Electricity and Magnetism
Gauss's Law
Electric Potential Energy

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Last time

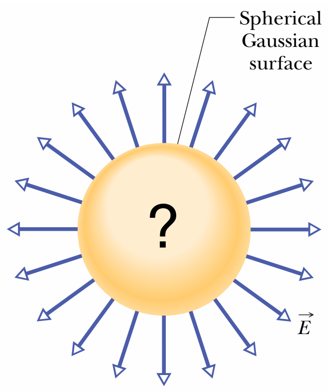
- motion of charges in electric fields

Overview

- electric flux
- Gauss's law
- implications of Gauss's law
- Coulomb's law from Gauss's law
- some rules to help solve problems

Gauss's Law basic idea

Gauss's law relates the electric field across a closed surface (*eg.* a sphere) to the amount of net charge enclosed by the surface.

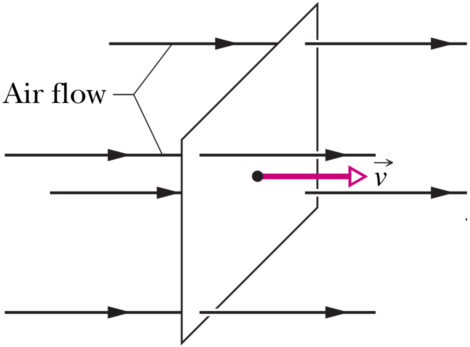


Can we quantify the "electric field across a boundary"?

Flux

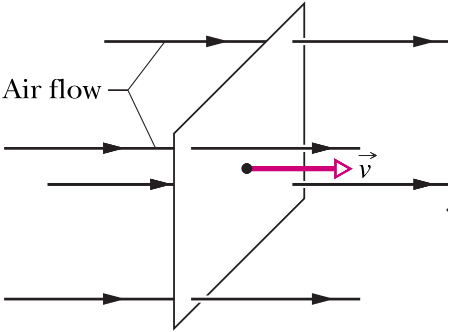
Flux is a quantity that makes the idea of the "electric field through some region" precise.

Flux is a flow rate through an area.



Flux

Imagine air blowing directly through a square loop of wire of area A.



The volume of air that passes through in 1 s is $V = A \times v \times (1\text{ s})$, where v is the speed of the air.

The *rate* of flow would be $V/t = Av$.

Flux

Now consider a more general situation: the air does not blow directly through the loop, but at some angle θ.

If $\theta = 90^\circ$, what is the flow rate (flux) through the loop?

Flux

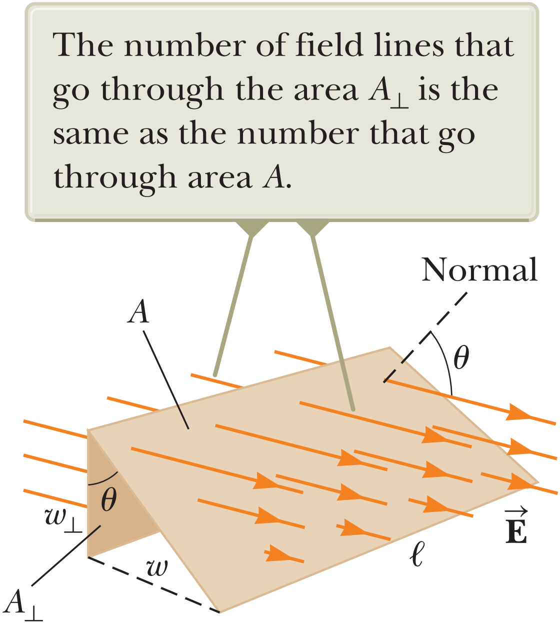
Now consider a more general situation: the air does not blow directly through the loop, but at some angle θ.

If $\theta = 90^\circ$, what is the flow rate (flux) through the loop? Zero!

In that case there is no flow through the loop. The air goes around the loop.

The flux depends on the angle that the flow makes to the loop / area.

