Physics 4A Fall 2017 Final Exam

Name: Key

Dec 12, 2017

Please show your work! Answers are not complete without clear reasoning. When asked for an expression, you must give your answer in terms of the variables given in the question and/or fundamental constants.

Answer as many questions as you can, in any order. Do not forget to include appropriate units when giving a number as an answer. Calculators are allowed. Notes, books, and internet-connectable devices are not allowed. If you detach any pages from the test, please write your name on every page.

Constants

$G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}

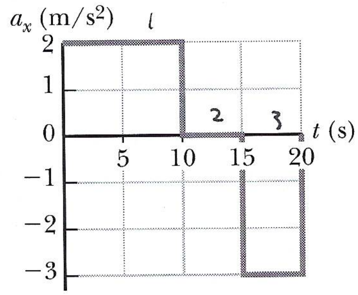
$g = 9.8$ m s^{-2}

1. A particle starts from rest and accelerates as shown, moving in the x-direction.

   (a) During which time interval(s) is the speed of the particle constant? [1 pt]

   (b) Determine the particle's speed at $t = 5.0$ s, $t = 10.0$ s, and at $t = 20.0$ s. [6 pts]

   (c) Determine the distance traveled in the 20.0 s shown. [7 pts]



a) $v = $ const. when accel. $a = 0$ $\left(a = \frac{dv}{dt}\right)$

↳ t: $\underline{10s < t < 15s}$

b) $v(t) = v_i + at$ (a const) or Since $v_i = 0$, $v =$ Area under a-t graph

$v(t=5) = \cancel{v_i}^{\,0} + (2m/s^2)(5s)$

$\underline{V(t=5) = 10 \text{ m/s}}$

$v(t=5) = (2m/s^2)(5s)$

$= 10 m/s$

$v(t=10s) = 0 + (2m/s^2)(10s)$

$\underline{v(t=10) = 20 m/s}$

$v(t=20s) = \cancel{v_i}^{\,0} + a_1t_1 + \cancel{a_2}^{\,0}t_2 + a_3t_3$

$= (2m/s^2)(10s) + (-3m/s^2)(5s)$

$\underline{v(t=20) = 5 m/s}$

c) $\Delta X = \left(\frac{V_i + V_f}{2}\right)t$ or $\Delta X = V_i t + \frac{1}{2}at^2$ ← when $a = $ const

$\Delta X_{0\to10} = 0 + \frac{1}{2}(2m/s^2)(10s)^2$

$= 100 m$

$\Delta X_{10\to15} = Vt$

$= (20m/s)(5s)$

$= 100 m$

$\Delta X_{15\to20} = vt + \frac{1}{2}a_3t^2$

$= (20m/s)(5s) + \frac{1}{2}(-3m/s^2)(5s)^2$

$= 62.5 m$

$\Delta X = \Delta X_{0\to10} + \Delta X_{10\to15} + \Delta X_{15\to20}$

$= 262.5 m$

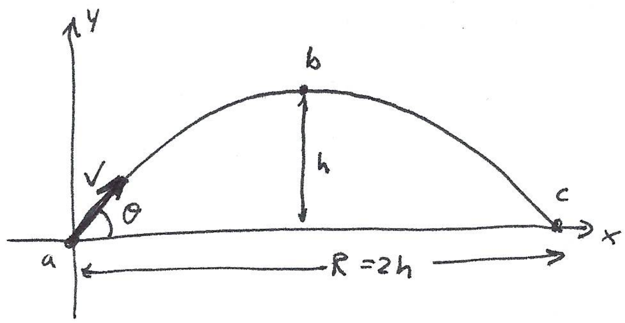
$= \underline{263 m}$ (3 sig figs)

2. A projectile is fired in such a way that its horizontal range (landing at the same height it is launched from) is equal to twice its maximum height.

