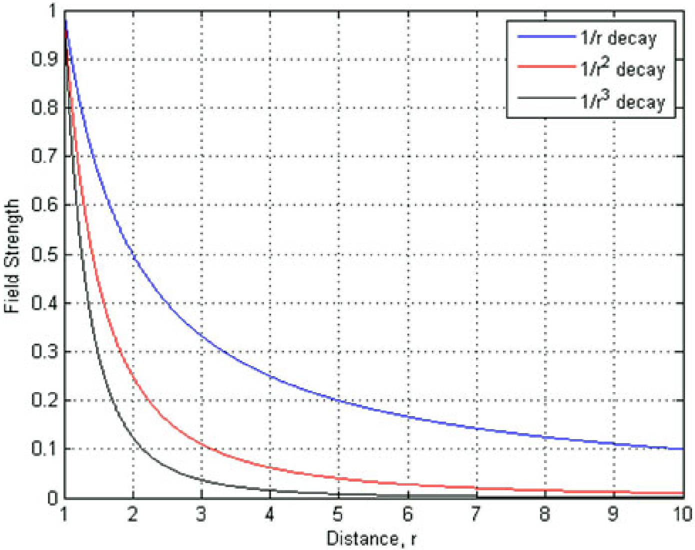
&= \frac{2kp}{z^3}
\end{aligned}
$$

where we assumed $z >> d$

The effect of the dipole falls off as $1/z^3$ - means the charges largely, but not entirely cancel each other out.

r-inverse decays



[1] Figure by Neeraj Sood, from rfidjournal.com.

Continuous distribution of charge

In previous examples, we added up the field from each point charge.

But what about the case of a charged object, like a plate or a wire?

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Solution: treat the charge as a continuous distribution with some charge density.

Charge Density

charge density

The amount of charge in per unit 'volume' of an object.

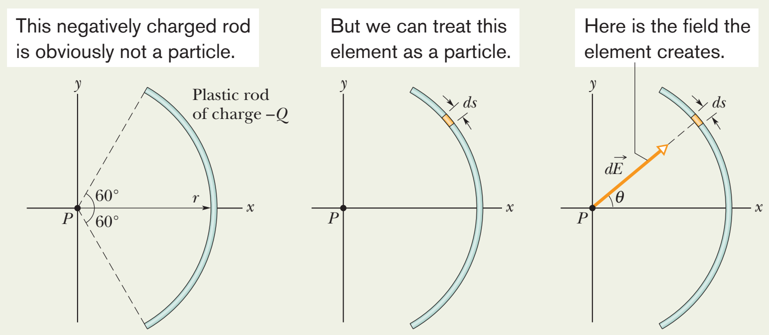
(Here 'volume' could be volume, area, or length)

By convention, different symbols can be used in different cases:

symbol	description	units
λ	charge per unit length	C m^{-1}
σ	charge per unit area	C m^{-2}
ρ	charge per unit volume	C m^{-3}

For a wire, usually use charge per length.
For a plate, charge per area.

Continuous distribution of charge



We need to add up the charge of each little "particle" ds. Each has charge $\lambda\,\mathrm{ds}$.

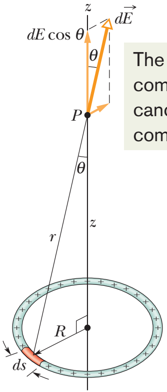
To be perfectly accurate, we would make the length of ds $\to 0$. This is an integral: $\sum \lambda \Delta s \to \int \lambda\,\mathrm{ds}$

The main trick

All this does not mean you have to be able to do integrals.

If you understand that you sum up the effect of charges, you can still figure out what the net field at many points is just by symmetry.

Example: Field from a ring of charge



The perpendicular components just cancel but the parallel components add.

