Electricity and Magnetism
Isolated Conductors and Potential
Capacitance

Lana Sheridan

De Anza College

Oct 15, 2015

Last time

- electric potential
- Electric potential from many charges

Overview

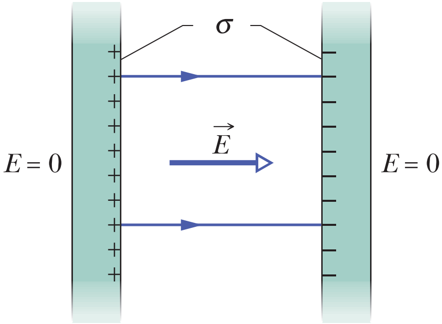
- Electric potential between charged plates
- Potential of charged conductor
- Conductor in a field
- Capacitance
- Capacitance of a parallel plate capacitor

Potential Difference across a pair of charged plates

Earlier we found:

$$\Delta V = -E\, d \cos\theta$$

If we have a pair of charged plates at a separation, d, there is a uniform E-field between them: $E = \frac{\sigma}{\epsilon_0}$.



Potential Difference across a pair of charged plates

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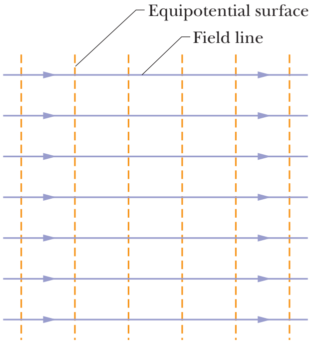
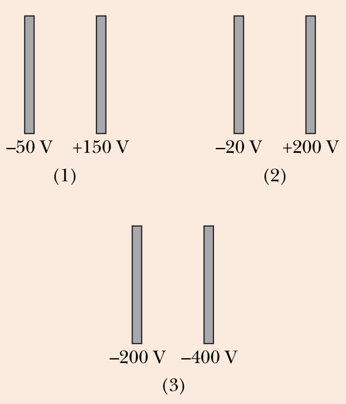


The potential difference between the two plates, separation, d:

$$|\Delta V| = E\,d$$

Question

Consider three pairs of parallel plates with the same separation. The electric field between the plates is uniform and perpendicular to the plates.
(a) Rank the pairs according to the magnitude of the electric field between the plates, greatest first.



(A) 1, 2, 3
(B) (1 and 3), 2
(C) 2, (1 and 3)
(D) 3, 2, 1

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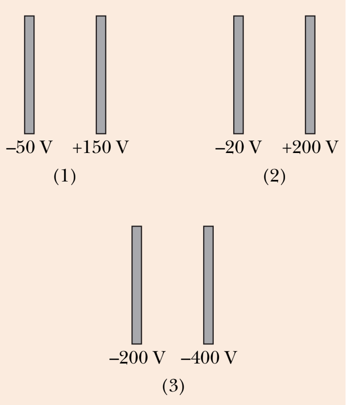

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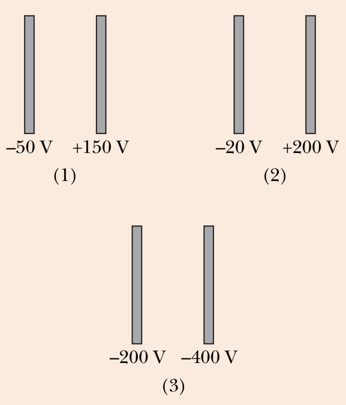
(b) For which pair is the electric field pointing rightward?



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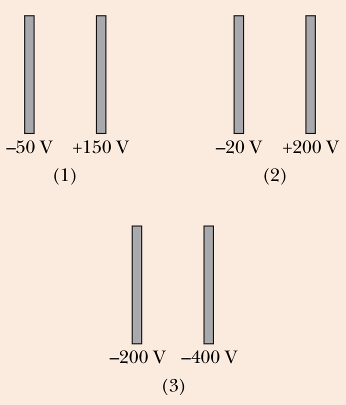
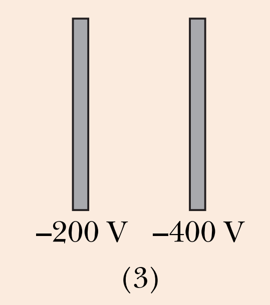