   (a) What is the angle of projection? [7 pts]

   (b) What is the ratio of the x-component of the initial velocity to the initial speed? [3 pts]

For both parts, your answer should be a number, not an expression.



a) $\theta = ?$

a → b

$$V_{fy}^2 = V_{iy}^2 - 2g\,\Delta y$$

$$0 = V_{iy}^2 - 2gh$$

$$V_i^2 \sin^2\theta = 2gh$$

$$2h = \frac{V_i^2 \sin^2\theta}{g} \quad —(1)$$

$$R = 2h = V_x t_{f1}$$

$$\Delta y = V_{iy} t - \frac{1}{2} g t^2$$

a → c

$$0 = V_{iy} \cancel{t_{f1}} - \frac{1}{2} g\, t_{f1}^{\cancel{2}}$$

$$t_{f1} = \frac{2V_i \sin\theta}{g}$$

$$2h = V_i \cos\theta \left(\frac{2V_i \sin\theta}{g}\right) \quad —(2)$$

(1) = (2):

$$\frac{\cancel{V_i^2} \sin^{\cancel{2}}\theta}{\cancel{g}} = \frac{2\cancel{V_i^2} \cos\theta\, \cancel{\sin\theta}}{\cancel{g}}$$

$$\frac{\sin\theta}{\cos\theta} = 2$$

$$\tan\theta = 2$$

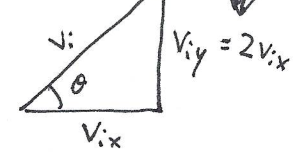
$$\theta = \tan^{-1}(2)$$

$$\underline{\theta = 63.4°}$$

b) $\frac{V_{ix}}{V_i} = ?$

$\tan\theta = 2$

⇓



$$V_i = \sqrt{V_{ix}^2 + (2V_{ix})^2}$$

$$V_i = V_{ix}\sqrt{5}$$

$$\frac{V_{ix}}{V_x} = \underline{\frac{1}{\sqrt{5}}} = \underline{0.447}$$

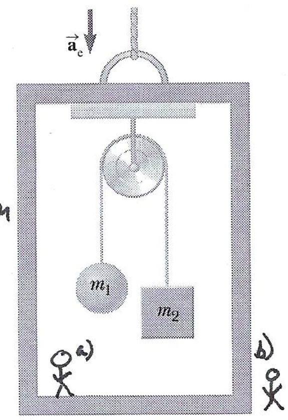
3. An Atwood machine is placed in an elevator that accelerates downward with an acceleration of magnitude a_e. You may assume $m_2 > m_1$ and the pulley is massless and frictionless.

(a) What is the acceleration of m_1 as seen by an observer in the elevator? [7 pts]

(b) What is the acceleration of m_1 as seen by an observer on the the ground outside the elevator? [2 pts]

(c) What is the tension in the rope connecting the masses? [3 pts]

You must show an argument that considers the forces on the masses to get full credit. (You cannot just write down the answers.)

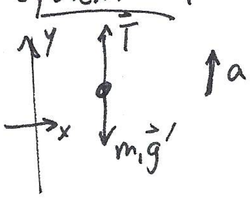


a) Inside the elevator the situation is indistinguishable from a planet where the acceleration due to gravity is reduced:

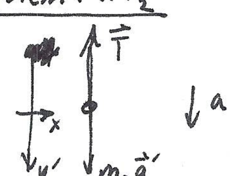
$$g' = g - a_e$$

The standard derivation for the Atwood machine still applies, replacing $g \to g'$.

System: m_1



System: m_2



$$F_{net,y} = m_1 a_{1y} \qquad\qquad F_{net,y'} = m_2 a_{2y'}$$

$$T - m_1 g' = m_1 a \text{ —— (1)} \qquad m_2 g' - T = m_2 a \text{ —— (2)}$$

(1) + (2):

$$m_2 g' - m_1 g' = (m_1 + m_2) a$$

$$a = \left(\frac{m_2 - m_1}{m_1 + m_2}\right) g'$$

$$a = \frac{(m_2 - m_1)}{(m_1 + m_2)} (g - a_e)$$

$$\vec{a}_1 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)(g - a_e)\,\vec{j}$$

b) Let a_b be the accel. seen by the observer outside the elevator.