Vertical components? From each charge $\lambda\, \mathrm{ds}$:

$$
\begin{aligned}
dE_y &= dE\cos\theta \\
&= \left(\frac{k\lambda\, ds}{r^2}\right)\cos\theta \\
&= \left(\frac{k\lambda\, ds}{(R^2+z^2)}\right)\frac{z}{\sqrt{R^2+z^2}} \\
&= \frac{k\, z\, \lambda\, ds}{(R^2+z^2)^{3/2}}
\end{aligned}
$$

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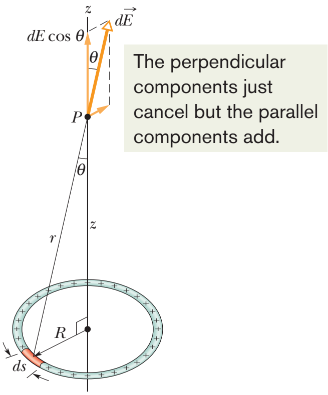


$$dE_y = \frac{k\,z\,\lambda\,ds}{(R^2+z^2)^{3/2}}$$

There are $2\pi R$-worth of little lengths ds. Adding the field for all together:

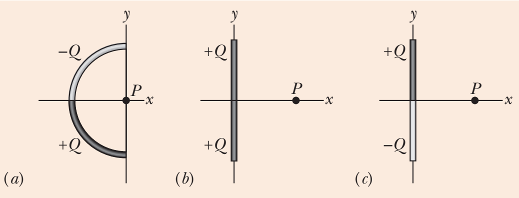
$$E_y = \frac{k\,z\,\lambda(2\pi R)}{(R^2+z^2)^{3/2}}$$

$$= \frac{k\,q\,z}{(R^2+z^2)^{3/2}}$$

since total charge $q = 2\pi R\lambda$ by definition.

Question

The figure here shows three nonconducting rods, one circular and two straight. Each has a uniform charge of magnitude Q along its top half and another along its bottom half. For each rod, what is the direction of the net electric field at point P?



For (a) it is:

(A) up

(B) down

(C) left

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[1] Page 590, Halliday, Resnick, Walker.

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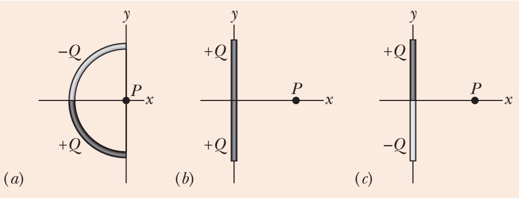

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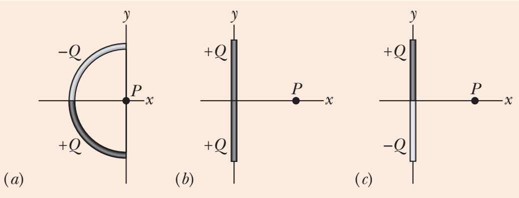

For (b) it is:

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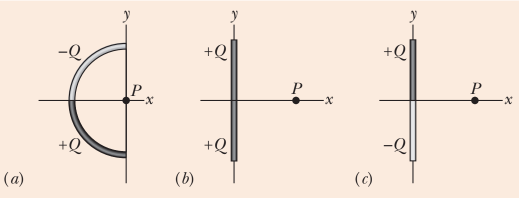

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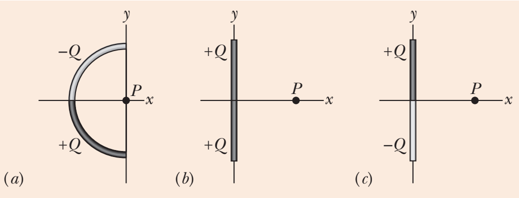

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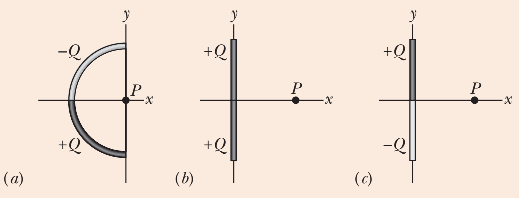

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Summary

- E-field from many charges
- electric fields of charge distribution

Homework

- E-fields worksheet

Halliday, Resnick, Walker:

- Ch 22, onward from page 597. Problems: 2, 5, 7, 9, 23