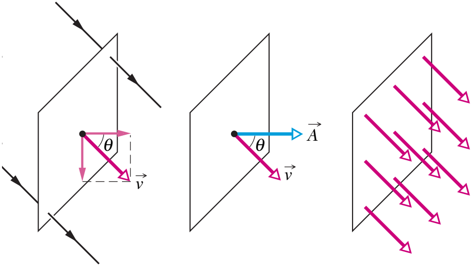
Flux


The number of field lines that go through the area $A_\perp$ is the same as the number that go through area A.
Normal
A
θ
θ
$w_\perp$
w
ℓ
$\vec{\mathbf{E}}$
$A_\perp$

Flux

The area $A_\perp = A\cos\theta$.

For other values of θ the flux of air that move through is $vA\cos\theta$.



We can define flux:

$$\Phi = vA\cos\theta$$

Electric Flux

The electric flux, Φ_E, through an area A is

$$\Phi_E = EA\cos\theta$$

where θ is the angle between the electric field vector at the surface and the normal vector to the surface.

This can be written:

$$\Phi_E = \mathbf{E} \cdot \mathbf{A}$$

The direction of $\mathbf{A}$ is $\perp$ to the surface, and the magnitude is the area of the surface.

Gaussian Surface

Gaussian surface

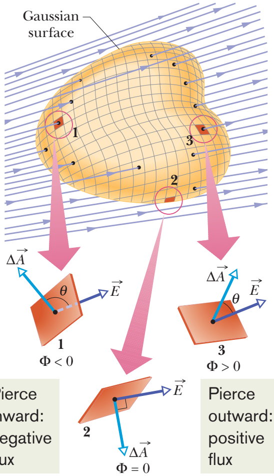
An imaginary boundary (close surface) drawn around some region of space in order to study electric charge and field.

The surface can be any shape you like!

It is just a tool for calculating charge or field.

Electric Flux

The electric flux Φ_E through a Gaussian surface is proportional to the net number of electric field lines passing through that surface.



Pierce inward: negative flux

Pierce outward: positive flux

Skim: zero flux

Consider small areas of the Gaussian surface ΔA.

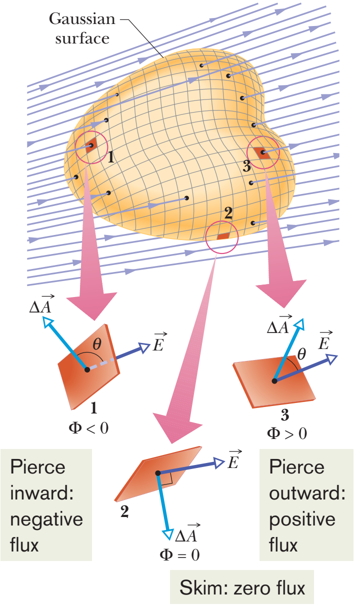
Total flux through the surface:

$$\Phi_E = \sum \mathbf{E} \cdot (\Delta \mathbf{A})$$

just the sum of all the flux through each little area.

$$\left(\text{Formally, } \Phi_E = \oint \mathbf{E} \cdot d\mathbf{A}\right)$$

Electric Flux through Gaussian Surfaces



- The flux is **positive** where the field vector point out of the surface.
- The flux is **negative** where the field vector point into the surface.

For a closed surface:

$$\Phi_E = \oint \mathbf{E} \cdot \mathrm{d}\mathbf{A}$$

Gauss's Law

The net flux through a surface is directly proportional to the net charge enclosed by the surface.

$$\epsilon_0 \Phi_E = q_{\text{enc}}$$

We can also write this same thing as an integral:

$$\oint \mathbf{E} \cdot \mathrm{d}\mathbf{A} = \frac{q_{\text{enc}}}{\epsilon_0}$$

Gauss's Law

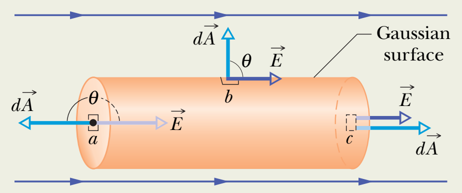
The net flux through a surface is directly proportional to the net charge enclosed by the surface.

$$\epsilon_0 \Phi = q_{\text{enc}}$$

Example: uniform field

Let's draw a Gaussian surface in a uniform field.