$$\vec{a}_b = \vec{a}_1 + \vec{a}_e$$

$$\vec{a}_b = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)(g - a_e)\vec{j} - a_e\vec{j}$$

$$\left(= \left[\left(\frac{m_2 - m_1}{m_1 + m_2}\right) g - \left(\frac{2m_2}{m_1 + m_2}\right) a_e\right] \vec{j} \right)$$

c) Using (1):

$$T - m_1 g' = m_1 a$$

$$T = m_1 (g' + a)$$

$$= m_1\left(g' + \left(\frac{m_2 - m_1}{m_1 + m_2}\right) g'\right)$$

$$= m_1\left(\frac{\cancel{m_1} + m_2 + m_2 - \cancel{m_1}}{m_1 + m_2}\right) g'$$

$$T = \frac{2 m_1 m_2}{m_1 + m_2}(g - a_e)$$

4. A block of mass m is attached to a spring of force constant k as shown. The block is pulled to a position x_i to the right of equilibrium and released from rest at time $t = 0$.

If the surface is frictionless:

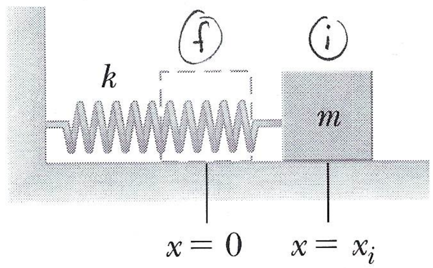
(a) Write down an expression for the equation of motion of the block (that is, specify how its position depends on time). If your expression uses other variables not given in this question, you must give their values. (You do not have to derive this expression.) [4 pts]

(b) What is the time period for this motion? [1 pt]

(c) Find the speed the block has as it passes through equilibrium. [4 pts]

If the surface has friction, and the speed that the block has when it first passes its equilibrium position is one-quarter the value found in part (c):

(d) Find an expression for the coefficient of kinetic friction μ_k. [5 pts]



a) Mass on a spring:

$x(t) = x_{max}\cos(\omega t + \phi)$ $\omega = \sqrt{\frac{k}{m}}$

released from rest at $x = x_i$:

$\underline{x(t) = x_i \cos\left(\sqrt{\frac{k}{m}}\, t\right)}$

b) $\underline{T = 2\pi\sqrt{\frac{m}{k}}}$ $\left(T = \frac{2\pi}{\omega}\right)$

c) Two ways: Conservation of energy

or using $v = \frac{dx}{dt} = \underbrace{-\omega x_i}_{= v_{max}} \sin(\omega t)$.

Cons. of Energy:

System: mass + spring (isolated)

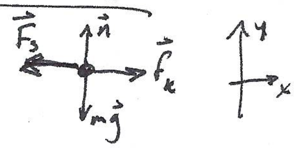
$\Delta K + \Delta U = 0$

$K_f - \cancel{K_i}^{0} + \cancel{U_f}^{0} - U_i = 0$

$\frac{1}{2} m \underset{v_{eq}}{\overset{\|}{v_f^2}} - \frac{1}{2} k x_i^2 = 0$

$\underline{v_{eq} = \sqrt{\frac{k}{m}}\, x_i}$

d) System: block



$F_{net,y} = m \cancel{a_y}^{0}$

$\Rightarrow n = mg$

$\Rightarrow f_k = \mu_k mg$

(i) → (f) System: block, spring, block + surface internal degr. of freedom

$\Delta K + \Delta U + \Delta E_{int} = 0$

$K_f - \cancel{K_i}^{0} + \cancel{U_f}^{0} - U_i + f_k x_i = 0$

$\frac{1}{2} m v_f^2 - \frac{1}{2} k x_i^2 + \mu_k m g x_i = 0$

$\frac{1}{2} m \left(\frac{v_{eq}}{4}\right)^2 - \frac{1}{2} k x_i^2 + \mu_k m g x_i = 0$

$\mu_k m g x_i = \frac{1}{2}\left(-\frac{\cancel{m}}{16}\left(\frac{k}{\cancel{m}} x_i^2\right) + k x_i^2\right)$

$\mu_k = \frac{\frac{15}{16} k x_i^2}{2 m g x_i}$

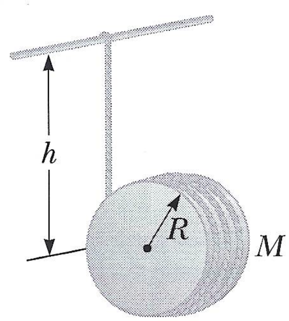
$\underline{\mu_k = \frac{15 k x_i}{32 m g}}$

5. A light string is wound around a uniform disk of radius R and mass M. The disk is released from rest with the string vertical and its top end tied to a fixed bar.