(A) 1
(B) 2
(C) 3 $\leftarrow$

Question

Consider three pairs of parallel plates with the same separation. The electric field between the plates is uniform and perpendicular to the plates.
(c) If an **electron** is released midway between the third pair of plates, does it



(A) remain there
(B) move at constant speed
(C) accelerate rightward, or
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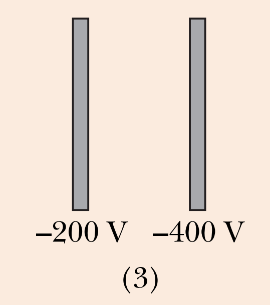

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From earlier: some Implications of Gauss's Law

- If an excess charge is placed on an isolated conductor, that amount of charge will move entirely to the surface of the conductor. None of the excess charge will be found within the body of the conductor.

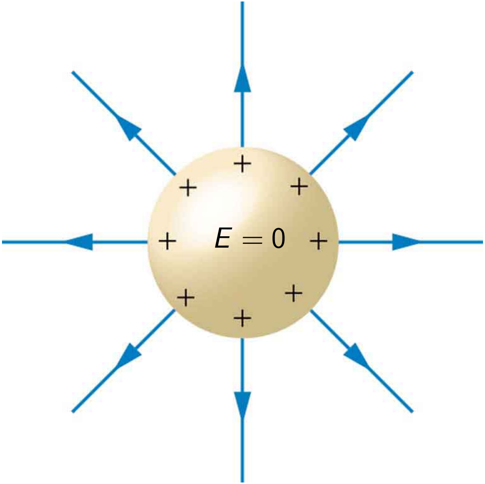
- A shell of uniform charge attracts or repels a charged particle that is outside the shell as if all the shell's charge were concentrated at the center of the shell.

- If a charged particle is located inside a shell of uniform charge, there is no electrostatic force on the particle from the shell.

Conductor in an Electric field

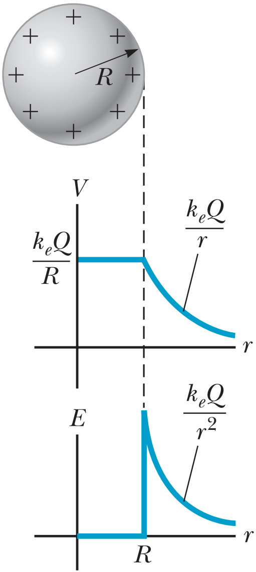
The E-field inside an isolated conductor at equilibrium is zero.

eg. an isolated conductor with excess charge:



[1]Figure from Openstax College Physics.

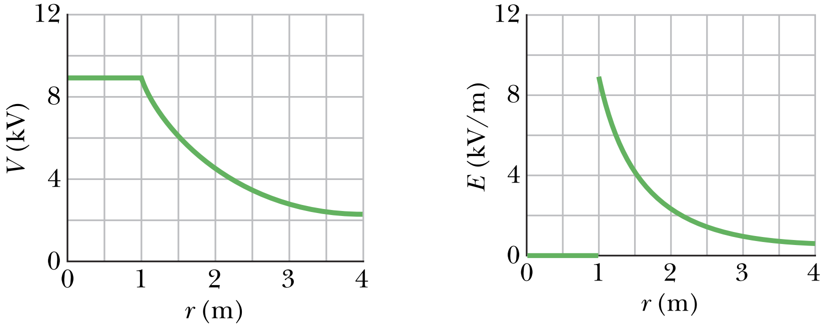
Potential due to an Isolated Charged Conductor



[1]Figure from Serway & Jewett, 9th ed.

Potential due to an Isolated Charged Conductor

Because all excess charge flows to the outside, in the interior, the electric field is zero.

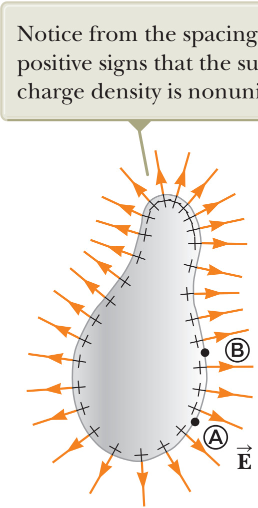


Since $\Delta V = -E\, d \cos\theta$ the potential inside the conductor is **constant**.

[1] Figure from Halliday, Resnick, Walker, 9th ed.

