LAB 2: Behr Free Fall*

Equipment List:

Behr Free Fall Apparatus and spark timer (shared by class)
two-meter stick
masking tape
wax paper tape with Behr Free Fall data on it produced by the apparatus.

Purpose: To investigate the acceleration due to gravity in free fall and understand two different graphical methods for finding instantaneous speeds. Even though the position of the falling object is not linearly dependent on time, it will still be possible to determine the instantaneous velocity (the derivative of the position-time curve) accurately. In this lab all graphs will be plotted by hand.

Introduction: The Behr Free Fall apparatus produces a written record of a freely falling object's position at equal intervals of time. The falling object is called the "bob". The bob falls a distance of about two meters. Falling from rest this would allow for a total time of flight $t = \sqrt{2y/g} \approx 0.6$ seconds, where y is the distance of fall (derive and confirm this). Since the bob is in flight for about half a second, for the bob to leave a trace of its position at equal intervals of time, the intervals of time must be very short. These small time intervals are produced by a "spark timer".

A spark timer is a high voltage mechanism that produces a spark which, as the bob falls, arcs across the bob and through a waxed paper tape making a small burn hole (a dot) in the paper. To produce the thirty or so total dots in the paper when only a total time of 0.6 seconds is allowed, sparks must be generated every one sixtieth of a second. We will assume the spark timer produces a spark exactly every one sixtieth of a second. So each dot is separated by an equal interval of time.

Although the initial speed of the bob is zero (it does fall from rest), the first dot produced on your paper tape is not created while the bob is at rest. Since the bob has to travel some distance from its rest position to where the first dot is made, the bob must have a non-zero speed when the first dot is created. We will determine this initial speed.

Since it is difficult to produce an arc through the paper, every now and then your paper tape may have a dot (or many dots) missing. When you receive your paper tape, inspect it immediately for missing dots. You should have thirty dots on your paper. Even if you are missing some dots, your paper tape may still be usable as discussed below. What really matters is that you have at least five positions each separated by equal intervals of time.

Note: In this lab the bob falls straight down and does not change direction. Therefore, the average speed and the magnitude average velocity will be the same.

*Based on the lab by Prof. Newton.

Procedure:

1. Under the direction of your instructor, obtain one paper tape "run". Check the tape and replace it if there are dots missing. Be careful during the operation of the apparatus since very high voltage is used to create the arc that burns the wax paper tape.

2. With the masking tape provided, tape down each end of the paper tape on your lab table so that it is taut and lies flat with the light colored side facing up showing all the small burned dots.

3. Starting from the beginning of the tape where the dots are close together, draw a circle around the first dot and then every sixth dot after that for a total of five circled dots. (If you are missing dots, you may need to adapt this: perhaps use every 4th dot up to a total of five dots instead, etc.) Be careful circling your dots; it is easy to miss one and that would throw all your data measurements off. These five circled dots will be your only data points for the entire experiment. Since you are not using all thirty dots produced by the apparatus, you need not have all thirty dots present on your paper tape for it to be acceptable. Your instructor may help determine if your paper tape is still acceptable with some missing dots.

4. On the paper tape, label each of the five circled dots X_0 through X_4, where X_0 refers to the first dot made by the spark.

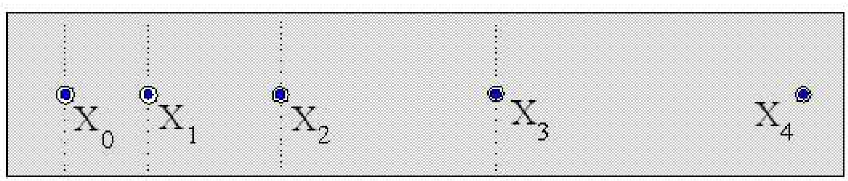

Figure 1: Circle every sixth dot for a total of five circled dots. The time interval between every sixth dot is $6 \times 1/60 = 1/10$ of a second or 0.1 seconds.

   For each circled dot, draw a thin line through the dot such that the line is perpendicular to the lengthwise direction of the tape. Draw the line across the entire width of the tape (see figure 1, above).