(a) Show that the tension in the string is one third of the weight of the disk. [7 pts]

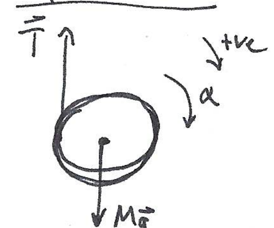
(b) Find the magnitude of the acceleration of the center of mass of the disk. [2 pts]

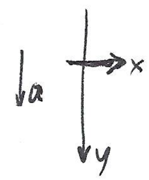
(c) Find the speed of the center of mass after the disk has descended through distance h. [3 pts]



a) system : disk

unrolls $\Rightarrow a = \alpha R$





$$F_{net,y} = Ma$$

$$Mg - T = Ma \quad \text{—(1)}$$

$$\tau_{net} = I\alpha$$

$$\cancel{R}T = \left(\tfrac{1}{2}MR^{\cancel{2}}\right)\alpha$$

$$T = \tfrac{1}{2}M(\alpha R)$$

$$T = \tfrac{1}{2}Ma$$

$$a = \frac{2T}{M} \quad \text{—(2)}$$

(2) into (1):

$$Mg - T = \cancel{M}\left(\frac{2T}{\cancel{M}}\right)$$

$$Mg = 3T$$

$$\underline{T = \frac{Mg}{3}}$$

b) $a_{cm} = a$

(2): $a = \dfrac{2T}{M}$

$$a = \frac{2}{\cancel{M}}\left(\frac{\cancel{M}g}{3}\right)$$

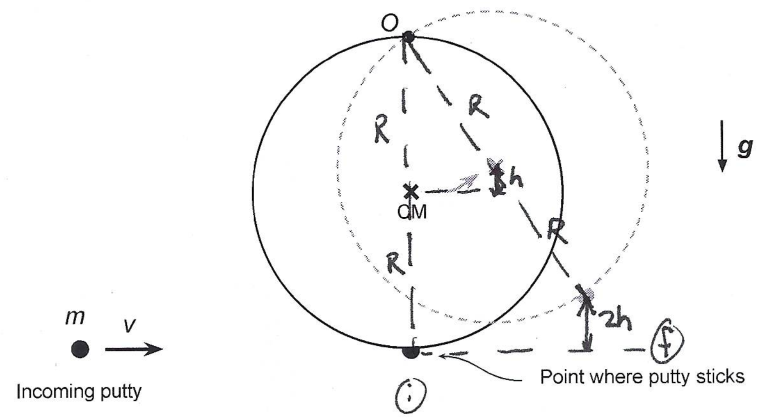
$$\underline{a_{cm} = \frac{2g}{3}}$$

c) $V_{cm}^2 = \cancel{V_{cm_i}^2}^{\,0} + 2a\,\Delta y$

$$V_{cm}^2 = 2\left(\frac{2g}{3}\right)h$$

$$\underline{V_{cm} = \sqrt{\frac{4gh}{3}}}$$

6. A small lump of putty, of mass m, moving initially with velocity $\mathbf{v}$, strikes the bottom outside edge of a uniform circular disk of mass $M = 8m$ and radius R that hangs vertically downward from a frictionless pivot at the point O on its top edge as shown. The initial velocity $\mathbf{v}$ is perpendicular to a line drawn downward from O through the center of mass of the disk. The putty sticks to the disk after striking it.



CM rises a vertical distance h.

Putty rises a distance $2h$.

Without calculating:

(a) Is linear momentum conserved in the collision? Why or why not? [2 pts]

(b) Is angular momentum conserved about axis O in the collision? Why or why not? [2 pts]

(c) Is kinetic energy conserved in the collision? Why or why not? [2 pts]

After being struck the disk swings to the right.

System is disk + putty.