For a cylinder, there is some symmetry: easy to calculate flux!



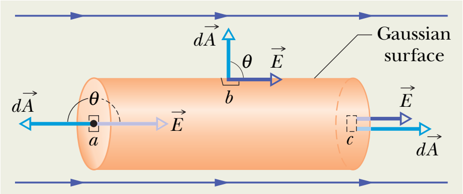
The sides of the cylinder are $\|$ to $\mathbf{E} \;\Rightarrow\; \Phi = 0$.

We only need to consider the ends.

$$\Phi = E(\pi r^2)\cos(0) + E(\pi r^2)\cos(180^\circ) = 0$$

Example: uniform field

The total flux across the boundary is zero!



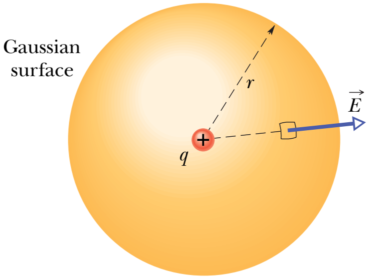
From Gauss's Law $\epsilon_0 \Phi = q_{\text{enc}}$, we know:

$$q_{enc} = 0$$

This is always true for any Gaussian surface in a uniform electric field.

Gauss's Law for a Point Charge

For a point charge, we can imagine a spherical Gaussian surface. The field will be perpendicular to the surface at every point.



$$\Phi = 4\pi r^2 E$$

Gauss's law:

$$\epsilon_0 \Phi = 4\pi r^2 E = q$$

so,

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} = \frac{kq}{r^2}$$

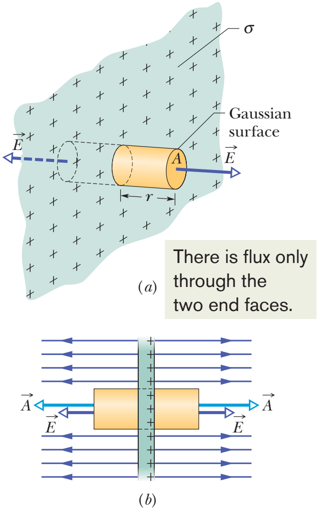
Same as from Coulomb's law!

Question

Imagine a Gaussian surface enclosing a dipole.

What is the net flux through the surface?

Nonconducting sheet of charge

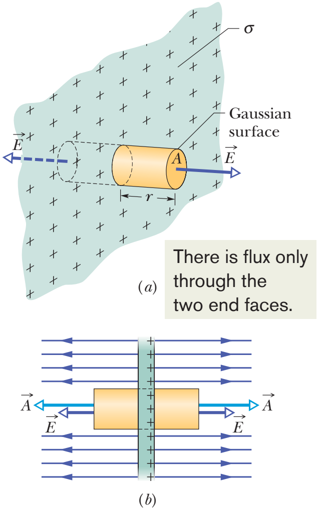


Again, the sides of the cylinder are $\parallel$ to $\mathbf{E} \;\Rightarrow\; \Phi = 0$.

We only need to consider the ends.

$$\begin{aligned}\Phi &= E(\pi r^2)\cos(0) + E(\pi r^2)\cos(0)\\ &= 2\pi r^2 E\end{aligned}$$

Nonconducting sheet of charge



Again, the sides of the cylinder are $\parallel$ to $\mathbf{E} \Rightarrow \Phi = 0$.

We only need to consider the ends.

