Electricity and Magnetism
Capacitors and Dielectrics

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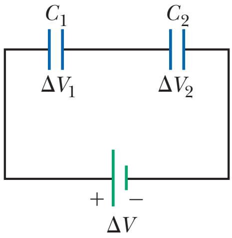
Oct 20, 2015

Last time

- circuits
- capacitors in series and parallel

Warm Up Question

Two capacitors of values 4.0 nF and 6.0 nF are connected in a circuit as shown:



(A) 4.0 nF

(B) 6.0 nF

(C) 10 nF

(D) 2.4 nF

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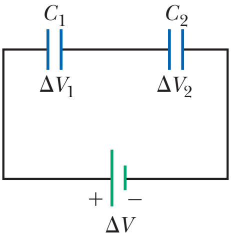

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Overview

- practice with capacitors in circuits
- energy stored in a capacitor
- dielectrics

Capacitors in Series and Parallel

In general, for any number n of capacitors in **series**, we can always relate the effective capacitance of them all together to the individual capacitances by:

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + ... + \frac{1}{C_n} = \sum_{i=1}^{n} \frac{1}{C_i}$$

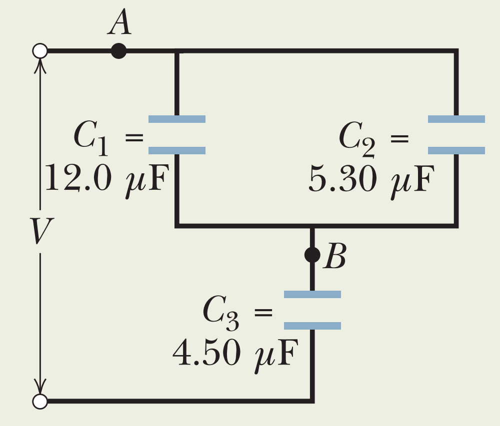
The equivalent capacitance of capacitors in series is always less than the smallest capacitance in the series.

And a reminder, in capacitors in **parallel**:

$$C_{\text{eq}} = C_1 + C_2 + ... + C_n = \sum_{i=1}^{n} C_i$$

More Practice with Multiple Capacitors

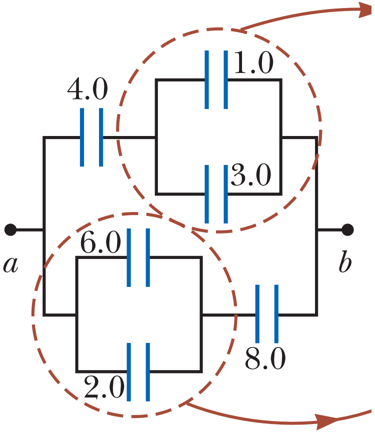
What is the equivalent capacitance of this arrangement?



More Practice

When solving this type of problem, take an iterative approach.

Identify sets of capacitors that are in parallel, then series, then parallel, etc. and at each step replace with the equivalent capacitance:



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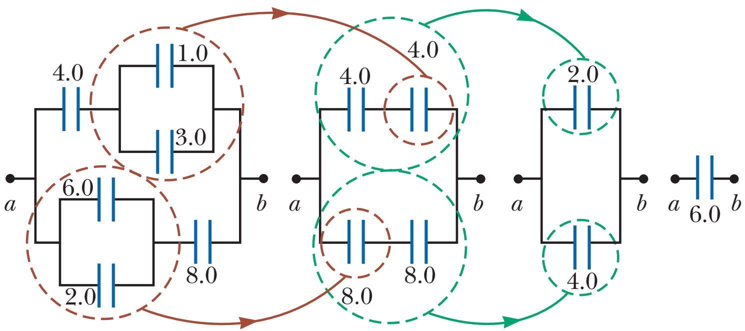

