LAB 9: Extending the Ballistic Pendulum Analysis of Lab 7*

Equipment List:
ballistic pendulum assembly
pen
ruler
calculator

Purpose: To apply our knowledge of moment of inertia and energy of rotation to the ballistic pendulum.

Introduction: No new data will be taken in this short lab. You will simply refine your calculation from Lab 7 using data you took during that lab. You will incorporate this analysis into the analysis and conclusion of your lab report for Lab 7.

Theory: Let h be the maximum height through which the center of mass of the pendulum arm rose in your experiment.

You will derive and expression for R', the predicted range of the ball fired from the ballistic pendulum gun using the conservation of angular momentum.

Step 1: Using your measurement of the period of the pendulum, T, derive an expression for the moment of inertia of the pendulum arm with the ball in the catcher, I, by the following method. Refer to Chapter 15, page 466 of Serway and Jewett, 9th edition. For an oscillating pendulum with moment of inertia I:

$$\tau_{\mathrm{net}} = I\alpha \;\Rightarrow\; -Mg\ell\sin\theta = I\frac{\mathrm{d}^2\theta}{\mathrm{dt}^2}$$

where ℓ is the distance from the pivot to the center of mass and $M = m_p + m_b$ is the total mass of the pendulum arm and ball. First use the approximation that for small angles $\sin\theta \approx \theta$. Then, suggest a solution, $\theta(t)$, to this differential equation and expression for ω, the angular frequency and T the time period, in terms of M, g, ℓ, and I. Now, rearrange that expression so that it is for I in terms of T.

Step 2: Derive a new expression for v_b the launch speed of the ball, in terms of I, M, m_b, g, h, and ℓ', where ℓ' is the distance from the pivot point to the place the ball strikes the pendulum arm ($\ell' > \ell$). To do this, you will need to equate angular momentum before and after the collision of the ball with the pendulum arm to find v_b in terms of the angular speed ω of the arm after the collision. Then use the conservation of energy (assuming all rotational kinetic energy becomes gravitational potential energy of the center-of-mass of the pendulum arm) to find an expression for ω just after the collision and use that to eliminate ω from the expression for v_b.

*Partially based on the Pasco Ballistic Pendulum Manual.

Step 3: Use your expression for I in terms of T to get an expression for v_b that depends on T instead of I.

Step 4: Using the projectile motion equations, find an expression for the range R' of a projectile, launched horizontally from a height d with a speed v_b. (You already did this in Lab 7; the expression will be the same.)

Step 5: Put these expressions together to obtain an expression for R' in terms of T, M, h, m_b, d, ℓ, ℓ', and g.

Step 6: Calculate a numerical value of R' in terms of those quantities that you measured in Lab 7.

More Analysis and Conclusion:

Find the percentage difference between the values $\bar{R}_{\text{meas}}$ and R'. Are they closer than $\bar{R}_{\text{meas}}$ and R_{pred}? (Remember, R_{pred} was the calculated theoretical value from Lab 7, assuming that all the mass in the pendulum arm was located at the center of mass.) How close are R_{pred} and R'?

What effect do you think the mass distribution of the pendulum has on the results? Was it the major source of error in this lab or were there other factors that were more important that effected the value of $\bar{R}_{\text{meas}}$?