# LAB 1: Measuring Devices, Uncertainty, and Density*

**Equipment List:**
- one aluminum block
- meter stick
- metric ruler
- triple-beam balance
- digital balance
- vernier calipers
- micrometer

**Purpose:** To familiarize you with various measuring devices and measurement uncertainty. You will gain familiarity with how to perform error propagation to understand how uncertainty in individual measurements leads to a range of possible values for a quantity calculated from these measurement. You will also get hands-on experience calculating a density.

**Introduction:** In this experiment you will measure the density of an aluminum block and find the uncertainty in your value. The density, $\rho$ ("rho"), of a uniform object is the ratio of the object's mass to its volume, $\rho = \frac{m}{V}$.

Use the following general error propagation equation to analyze the errors involved in making calculations involving measurements with their own uncertainty. If $f$ is a function of the values $x,\ y,\ z$ each corresponding uncertainty $\delta x,\ \delta y,\ \delta z$, then the uncertainty in $f$ is given by

$$\delta f = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 (\delta x)^2 + \left(\frac{\partial f}{\partial y}\right)^2 (\delta y)^2 + \left(\frac{\partial f}{\partial z}\right)^2 (\delta z)^2}\,. \tag{1}$$

Strictly speaking, this equation should be written in terms of the standard deviations of the measurements. An individual measurement, say of length, would be repeated a very large number of times and the best estimate for the true value of the length would be the average of all the measurements. The standard deviation could then be interpreted as the uncertainty of the value and the standard deviation of $f$ would be related to the standard deviations of $x,\ y$, and $z$ as in equation (1). To save time in this lab, we will instead take each measurement only once, and assume that the imperfect precision of the measuring device is the biggest source of variation and uncertainty. We can use this formula because we assume the errors caused by imprecise measurements are random.

As an example of how to use equation (1), suppose the area $A$ of a rectangle is calculated from measurements of its length $x$ and width $y$. The area of a rectangle is given by

$$A = xy$$

*Based on labs by Prof. Luna and Prof. Newton.

The partial derivatives $\frac{\partial A}{\partial x} = y$ and $\frac{\partial A}{\partial y} = x$ so that the error propagation equation for the uncertainty in $A$ becomes

$$\delta A = \sqrt{y^2 (\delta x)^2 + x^2 (\delta y)^2}\,.$$

The uncertainty in the area can now be calculated from the measured values $x$ and $y$ and their uncertainties $\delta x$ and $\delta y$.

**Note**: for this lab any measurements and calculations should be stated in the standard form of: measurement value $= x_{\text{meas}} \pm \delta x$.

**Procedure:**

1. Write down the measurement uncertainties for a measurement of length using a metric ruler, vernier calipers, and a micrometer, and for a measurement of mass using a triple-beam balance and a digital balance.
2. Use the error propagation equation derive an expression for the uncertainty in the volume of a rectangular object, $\delta V$, in terms of the uncertainties of three length measurements.
3. Measure the dimensions of the aluminum block with the metric ruler, vernier calipers, and micrometers.
4. Measure the mass with the block with the digital balance and triple-beam balance.
5. Calculate the volume and uncertainty of the block using the dimensions obtained from the metric ruler, vernier calipers, and micrometers. (Calculate the uncertainty in the volume of the block $\delta V$ using the equation you derived earlier.)
6. Using the error propagation equation derive an expression for the uncertainty $\delta\rho$ for the density of the block in terms of the uncertainty of the mass $\delta m$ and the uncertainty of the volume $\delta V$.
7. Calculate the density and uncertainty of the block by using the measurements obtained from the triple-beam balance and metric ruler.
8. Calculate the density and uncertainty of the block by using the measurements obtained from the digital balance and vernier caliper.
9. Calculate the density and uncertainty of the block by using the measurements obtained from the digital balance and micrometer.

**Analysis and Conclusion:**

1. Calculate the percentage error between your calculate value of density and the expected value of 2.699 g/cm$^3$.
2. Which of the 3 densities gave the most accurate answer? Was this what you expected why or why not? Explain.
3. Was the propagating error involved in calculating the density significant with any combination of the measuring devices? Explain.

4. What were the random errors involved and how did they affect the density and uncertainty calculation?

5. What systematic errors were involved?

6. Comment on any other sources of error that could have been involved.

7. Would repeating each measurement 10 times have any advantages for improving the accuracy and/or precision of the final density value?

8. Can you think of any ways to improve or extend this experiment?