5. **Please read this next section entirely before recording any data.**

   Take your two-meter stick and place it on edge (to minimize parallax error) aligning it with the length of the paper tape. Place the meter stick near the dots but do not cover the dots. The two-meter stick should be kept parallel to the line made by all thirty dots. Place the meter stick down at an arbitrary position so the position of the first dot will be the smallest value of your five positions. Do **not** place the meter stick at a pre-chosen position like its end edge for the first point, doing so merely prejudices your data measurements, something to be avoided.

   Draw a position versus time graph while you take each data point and check to see if you have skipped any dots. Before you start taking your data, plan the scale of your graph so that it will cover at least a half page of your lab book.

Relative to the meter stick, note the positions of the first and last circled dots so you can approximate the vertical axis scaling, and for the horizontal axis scaling, recall that the time interval between each circled dot is 0.1 seconds. You will know if you skipped any dots if the curve described by the data points becomes non-parabolic. **Record and graph the position of the five circled dots.** You should interpolate the measurement on your meter stick to read positions to the one hundredth of a centimeter. Write down the absolute uncertainty associated with your measurement. Let the first position be graphed at time $t = 0$; the first position itself should not be a zero value but should be the value you measured on your meter stick.

Analysis and Conclusion:

Part I: Finding instantaneous speeds from a tangent line on a position versus time graph. On your position versus time graph drawn while you took your data, use your ruler to draw a tangent line by eye to the parabolic curve at each position X_0, X_2, and X_4. Measure the slope of each of your three straight lines. Interpret the physical meaning of each line.

Part II: Finding the instantaneous speed V_0 at the initial position X_0 and calculating g by plotting a graph. The following method is important to understand as it forms the basis for the analysis of parts II, III, and IV. The goal of this analysis is to show a limit process by graphical methods and, by extrapolating the graph, to find an instantaneous speed. In this part we will also determine the value of g from the graph.

We know the definition of instantaneous speed as a limit:

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

If we plot **average speeds** on the y-axis and corresponding **time intervals** on the x-axis, then it is possible to see that the average speed approaches an instantaneous speed as the time intervals approach zero. On the graph the time intervals approach zero as the data points approach the vertical axis. Understand that this kind of graph is not the classic instantaneous speed versus time graph that is typically discussed in kinematics; the graph we will use is an average speed versus a time interval. Study the graph on the next page to help understand this.

As an example, a sample data/calculation table for a hypothetical Behr Free Fall experiment is given on the next page. *The acceleration that produced this sample data is not equal to g and the time interval between dots is 0.5 seconds not 0.1, but the calculations used are identical to those needed in constructing your graphs.* These calculations would be used to find the initial instantaneous speed V_0 at the position X_0. When you graph $\bar{V}_{ij}$ versus t_{ij} (you will have four data points to graph from five measured positions) a linear relation should be observed. From the table, you should observe that the first average speed is calculated between the first two circled dots at positions 2.25 and 2.53 cm. Since the time interval between these positions is half a second, the average speed is:

$$\bar{V}_{01} = \frac{(X_1 - X_0)}{\Delta t_{01}} = \frac{(2.53 - 2.25)}{0.5} = 0.56 \text{ cm/s}$$

You should confirm the other calculations in the table to test your understanding. Your graph is a graph of $\bar{V}_{ij}$ versus t_{ij}. Notice that in calculating the different average speeds the position of the dot where we want to find the instantaneous speed is always part of the calculation. Notice also that the average speeds increase as the position between the dots increases and the time interval between the dots increases; this is natural since the bob is speeding up as it falls - its average speed is increasing. To further aid your understanding, the next data point in the graph would be calculated as follows:

$$\bar{V}_{02} = \frac{(X_2 - X_0)}{\Delta t_{02}} = \frac{(3.35 - 2.25)}{1.0} = 1.10 \text{ cm/s}$$

Sample data/calculation table (with different time intervals and acceleration than in your experiment)

