LAB 5: Projectile Motion

Equipment List:

Pasco projectile launcher, ramrod, and ball
safety glasses
table clamp
two meter sticks
contact paper (8 sheets)
plumb line
masking tap

Purpose: To investigate the effect of the launch angle on the range of a projectile that lands at a lower height than the launch height.

Introduction: In this experiment you will study projectile motion when the projectile is launched from a height above the ground. The angle of launch will be varied while launch height is held constant.

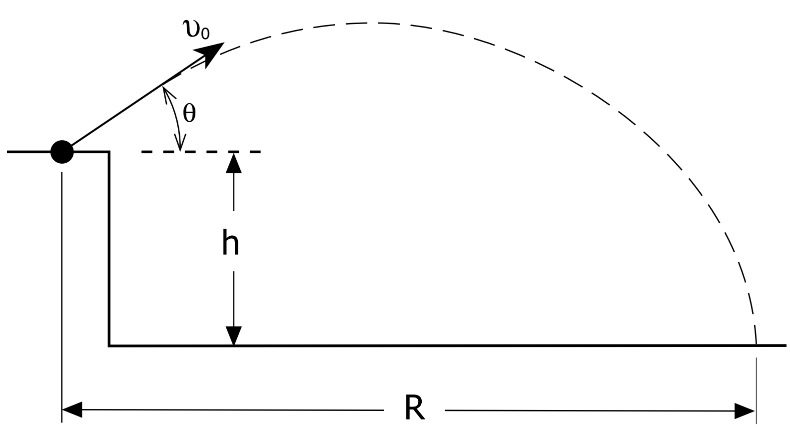
Theory:

For a projectile near the surface of the earth, we can describe its motion using the equations:

$$x = v_i \cos\theta\, t \tag{1}$$

$$y = h + v_i \sin\theta\, t - \frac{1}{2}gt^2 \tag{2}$$

where the y-axis is the vertical direction, the initial speed is v_i, the launch angle is θ, and the launch point of the projectile is $(0, h)$.



In homework 1, you already have shown that the range of such a projectile is a function of the launch angle θ as:

$$R(\theta) = \frac{v_i^2 \cos(\theta)}{g} \left(\sin\theta + \sqrt{\sin^2\theta + \frac{2gh}{v_i^2}} \right) \tag{3}$$

Show clearly in your lab book that for $\theta = 0°$, equation (3) simplifies to

$$R(0°) = v_i \sqrt{\frac{2h}{g}}$$

and so

$$v_i = R(0°) \sqrt{\frac{g}{2h}}\,. \tag{4}$$

You also showed for homework that equation (3) could be simplified to:

$$R(\theta) = \frac{v_i^2 \sin(2\theta)}{2g} \left(1 + \sqrt{1 + \frac{2gh}{v_i^2 \sin^2\theta}} \right) \tag{5}$$

when $\theta \neq 0°$. Recall that the angle θ that maximizes the range is

$$\theta_{\max} = \cot^{-1} \sqrt{1 + \frac{2gh}{v_i^2}} \tag{6}$$

and the maximum range is

$$R_{\max} = \frac{v_i^2}{g} \sqrt{1 + \frac{2gh}{v_i^2}}\,. \tag{7}$$

Procedure:

1. Fix the base the projectile launcher to the lab bench top with a table clamp, so that the muzzle of the launcher sticks out just beyond the edge of the table. Use the plumb line to find the spot directly below the launcher muzzle on the floor. Put masking tape down on the spot and mark an X on it directly below the muzzle.

2. Use a two meter stick to find the height of the muzzle of the launcher. You should measure from the floor to the height of the point on the barrel labeled "bottom of ball". Record this value as h.

3. Lay the strip of contact paper pages along the floor in line with the barrel of the launcher. Do this by standing 3 m in front of the barrel of the unloaded launcher and sighting along its flat side. This will help you align the paper. Try to make the center line of the paper directly along the projectile's trajectory. Do not step on the paper.

4. Once the paper is on the floor, check that the start of your strip is about 85 cm from the point you marked on the floor with an X and that the paper continues to a distance of 3 m from the X. Use masking tape to secure it in place. Masking tape will also be on the floor to help you mark where your paper strip goes.

5. Using the two meter stick, measure a distance 1 m from the X and mark that point on your paper. Draw a line across the paper at this point. This will serve as a reference for subsequent measurements.

6. Set the launcher to an angle of zero for a horizontal launch.

7. One person should wear safety glasses and stand **to the side of the paper out of the launch path** downrange from the launcher. The other person loads the launcher carefully using the ramrod to the "short range" mark. With the launcher set horizontally, the ball might shift. You should feel it just lock into place when you load it, then withdraw the ramrod gently so the ball stays put against the spring.

8. The person operating the launcher will launch the projectile by pulling on the trigger line, while the person downrange watches to see where the projectile will first strike the paper. The projectile will leave a mark on the contact paper where it hits. If you can do so safely, catch the ball before it strikes the paper a second time. With a pen, circle the mark on the paper and label it 1. If the ball hits the paper again before you can retrieve it, put a cross through the mark(s) left by the second strike, so that you know it is not a data point.

9. Repeat the previous step nine more times, labeling the strike points 2 through 10.

10. Make a circle around all the points and label them 0°.

11. Remove the paper from the floor, so another group can use the launcher.

12. Measure the distances of each of the impacts from the marked line on the paper and add 1.00 m to each to get the distance of each impact from the launch point. These ten values are $R_i(0°)$ where $i \in \{1, 2, ..., 10\}$. Find the average of these values, $\bar{R}(0°)$. Now use equation (4) in the Theory section to find v_i, the launch speed.

13. Using v_i find the angle that theoretically should maximize the range, $\theta_{\max}$ and find that range, $R_{\max}$, using equations (6) and (7), respectively.

14. Replace the paper on the floor exactly as it was before, using the tape and your marked line as a guide.

15. Now set the launch angle to each of the angles 15°, 30°, 45°, 60°, 75°, and the angle that is your predicted maximum-range angle and fire the projectile five times at each angle, marking the impact locations as before.

16. To measure the ranges for each of these angle, find the largest and smallest range for each angle, R_l and R_s and record them in your lab book.

Analysis:

Using equation (5) calculate the theoretical ranges $R_{\text{th}}(\theta)$ for the angles 15°, 30°, 45°, 60°, and 75°.

From your measured range values R_l and R_s at each angle (including $\bar{\theta}$), calculate

$$\bar{R} = \frac{R_l + R_s}{2}$$

and use this as the average experimental range for that angle. Also calculate the uncertainty for each value as:

$$\Delta R = \frac{R_l - R_s}{2}$$

Put this data clearly in table showing each angle and the corresponding theoretical value and the corresponding average experimental range value and uncertainty.

Plot a graph of the angle, θ on the x-axis against the theoretical ranges R_{th} on the y-axis. Draw a curve through this data. Now add to your plot the average experimental ranges at for each angle $\bar{R}(\theta)$ on the y-axis drawing on the uncertainties as error bars.

Conclusion:

1. Did your predicted maximum value for the angle correspond to the largest range? Did the theoretical values for the ranges fall within your experimental error bars?
2. What sources of error were there in this experiment? Were they systematic or random? Do your results show the effects of any of these errors?
3. How accurate do you think your value of v_i is? What effect would it have if your measured value was too big? Too small?
4. Do you think that the launch speed is exactly the same for all launch angles? Think about the effect of gravity on the ball while it is still inside the barrel of the launcher.
5. What other questions could you investigate with this equipment? In what other ways might you experimentally study projectile motion?