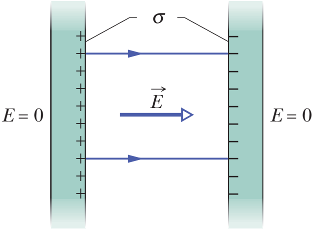
$$\begin{aligned} \Phi &= E(\pi r^2)\cos(0) + E(\pi r^2)\cos(0) \\ &= 2\pi r^2 E \end{aligned}$$

Then, using Gauss's law:

$$\begin{aligned} \epsilon_0(2\pi r^2 E) &= \sigma(\pi r^2) \\ E &= \frac{\sigma}{2\epsilon_0} \end{aligned}$$

which is what I claimed in the previous lecture.

Field between conducting plates



From Gauss's Law we can also find the field between conducting plates with an air (or vacuum) gap separating them:

$$E = \frac{\sigma}{\epsilon_0}$$

Some Implications of Gauss's Law

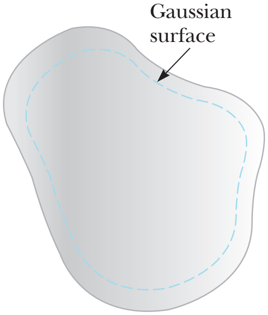
Rules that make calculating easier!

- If an excess charge is placed on an isolated conductor, that amount of charge will move entirely to the surface of the conductor. None of the excess charge will be found within the body of the conductor.
- A shell of uniform charge attracts or repels a charged particle that is outside the shell as if all the shell's charge were concentrated at the center of the shell.
- If a charged particle is located inside a shell of uniform charge, there is no electrostatic force on the particle from the shell.

Some Implications of Gauss's Law

Excess Charge on a Conductor

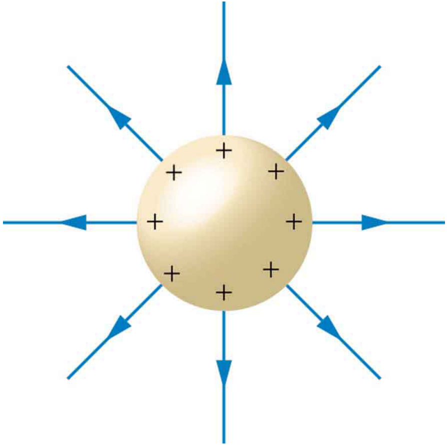
If an excess charge is placed on an isolated conductor, that amount of charge will move entirely to the surface of the conductor. None of the excess charge will be found within the body of the conductor.



$\mathbf{E} = 0$ inside the conductor, so the Gaussian surface shown cannot enclose a net charge.

Charges and Conductors

Excess charge sits on the outside surface of a conductor.



Close to the surface, the electric field lines are perpendicular to the surface.

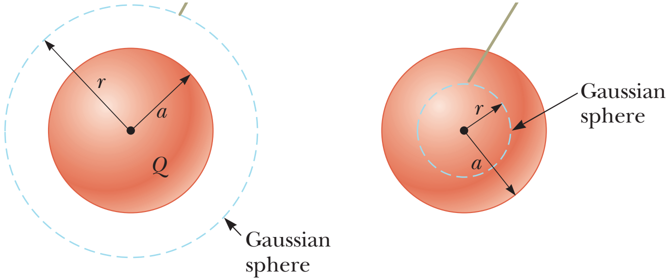
[1] Figure from OpenStax College Physics.

Some Implications of Gauss's Law

Uniform Shell of Charge

- A shell of uniform charge attracts or repels a charged particle that is outside the shell as if all the shell's charge were concentrated at the center of the shell.
- If a charged particle is located inside a shell of uniform charge, there is no electrostatic force on the particle from the shell.

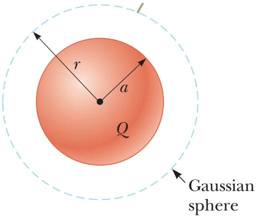
Uniform Sphere of Charge

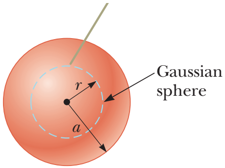


Consider a uniform insulating sphere of charge, radius a, charge density ρ, total charge Q.

How does the electric field strength change with distance from the center?