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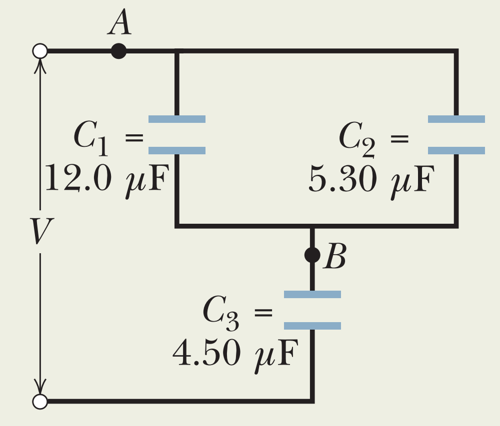

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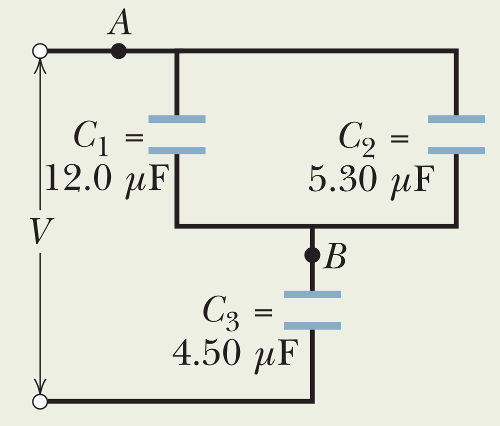

$C_{eq} = 3.57\,\mu F$.

Energy Stored in a Capacitor

A charged capacitor has an electric field between the plates. This field can be thought of as storing potential energy.

The energy stored in a capacitor with charge q and capacitance C is

$$U = \frac{1}{2}\left(\frac{q^2}{C}\right)$$

Since $q = CV$ we can also write this as:

$$U = \frac{1}{2}C\left(\Delta V\right)^2$$

Stored Energy Example

Suppose a capacitor with a capacitance 12 pF is connected to a 9.0 V battery.

What is the energy stored in the capacitor's electric field once the capacitor is fully charged?

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$$U_E = 4.9 \times 10^{-10} \text{ J}$$

Energy Density

It is sometimes useful to be able to compare the energy stored in different charged capacitors by their stored energy per unit volume.

We can link energy density to electric field strength.

This will make concrete the assertion that energy is stored in the field.

For a parallel plate capacitor, energy density u is:

$$u = \frac{U}{Ad}$$

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(Ad is the volume between the capacitor plates.)

Energy Density and Electric Field

$$\begin{aligned} u_E &= \frac{U_E}{Ad} \\ &= \frac{C(\Delta V)^2}{2Ad} \end{aligned}$$

Energy Density and Electric Field

$$u_E = \frac{U_E}{Ad} = \frac{C(\Delta V)^2}{2Ad}$$

Replace $C = \frac{\epsilon_0 A}{d}$:

$$u_E = \frac{\epsilon_0 A}{d}\frac{\Delta V^2}{2Ad} = \frac{\epsilon_0}{2}\left(\frac{\Delta V}{d}\right)^2$$

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Lastly, remember $|\Delta V| = Ed$ in a parallel plate capacitor, so:

$$u_E = \frac{1}{2}\epsilon_0 E^2$$

Dielectrics

dielectric

an insulating material that can affects the strength of an electric field passing through it

Different materials have different **dielectric constants**, κ.

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Different materials have different **dielectric constants**, κ.

κ tells us how the capacitance of a capacitor changes if the material between the plates is changed.

For air $\kappa \approx 1$. (It is 1 for a perfect vacuum.)

κ is never less than 1. It can be very large > 100.

Dielectrics and Capacitance

dielectric

an insulating material that can affects the strength of an electric field passing through it

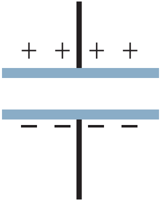
The effect of sandwiching a dielectric in a capacitor is to change the capacitance:

$$C \to \kappa C$$

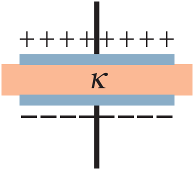
κ is the **dielectric constant**.