(d) To what height above its initial position does the center of mass of the disk rise before coming momentarily to rest? [11 pts]

a) No. External forces act from the pivot on the disk during the collision.

$$\vec{F}_{\substack{ext\\net}} = \frac{d\vec{p}}{dt}$$

b) Yes. There is no net external torque in this collision about O. The lever arms of the weight and the pivot forces are both zero. $\vec{\tau}_{\substack{net\\ext,O}} = \frac{d\vec{L}_O}{dt} = 0 \Rightarrow \Delta\vec{L}_O = 0$

c) No. This is a completely inellastic collision.

d) During collision:

$$\Delta\vec{L}_O = 0$$

$$\vec{L}_{i,O} = \vec{L}_{f,O}$$

$$mv(2R) = I_{tot}\,\omega_f \quad (\text{ccw } +\text{ve})$$

$$2mvR = \Big(\underbrace{\big(\tfrac{1}{2}MR^2 + MR^2\big)}_{\text{disk about } O\text{, using} \parallel \text{axis theorem.}} + m(2R)^2\Big)\omega$$

$$\omega = \frac{2\cancel{m}v\cancel{R}}{\frac{3}{2}(8\cancel{m})R^{\cancel{2}} + 4\cancel{m}R^{\cancel{2}}}$$

$$= \frac{v}{8R}$$

After collision, Mechanical energy is conserved.

$$\Delta K + \Delta U = 0 \quad (\text{system: disk, putty, Earth})$$

$$\left(0 - \tfrac{1}{2}I_{tot}\,\omega^2\right) + \left(Mgh + mg(2h) - 0\right) = 0$$

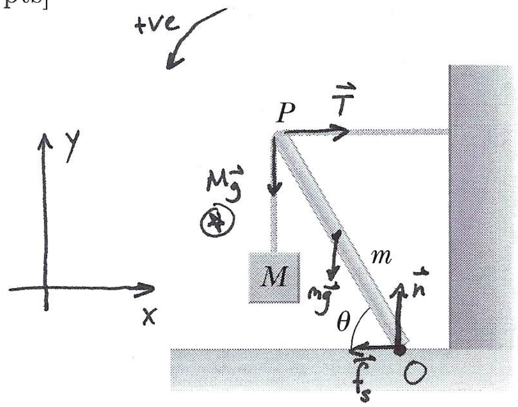
$$10\cancel{m}gh = \tfrac{1}{2}\left(\tfrac{3}{2}8\cancel{m}R^2 + 4\cancel{m}R^2\right)\left(\frac{v}{8R}\right)^2$$

$$\underline{h = \frac{v^2}{80g}}$$

7. A uniform beam of mass m is inclined at an angle θ to the horizontal. Its upper end (point P) produces a 90° bend in a very rough rope tied to a wall, and its lower end rests on a rough floor. Let μ_s represent the coefficient of static friction between beam and floor. In terms of m, M, and μ_s:

(a) find $\theta_{\min}$ the minimum value the angle θ can take if the beam is not to slip. [9 pts]

(b) find the magnitude of the reaction force on the beam at the floor, given $\theta = \theta_{\min}$. [2 pts]



System : mass M

$F_{net,y} = Ma_y^{\,0}$

$T' - Mg = 0$

$\Rightarrow T' = Mg$ (*)

a) system : beam in static equilibrium $a = 0,\ \alpha = 0$

$F_{net,x} = 0$

$T - f_s = 0$

$T = f_s$

at point of slipping

$f_s = f_s^{max} = \mu_s n$

$T = \mu_s n$ —(1)

$F_{net,y} = 0$

$n - Mg - mg = 0$

$n = (M+m)g$ —(2)

let l be the length of the beam.

$\tau_{net,O} = 0$

$Mgl\cos\theta - Tl\sin\theta + mg\frac{l}{2}\cos\theta = 0$

$(M + \frac{m}{2})g\cos\theta - T\sin\theta = 0$

using (1) and (2):

$(M+\frac{m}{2})g\cos\theta_{min} = \mu_s(M+m)g\sin\theta_{min}$

$$\tan\theta_{min} = \frac{(M+\frac{m}{2})}{\mu_s(M+m)}$$

$$\theta_{min} = \tan^{-1}\left(\frac{M+\frac{m}{2}}{\mu_s(M+m)}\right)$$

b) $R = \sqrt{n^2 + f_s^2}$ (R_y, R_x)

$= \sqrt{(M+m)^2g^2 + \mu_s^2(M+m)^2g^2}$ (using (2))