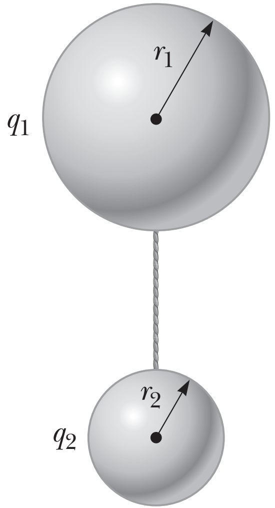
Charge distribution on a conductor

The electric potential is constant everywhere on a conductor (including the surface!), but the charge distribution may vary.



Charge distribution on a conductor

An illustrative example (25.8), electric field around conductor.



At all points on the object V is constant.

$$
\begin{aligned}
V_1 &= V_2 \\
\frac{k_e q_1}{r_1} &= \frac{k_e q_2}{r_2} \\
\frac{q_1}{q_2} &= \frac{r_1}{r_2}
\end{aligned}
$$

Since $r_2 < r_1$, $\boxed{q_1 > q_2}$.

And

$$\frac{\sigma_1}{\sigma_2} = \frac{r_2}{r_1}$$

$\Rightarrow$ sharper curvature of surface, higher charge density

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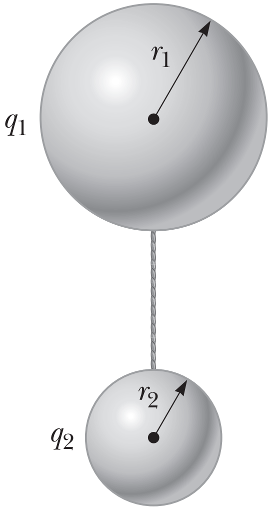

At all points on the object V is constant.

$$\begin{aligned} V_1 &= V_2 \\ E_1 r_1 &= E_2 r_2 \\ \frac{E_1}{E_2} &= \frac{r_2}{r_1} \end{aligned}$$

Since $r_2 < r_1$, $\boxed{E_1 < E_2}$.

$\Rightarrow$ sharper curvature of surface, stronger electric field

Corona Discharge

A corona discharge occurs when a conductor at a very high potential ionizes a fluid (*eg.* air) that surrounds it.

The fields that form around sharp edges of the conductor are high enough to form small plasma regions, but not full electric breakdown.

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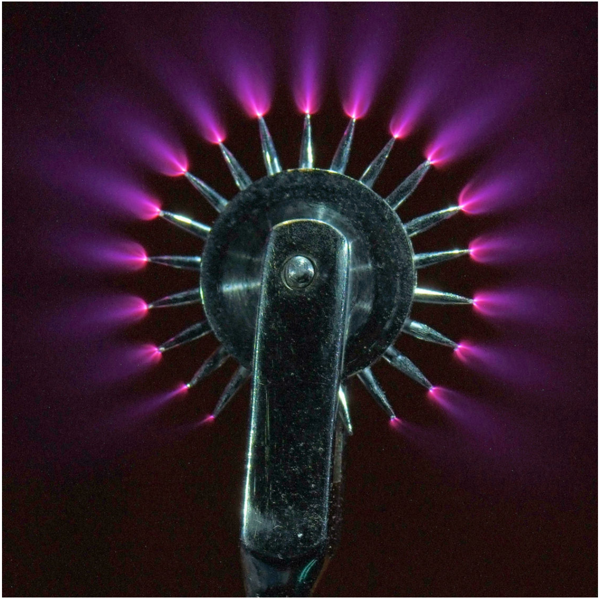
- responsible for significant power losses in high voltage lines
- useful for
  - pool sanitation
  - ozone manufacture
  - ionizers
  - air purifiers
  - nitrogen lasers (TEA lasers)

Coronal Discharge



[1]Photo "Wartenburg Pinwheel" by Giles Read. 30–50kV

Corona Discharge

Fork in a microwave.

(Microwave ovens generate electric fields.)

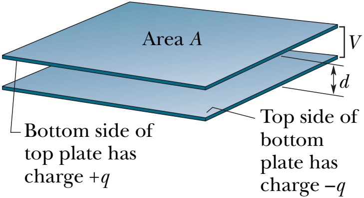
https://www.youtube.com/watch?v=b1MFWbX3Bfc

Capacitance

capacitor

Any two isolated conductors separated by some distance that store charges of equal magnitude and opposite sign.