Dielectric in a Capacitor

Capacitance C



Capacitance κC



Adding a dielectric increases the capacitance.

Effect of a Dielectric

The most straightforward way of tracking quantities that will change when a dielectric is added is by replacing ϵ_0 in all equations with ϵ using this relation:

$$\epsilon = \kappa\epsilon_0$$

(Or just think of the effect of the dielectric being $\epsilon_0 \rightarrow \kappa\epsilon_0$.)

The electrical permittivity increases.

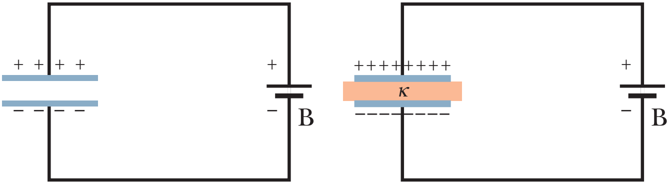
Dielectric in a Capacitor

For a parallel plate capacitor with a dielectric, the capacitance is now:

$$C = \frac{\kappa \epsilon_0 A}{d}$$

Dielectric in a Capacitor

If we add a dielectric while the capacitor is connected to a battery:



V = a constant

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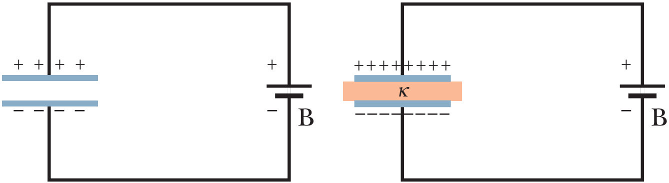
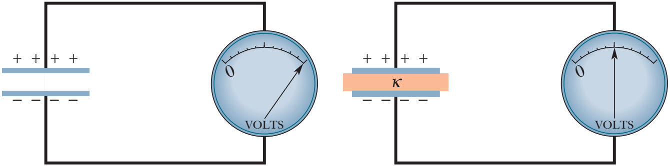


V = a constant

- q will increase. $(q = CV)$
- U will increase. $(U = \frac{1}{2}CV^2)$

Dielectric in a Capacitor

If we add a dielectric while the capacitor is isolated so charge cannot leave the plates:



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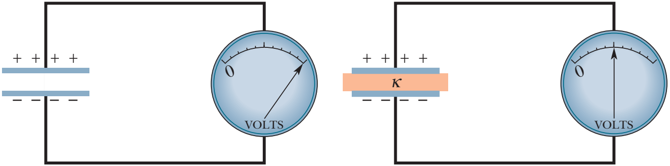
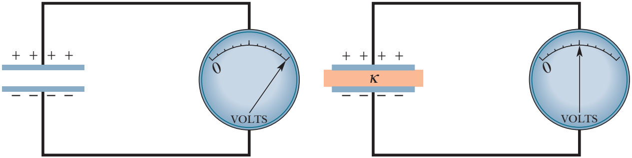


q = a constant

- V will decrease. $(V = \frac{q}{C})$
- U will decrease. $(U = \frac{q^2}{2C})$

Effect of a Dielectric on Field

Imagine again the isolated conductor: charge density σ is constant.



q = a constant

The electric field between the plates is $E = \frac{\sigma}{\epsilon_0}$ originally.

With dielectric added: $E \rightarrow \frac{\sigma}{\kappa\epsilon_0}$.

The field strength decreases! $E \rightarrow \frac{E}{\kappa}$

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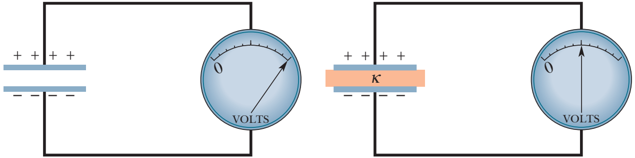

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What happens to the energy density u?

