2D Kinematics
Relative Motion

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Last Time

- projectile trajectory equation
- projectile examples

Overview

- relative motion equations
- relative motion examples

Relative Motion

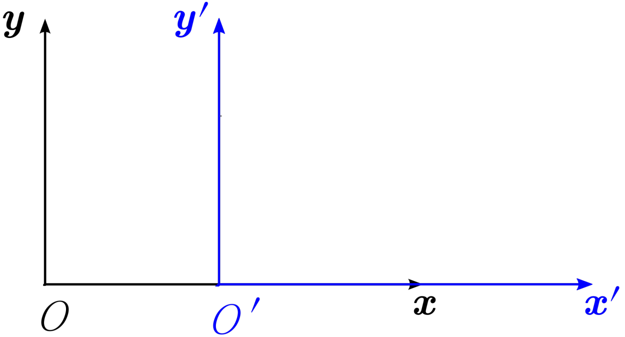
One *very* useful technique for physical reasoning is considering other *frames of reference*.

A reference frame is a coordinate system that an observer adopts.

Different observers may have different axes: different frames of reference.

Reference Frames

Two observers could agree to choose directions as North (y) and East (x). However, they might pick different origins, O and O', for their axes.

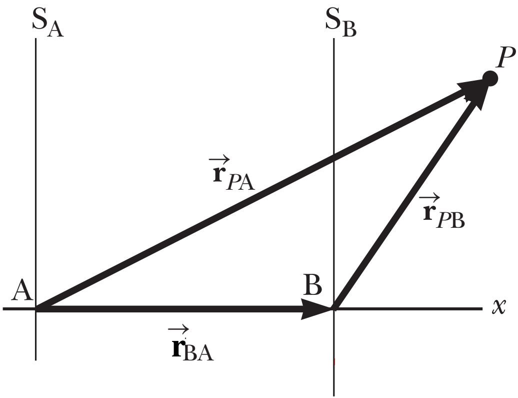


In this case, each person would describe the location of a particle slightly differently. How can we relate those descriptions?

[1] Image modified from work of Wikipedia user Krea.

Reference Frames: Fixed Relative Positions

Observers A and B have different descriptions of the location of particle P. We can relate these descriptions using **vector addition**.



$$\vec{\mathbf{r}}_{PA} = \vec{\mathbf{r}}_{PB} + \vec{\mathbf{r}}_{BA}$$

$\vec{\mathbf{r}}_{PA}$ is the position of particle P *relative to* frame A.

Relative Motion

We can use the notion of motion in 2 dimensions to consider how one object moves **relative** to something else.

All motion is relative.

Our **reference frame** tells us what is a fixed position.

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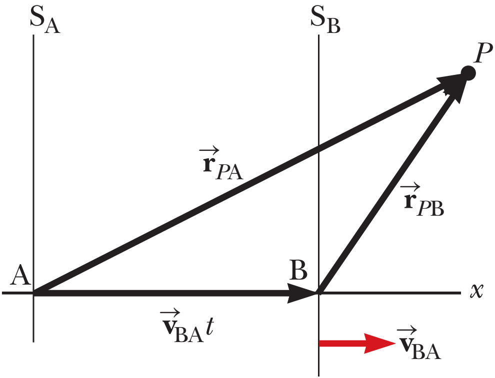
An example of a reference might be picking an object, declaring that it is at rest, and describing the motion of all objects relative to that.

Two different people could pick different reference objects and end up with two reference frames *moving* relative to each other.

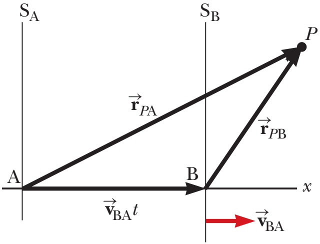
Frames of Reference: Const Relative Velocity

Now consider observer B moving with constant velocity $\vec{\mathbf{v}}_{BA}$ relative to A. Both observe a particle P.



