## Physics 4A: Assignment 3 Winter 2019

Please do not write your solutions on this question paper. You might like to use a separate piece of paper for each question. Solutions are not considered complete without the logical argument and/or full calculation.

1. Alice and Bob sit in a 35.0 kg canoe floating on a lake. Bob has mass 75.0 kg and Alice is lighter that Bob. When the canoe is at rest in still water, they switch seats, which are 3.00 m apart and symmetrically located with respect to the canoe's center. If the canoe moves 38.0 cm horizontally relative to a fixed point on the ground, what is Alice's mass?

(see next page for question 2)

## 2. (a) Prove that

$$\frac{\omega_f^2 - \omega_i^2}{2} = \int_{\theta_i}^{\theta_f} \alpha \, \mathrm{d}\theta$$

where  $\alpha$  is the angular acceleration, and  $\omega_i$  and  $\omega_f$  are the initial and final angular velocities, respectively, at any initial and final angles  $\theta_i$  and  $\theta_f$ .

For the rest of the question, consider a particle of mass m that is attached to the end of a light rod of length r, as shown in the diagram. One end of the rod is fixed in place and the rod can rotate about that point. The rod starts out vertically upward at rest, and is perturbed just slightly so that the mass and rod rotate downwards under the influence of gravity.



- (b) Find an expression for  $\alpha$  for the mass-and-rod system as a function of  $\theta$ . (Hint: consider the tangential acceleration.)
- (c) Find the final angular velocity  $\omega$  of the mass-and-rod system when it has swung all the way down and  $\theta = \pi$ .
- (d) Suppose the kinetic energy of a rigid, rotating system can be expressed as  $K = \frac{1}{2}I\omega^2$ , where I is some constant for this system. By considering the conservation of mechanical energy, find an expression for I for this system in terms of m and r.