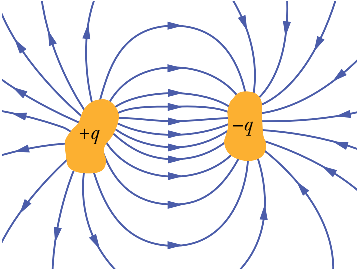
(When the capacitor is discharged this stored charge is 0.)



The **capacitance** of a capacitor relates the potential difference across the capacitor to its stored charge.

Capacitors

Usually capacitors are diagrammed and thought of as parallel sheets of equal area, but paired, isolated conductors of any shape can act as capacitors.

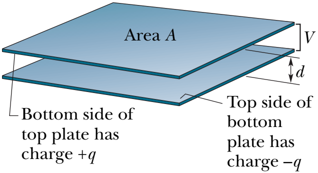


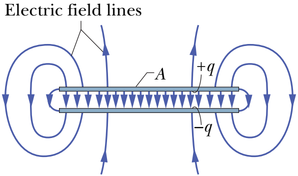
Capacitors

When a capacitor is **charged** is has a net charge $+q$ on one plate and a net charge $-q$ on the other plate.

An electric field exists between the plates.

For the case of parallel sheet plates, the field is uniform, except at the edges of the plates.





Charge of a Capacitor

The net charge on a capacitor is zero: $(+q) + (-q) = 0$.

However, when we speak of the **charge of a capacitor**, q, we mean that the absolute value of the charge on either plate is q.

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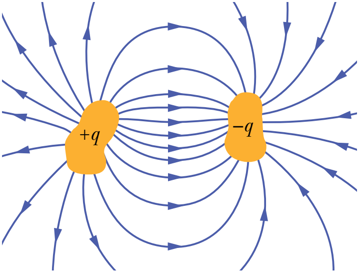
The charge on this capacitor is q:



Potential Difference

The **potential difference** between two points a and b is the difference between the electric potential at a and the potential at b.

$$\Delta V = V_b - V_a$$

This can be positive or negative, but very, very often, people also just are interested in the magnitude of it, so quote it as:

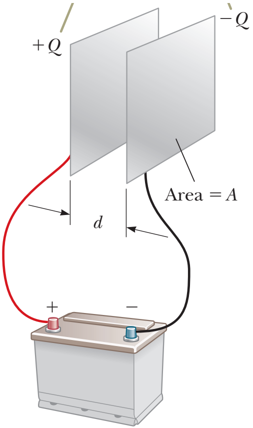
$$|\Delta V| = |V_b - V_a|$$

The book uses V instead of ΔV for the potential difference from here on out. We will stick with ΔV.

What the book does can be a bit confusing, but unfortunately it is almost universally done when talking about circuits.

Capacitance

When a battery is connected to a pair of plates so that one plate is connected to the positive terminal of the battery and the other is connected to the negative terminal, the plates become charged.



[1]Diagram from Serway & Jewett, 9th ed, page 778.

Capacitance

capacitance, C

The constant of proportionality relating the charge of the capacitor to the potential difference across it:

$$q = C\,|\Delta V|\ ; \quad C = \frac{Q}{|\Delta V|}$$

Capacitance is always positive by convention.

where ΔV is the potential difference between one plate of the capacitor and the other.

Capacitance is measured in Farads. $1\ \mathrm{F} = 1\ \mathrm{C/V}$.

C is a property of the geometry of the capacitor.

Capacitance Questions

A capacitor is altered so that the charge q is doubled on the plates, while the potential difference ΔV is held constant. Capacitance:

(A) increases

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[1] Halliday, Resnick, Walker, page 658.

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Capacitance Questions

If the potential difference is fixed, *eg.* the capacitor plates are charged by a constant 9 V battery, does capacitance

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when the separation of the plates d is doubled?

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Capacitance

$$q = C\,|\Delta V| \quad \Rightarrow \quad C = \frac{q}{|\Delta V|}$$

C is a property of the geometry of the capacitor.

A particular capacitor will have a particular fixed value of C, just like a given resistor will have a constant value of resistance R.

For a parallel plate capacitor:

$$C = \frac{\epsilon_0 A}{d}$$

where d is the separation distance of the plates and A is the area of each plate

Capacitance

Capacitors with different construction will have different values of C.

For example,
for a **cylinderical** capacitor of length L, inner radius a and outer radius b:

$$C = 2\pi\epsilon_0 \frac{L}{\ln(b/a)}$$

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for an **isolated charged sphere** of radius R:

$$C = 4\pi\epsilon_0 R$$

Parallel Plate Capacitor

Back to the parallel plate capacitor:

$$C = \frac{\epsilon_0 A}{d}$$

Let's justify why this expression should hold.