Effect of a Dielectric on Field

What happens to the energy density? Was: $u_0 = \frac{1}{2}\epsilon_0 E_0^2$.

$$u = \frac{1}{2}\left(\kappa\epsilon_0\right)\left(E\right)^2$$

Effect of a Dielectric on Field

What happens to the energy density? Was: $u_0 = \frac{1}{2}\epsilon_0 E_0^2$.

$$\begin{aligned} u &= \frac{1}{2}\left(\kappa\epsilon_0\right)\left(E\right)^2 \\ &= \frac{1}{2}\left(\kappa\epsilon_0\right)\left(\frac{\sigma}{\kappa\epsilon_0}\right)^2 \\ &= \frac{1}{2}\epsilon_0\kappa\left(\frac{1}{\kappa^2}\right)E_0^2 \\ &= \frac{1}{\kappa}\left(\frac{1}{2}\epsilon_0 E_0^2\right) \\ u &= \frac{u_0}{\kappa} \end{aligned}$$

Energy density decreases.

Dielectrics and Electric Field

Dielectrics effect the field around a charge

$$E \to \frac{E}{\kappa}$$

For example, for a point charge q in free space:

$$E_0 = \frac{k\,q}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

But in a dielectric, constant κ:

$$E = \frac{1}{4\pi(\kappa\epsilon_0)}\frac{q}{r^2} = \frac{E_0}{\kappa}$$

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But *how* does this happen?

Dielectrics and Electric Field

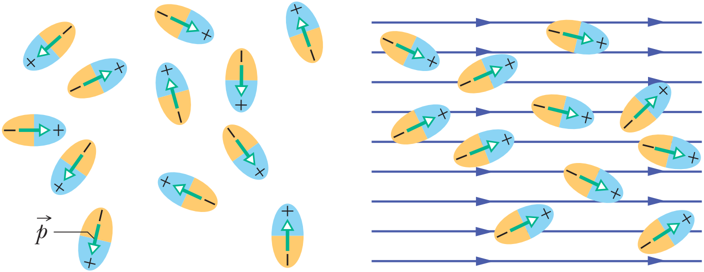
Dielectrics become polarized by the presence of an electric field.

There are two types of dielectrics, the process is a little different in each:

- polar dielectrics
- nonpolar dielectrics

Polar Dielectrics

The external electric field partially aligns the molecules of the dielectric with the field.



Since the dielectric is an insulator, there are no free charges to move through the substance, but molecules can align.

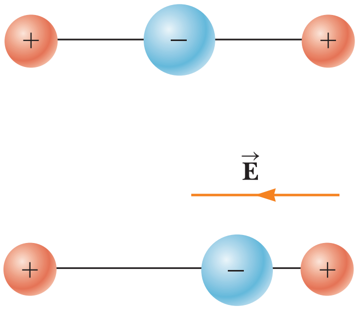
eg. distilled water

[1]Figures from Halliday, Resnick, Walker, 9th ed.

Nonpolar Dielectrics

Nonpolar dielectrics are composed of molecules which are not polar.

However, under the influence of a field, the distribution of the electrons in the molecules, or the shape of the molecule, is altered. Each molecule becomes slightly polarized.



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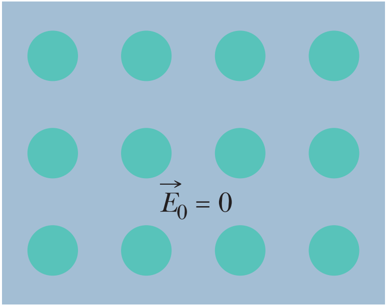
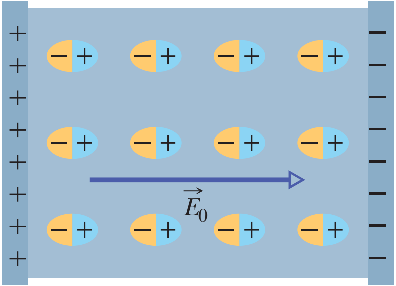
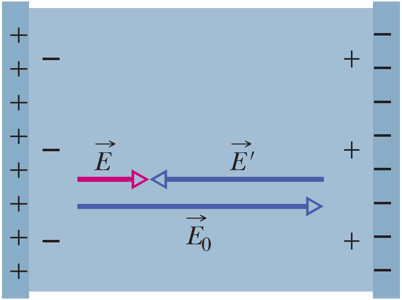




eg. nitrogen, benzene

Electric field inside the dielectric

The polarized dielectric contributes its own field, E'.