Uniform Sphere of Charge





$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{\text{enc}}}{\epsilon_0}$$

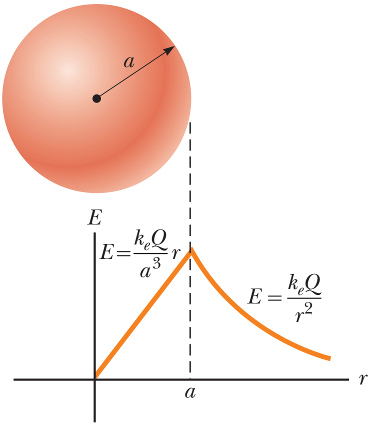
Outside sphere (for $r > a$):

$$\begin{aligned} 4\pi r^2 E &= \frac{1}{\epsilon_0} Q \\ E &= \frac{Q}{4\pi\epsilon_0 r^2} \\ E &= \frac{k_e Q}{r^2} \end{aligned}$$

Inside sphere (for $r < a$):

$$\begin{aligned} 4\pi r^2 E &= \frac{1}{\epsilon_0}\left(\frac{4}{3}\pi r^3 \rho\right) \\ E &= \frac{\rho r}{3\epsilon_0} \\ &= \frac{k_e Q r}{a^3} \end{aligned}$$

Uniform Sphere of Charge



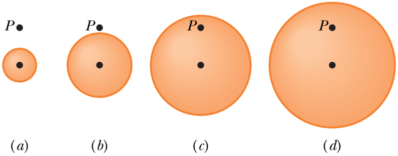
Outside the sphere, the electric field is the same as for a point charge, strength Q, located at the center of the sphere.

Inside the sphere, field varies linearly in the distance from the center and all charge outside the distance r cancels out!

Question

Page 621, #8

The figure shows four solid spheres, each with charge Q uniformly distributed through its volume.



Rank the spheres according to their volume charge density, greatest first. The figure also shows a point P for each sphere, all at the same distance from the center of the sphere.

(A) a, b, c, d

(B) d, c, b, a

(C) a and b, c, d

(D) a, b, c and d

[1] Halliday, Resnik, Walker

Question

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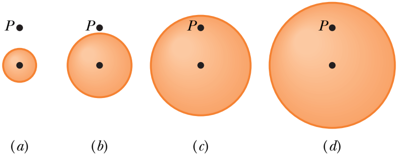

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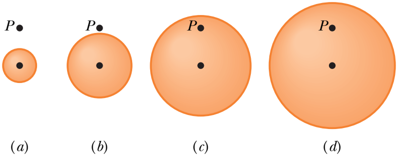

Rank the spheres according to the magnitude of the electric field they produce at point P, greatest first.

(A) a, b, c, d

(B) d, c, b, a

(C) a and b, c, d

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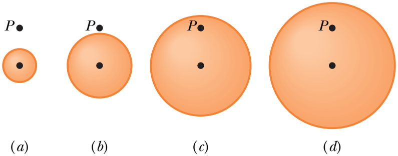

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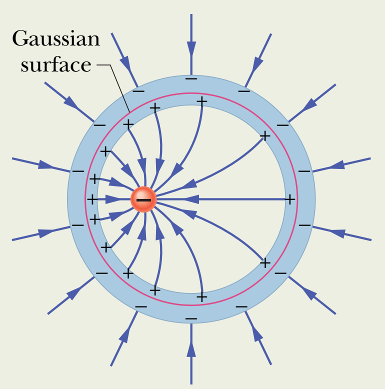
Some Implications of Gauss's Law

Faraday Ice Pail



A charge placed inside a conducting shell appears on the outside of the conductor.

($\mathbf{E} = 0$ for the Gaussian surface shown.)

Summary

- Electric flux
- Gauss's law idea
- Gauss's law implications
- Coulomb's law from Gauss's law
- problem solving tricks
- electric potential energy

Homework Halliday, Resnick, Walker:

- Ch 23, onward from page 622. Problems: 1, 57, 63, 65, 69
- Look ahead at Chapter 24, Electric potential.