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From Guass's law:

$$Q = \epsilon_0 \Phi_E$$

$$Q = \epsilon_0 E A$$

Parallel Plate Capacitor

Also:

$$|\Delta V| = Ed$$

Taking the ratio gives:

$$\frac{Q}{\Delta V} = \frac{\epsilon_0 A}{d}$$

Confirming that

$$C = \frac{\epsilon_0 A}{d}$$

since $C = Q/(\Delta V)$.

Parallel Plate Capacitor

In particular, notice that we used expressions for

the charge on a parallel plate capacitor:

$$q = \epsilon_0 EA$$

and the potential difference across the plates of a parallel plate capacitor:

$$|\Delta V| = Ed$$

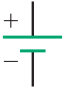
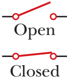
Circuits

Circuits consist of electrical **components** connected by wires.

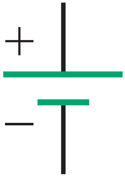
Some types of components: batteries, resistors, capacitors, lightbulbs, LEDs, diodes, inductors, transistors, chips, etc.

The wires in circuits can be thought of as channels for an electric field that distributes charge to (or charge flow through) the components.

Circuit component symbols

battery	ΔV	
capacitor	C	
switch	S	
resistor	R	
inductor	L	

Circuits: Batteries

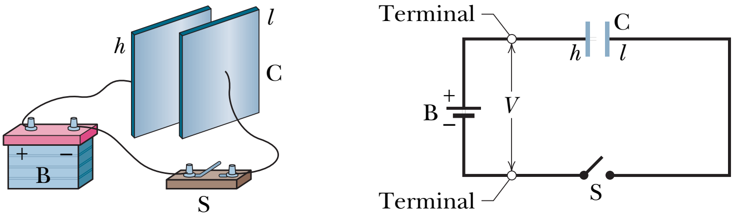


Batteries cause a potential difference between two parts of the circuit.

This drives a charge flow.

Circuits

The different elements can be combined together in various ways to make complete circuits: paths for current to flow from one terminal of a battery or power supply to the other.

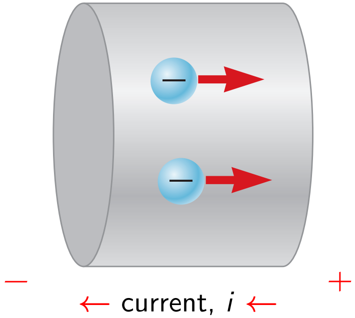


This circuit is said to be incomplete while the switch is open.

Flow of charge in a circuit

Conventional current is said to flow from the positive terminal to the negative terminal.

However, actually it is negatively charged electrons that flow through metal wires:

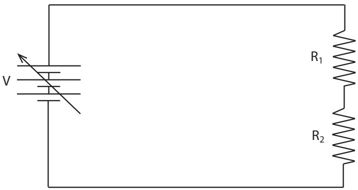


[1]Figure from Serway and Jewett, 9th ed.

Series and Parallel

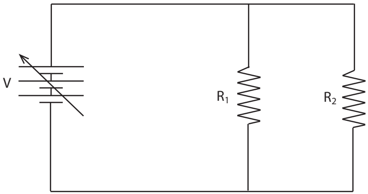
Series

When components are connected one after the other along a single path, they are connected in series.



Parallel

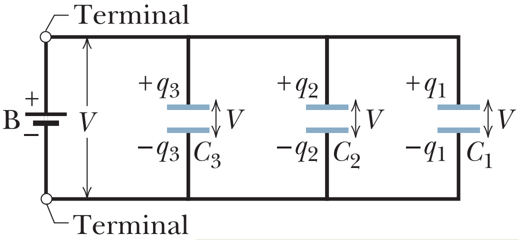
When components are connected side-by-side on different paths, they are connected in parallel.



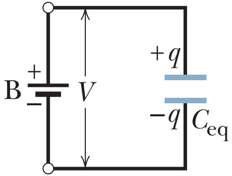
Capacitors in Parallel

Capacitors in parallel all have the **same potential difference** across them.

Three capacitors in parallel:



Equivalent circuit:



We could replace all three capacitors in the circuit with one equivalent capacitance. The current and potential difference in the rest of the circuit is unchanged by this.