X_i (cm)	t_i (s)	$\bar{V}_{ij}$ (cm/s)	Δt_{ij} (s)
2.25	0.0	——	——
2.53	0.5	$\bar{V}_{01} = 0.56$	0.5
3.35	1.0	$\bar{V}_{02} = 1.10$	1.0
4.73	1.5	$\bar{V}_{03} = 1.65$	1.5
6.65	2.0	$\bar{V}_{04} = 2.20$	2.0

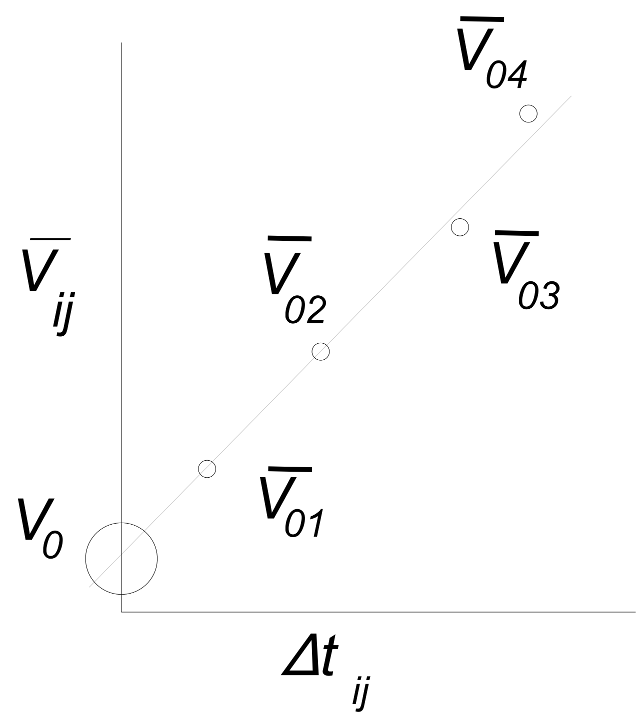

Figure 2: Sketch of a plot of average speed vs time interval. Your points should form a line.

From your graph determine the initial speed of the bob, V_0, at the first position X_0. Also from the slope of your graph calculate g (see the theory section below). Compare your calculated value of g with the known value using a discrepancy test.

Think about what you are doing and you should realize that you are using your graph to take a limit from the right.

Theory exercise: Using kinematics, show that the average velocity $\bar{V}$ is linear in t. As part of the derivation of your equation, show how the slope of this graph is related to the acceleration of the falling body. Interpret the physical meaning of the y-intercept.

Part III: Finding the instantaneous speed of the bob at its final position by plotting another graph. Repeat the methods of part II to find the instantaneous speed V_4 at the position X_4. To find the instantaneous speed at the final position, hold the final position constant while varying the other position and time values. You will need to find the values t_{i4} and $\bar{V}_{i4}$ where $i \in \{0, 1, 2, 3\}$.

Think about what you are doing and you should realize that you are using your graph to take a limit from the left.

Part IV: Find the instantaneous speed V_2 of the bob at the time midpoint t_2 at the position X_2 by plotting yet another graph. Again use the methods previously developed. In this part however, as you shrink your time intervals down around t_2 from the left and the right. You should find t_{i2} and $\bar{V}_{i2}$ where $i \in \{0, 1\}$ and t_{2i} and $\bar{V}_{2i}$ where $i \in \{3, 4\}$ and plot them.

Part V: Compare the three instantaneous speeds found in part I to those found in parts II, III, and IV. Also, can you calculate g from the graphs in parts III and IV? Error analysis has been ignored, how could you apply it meaningfully to this experiment?

Part VI: Show mathematically (*i.e.*, algebraically with no numbers) that the average speed over a time interval is equal to the instantaneous speed at the midpoint of that time interval if the acceleration is constant. Confirm this principle by taking the average speed $(V_0+V_4)/2$ and comparing the result to the instantaneous speed value V_2 at the time midpoint as calculated in Part I and Part IV.

Comment on any sources of error that could have been involved in this experiment. Did they make a significant impact on your data, or was the data you collected close to what you expected from your theoretical derivation?