$R = (M+m)g\sqrt{1+\mu_s^2}$

—Extra Workspace—

—Extra Workspace—

Equations

$x_f = x_i + v_i t + \frac{1}{2}at^2$

$v_f = v_i + at$

$v_{\text{avg}} = \frac{v_i + v_f}{2}$

$v_f^2 = v_i^2 + 2a\Delta x$

$x_f = x_i + v_{\text{avg}} t$

$\omega = \frac{2\pi}{T}$

$v = r\omega$

$a_t = r\alpha$

$a_c = \frac{v^2}{r}$

$v_f = v_i + v_e \ln\left(\frac{m_i}{m_f}\right)$

$\text{Thrust} = v_e \frac{\text{dm}}{\text{dt}}$

$\mathbf{r}_{\text{CM}} = \frac{1}{M_{\text{tot}}} \sum_i m_i \mathbf{r}_i$

$I = \sum_i m_i r_i^2$

$I' = I_{CM} + MD^2$

$T = 2\pi\sqrt{\frac{m}{k}}$

$T = 2\pi\sqrt{\frac{L}{g}}$

$T = 2\pi\sqrt{\frac{I}{mgd}}$

$x = A\cos(\omega t + \phi)$

$K = \frac{1}{2}mv^2$

$U_g = mgy$

$U_s = \frac{1}{2}kx^2$

$U_G = -\frac{Gm_1m_2}{r}$

$W = \int \boldsymbol{\tau} \cdot \text{d}\boldsymbol{\theta} = \int \boldsymbol{F} \cdot \text{ds}$

$P = \boldsymbol{\tau} \cdot \boldsymbol{\omega} = \mathbf{F} \cdot \mathbf{v}$

$F_x = -\frac{\text{dU}}{\text{dx}}$

$\mathbf{R} = -b\mathbf{v}$

$v(t) = v_T(1 - e^{-bt/m})$

$R = \frac{1}{2}D\rho A v^2$

$v(t) = v_T \tanh\left(\frac{g}{v_T}t\right)$

$\frac{\text{dA}}{\text{dt}} = \frac{L}{2M_p}$

$T^2 = \left(\frac{4\pi^2}{GM}\right)a^3$

$\mathbf{p} = m\mathbf{v}$

$\mathbf{F} = \frac{\text{d}\mathbf{p}}{\text{dt}}$

$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$

$\mathbf{L} = \mathbf{r} \times \mathbf{p}$

$\boldsymbol{\tau} = \frac{\text{d}\mathbf{L}}{\text{dt}}$

$\mathbf{I} = \int \mathbf{F}(t)\,\text{dt}$

$\Delta\mathbf{L} = \int \boldsymbol{\tau}\,\text{dt}$

$\mathbf{L} = I\boldsymbol{\omega}$

$L = mvR$

$\omega_p = \frac{Mg\,r_{\text{CM}}}{I\omega}$

$F_k = \mu_k n$

$F_{s,\max} = \mu_s n$

$\mathbf{F} = -k\mathbf{x}$

$\mathbf{F}_G = -\frac{Gm_1m_2}{r^2}\hat{\mathbf{r}}$

$E = -\frac{Gm_1m_2}{2a}$

Moments of Inertia

All objects listed here have mass M.

Thin rod, length L, axis through CM perpendicular to rod: $I = \frac{1}{12}ML^2$

Solid sphere, radius R, axis through CM: $I = \frac{2}{5}MR^2$

Cylinder or disc, radius R, axis through CM: $I = \frac{1}{2}MR^2$

Thin ring, radius R, axis through CM: $I = MR^2$

Trigonometric Identities

$\sin^2\theta + \cos^2\theta = 1$

$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$.

$\cos(2\theta) = \cos^2\theta - \sin^2\theta$

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$

$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$

$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha + \beta) + \sin(\alpha - \beta)]$

$\sin\left(\theta + \frac{\pi}{2}\right) = \cos\theta$

$\cos\left(\theta + \frac{\pi}{2}\right) = -\sin\theta$

$\sec\theta := \frac{1}{\cos\theta}$

$\csc\theta := \frac{1}{\sin\theta}$

$\cot\theta := \frac{1}{\tan\theta}$