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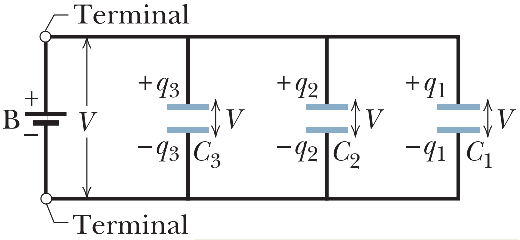

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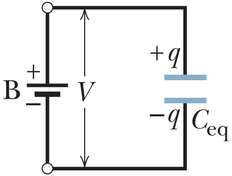

We could replace all three capacitors in the circuit with one equivalent capacitance. The current and potential difference in the rest of the circuit is unchanged by this.

What would be the capacitance of this equivalent capacitor?

Capacitors in Parallel

Capacitors in parallel all have the **same potential difference** across them.

$$\Delta V_1 = \Delta V_2 = \Delta V_3 = \Delta V$$

The total charge on the three capacitors is the **sum** of the charge on each.

$$q_{\text{net}} = q_1 + q_2 + q_3$$

where $q_1 = C_1\,\Delta V$.

Capacitance is $C = q/(\Delta V)$:

$$C_{\text{eq}} = \frac{q_{\text{net}}}{\Delta V}$$

Capacitors in Parallel

Equivalent capacitance:

$$\begin{aligned} C_{\mathrm{eq}} &= \frac{q_{\mathrm{net}}}{\Delta V} \\ &= \frac{q_1}{\Delta V} + \frac{q_2}{\Delta V} + \frac{q_3}{\Delta V} \\ &= C_1 + C_2 + C_3 \end{aligned}$$

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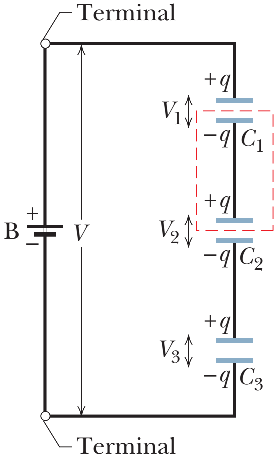
So in general, for any number n of capacitors in **parallel**, the effective capacitance of them all together is:

$$C_{\text{eq}} = C_1 + C_2 + ... + C_n = \sum_{i=1}^{n} C_i$$

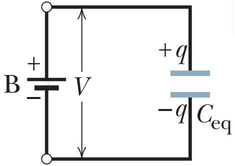
Capacitors in Series

Capacitors in series all store the **same charge**.

Three capacitors in series:



Equivalent circuit:



Capacitors in Series

Again, we could replace all three capacitors in the circuit with one equivalent capacitance and we can find the capacitance of this equivalent capacitor.

The **sum of the potential differences** across capacitors in series is V, the battery's supplied potential difference.

$$\Delta V = \Delta V_1 + \Delta V_2 + \Delta V_3$$

where $\Delta V_1 = q/C_1$, etc.
Then,

$$C_{\text{eq}} \quad = \quad \frac{q}{\Delta V}$$

Capacitors in Series

Equivalent capacitance:

$$
\begin{aligned}
C_{\text{eq}} &= \frac{q}{\Delta V} \\
&= \frac{q}{\Delta V_1 + \Delta V_2 + \Delta V_3} \\
&= \left[\frac{V_1 + V_2 + V_3}{q} \right]^{-1} \\
&= \left[\frac{\Delta V_1}{q} + \frac{\Delta V_2}{q} + \frac{\Delta V_3}{q} \right]^{-1} \\
&= \left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right]^{-1}
\end{aligned}
$$

Capacitors in Series

In general, for any number n of capacitors in **series**, we can always relate the effective capacitance of them all together to the individual capacitances by:

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + ... + \frac{1}{C_n} = \sum_{i=1}^{n} \frac{1}{C_i}$$

The equivalent capacitance of capacitors in series is always less than the smallest capacitance in the series.

Practice

A $5.0\,\mu F$ capacitor is connected in parallel with a $10\,\mu F$ capacitor. What is the equivalent capacitance of this arrangement?

Practice

A 5.0 μF capacitor is connected in parallel with a 10 μF capacitor. What is the equivalent capacitance of this arrangement?

$$C_{\text{eq}} = 15\ \mu\text{F}$$

Practice

A 5.0 μF capacitor is connected in parallel with a 10 μF capacitor. What is the equivalent capacitance of this arrangement?

$$C_{\mathrm{eq}} = 15\ \mu\mathrm{F}$$

A 5.0 μF capacitor is connected in series with a 10 μF capacitor. What is the equivalent capacitance of this arrangement?