This reduces the electric field from the charged plates alone E_0.

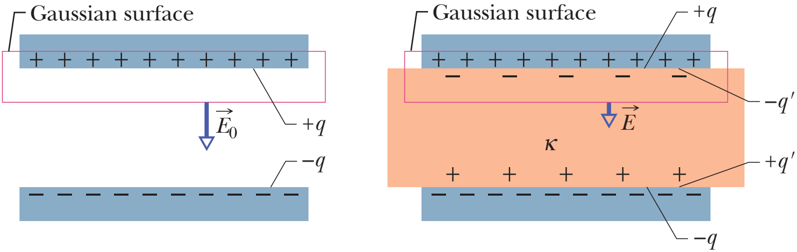
The resulting reduced field is $E = \frac{E_0}{\kappa}$

Guass's Law with dielectrics

$$\kappa \epsilon_0 \Phi_E = q_{\text{free}}$$

or:

$$\oint_A \mathbf{E} \cdot d\mathbf{A} = \frac{q_{\text{free}}}{\kappa \epsilon_0}$$



The charge $q_{\text{free}} = q$ in the diagram. It is just the charge on the plates, the charge that is free to move.

Electric Displacement

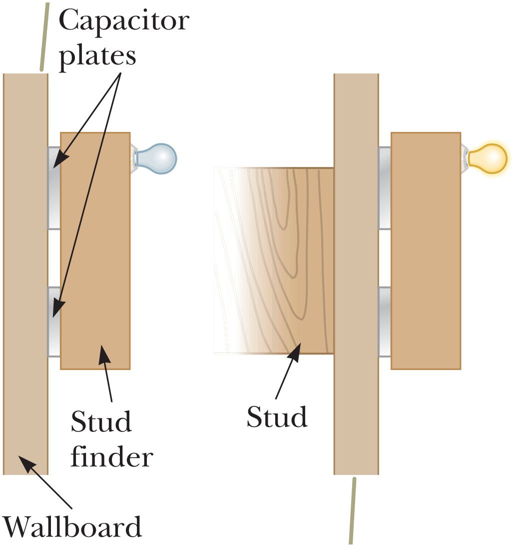
It is sometimes convenient to package the effect of the electric field together with the effect of the dielectric.

For this, we introduce a new quantity, **Electric Displacement**.

$$\mathbf{D} = \kappa \epsilon_0 \mathbf{E}$$

Gauss's law is very often written in terms of the electric displacement, rather than the electric field, if the field being studied is in a polarizable material.

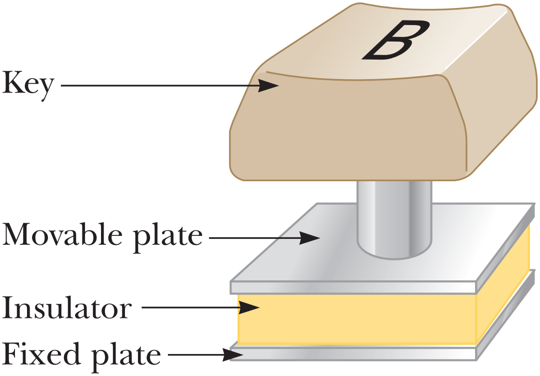
Uses of Dielectric Effects



[1] Figures from Serway & Jewett, 9th ed.

Uses of Dielectric Effects

Computer keyboard:



Summary

- practice with capacitors in circuits
- energy stored in a capacitor
- dielectrics

Homework Halliday, Resnick, Walker:

- PREVIOUS: Ch 25, onward from page 675. Questions: 1, 3, 5; Problems: 1, 3, 5
- NEW: Ch 25, Problems: 9, 11, 13, 19, 29, 31, 45