Practice

A $5.0\,\mu\text{F}$ capacitor is connected in parallel with a $10\,\mu\text{F}$ capacitor. What is the equivalent capacitance of this arrangement?

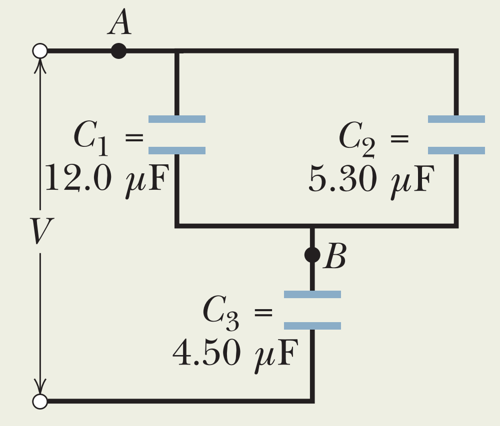
$$C_{\text{eq}} = 15\ \mu\text{F}$$

A $5.0\,\mu\text{F}$ capacitor is connected in series with a $10\,\mu\text{F}$ capacitor. What is the equivalent capacitance of this arrangement?

$$C_{\text{eq}} = 3.3\ \mu\text{F}$$

More Practice

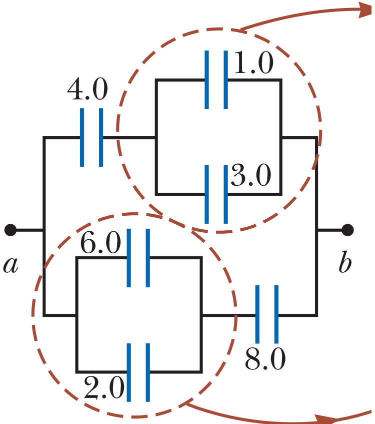
What is the equivalent capacitance of this arrangement?



More Practice

When solving this type of problem, take an iterative approach.

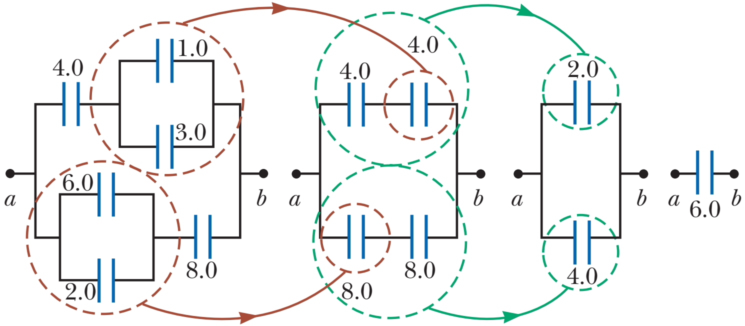
Identify sets of capacitors that are in parallel, then series, then parallel, etc. and at each step replace with the equivalent capacitance:



More Practice

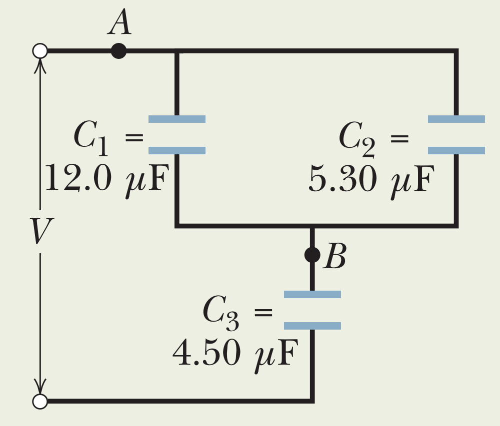
When solving this type of problem, take an iterative approach.

Identify sets of capacitors that are in parallel, then series, then parallel, etc. and at each step replace with the equivalent capacitance:



More Practice

What is the equivalent capacitance of this arrangement:



Summary

- Electric potential difference of charged plates
- electric potential and conductors
- capacitance

Homework

- worksheet

Halliday, Resnick, Walker:

- Ch 24, onward from page 651. Problems: 47, 59, 65, 73
- Ch 25, onward from page 675. Questions: 1; Problems: 1, 3, 5