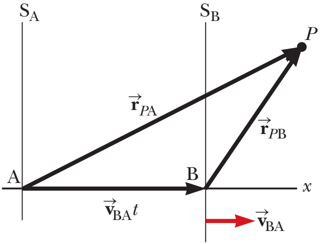
(In this diagram, the frames coincide at $t = 0$.)

Relative Motion: Const Relative Velocity



$$\vec{\mathbf{r}}_{PA} = \vec{\mathbf{r}}_{PB} + \vec{\mathbf{v}}_{BA}t$$

Relative Motion: Const Relative Velocity



$$\vec{\mathbf{r}}_{PA} = \vec{\mathbf{r}}_{PB} + \vec{\mathbf{v}}_{BA}t$$

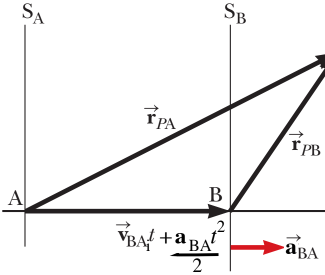
Differentiating, noting that $\vec{\mathbf{v}}_{BA}$ is a constant:

$$\begin{aligned} \frac{\mathrm{d}\,\vec{\mathbf{r}}_{\mathrm{PA}}}{\mathrm{dt}} &= \frac{\mathrm{d}\,\vec{\mathbf{r}}_{\mathrm{PB}}}{\mathrm{dt}} + \vec{\mathbf{v}}_{BA} \\ \vec{\mathbf{v}}_{PA} &= \vec{\mathbf{v}}_{PB} + \vec{\mathbf{v}}_{BA} \end{aligned}$$

Differentiating again shows that observers in two frames related by a constant relative velocity see the same accelerations.

Relative Motion: Const Relative Acceleration

What if $\vec{\mathbf{v}}_{BA}$ is not constant?

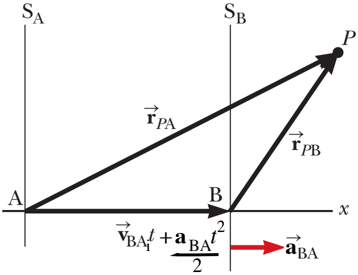


Suppose B **accelerates** relative to A with a constant acceleration.

$$\vec{\mathbf{r}}_{PA} = \vec{\mathbf{r}}_{PB} + \vec{\mathbf{v}}_{BA,i}\,t + \frac{1}{2}\vec{\mathbf{a}}_{BA}t^2$$

Relative Motion: Const Relative Acceleration

What if $\vec{\mathbf{v}}_{BA}$ is not constant?



Suppose B **accelerates** relative to A with a constant acceleration.

$$\vec{\mathbf{r}}_{PA} = \vec{\mathbf{r}}_{PB} + \vec{\mathbf{v}}_{BA,i} t + \frac{1}{2} \vec{\mathbf{a}}_{BA} t^2$$

Differentiating twice:

$$\vec{\mathbf{a}}_{PA} = \vec{\mathbf{a}}_{PB} + \vec{\mathbf{a}}_{BA}$$

Relative Motion

In summary, when comparing two frames moving relative to each other:

$$\vec{\mathbf{v}}_{PA} = \vec{\mathbf{v}}_{PB} + \vec{\mathbf{v}}_{BA}$$

$$\vec{\mathbf{a}}_{PA} = \vec{\mathbf{a}}_{PB} + \vec{\mathbf{a}}_{BA}$$

Relative Motion

In summary, when comparing two frames moving relative to each other:
Imagine

$$\vec{\mathbf{v}}_{PA} = \vec{\mathbf{v}}_{P\cancel{B}} + \vec{\mathbf{v}}_{\cancel{B}A}$$

$$\vec{\mathbf{a}}_{PA} = \vec{\mathbf{a}}_{P\cancel{B}} + \vec{\mathbf{a}}_{\cancel{B}A}$$

Relative Motion

In summary, when comparing two frames moving relative to each other:
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$$\vec{\mathbf{v}}_{\underline{PA}} = \vec{\mathbf{v}}_{\underline{P}\cancel{B}} + \vec{\mathbf{v}}_{\cancel{B}\underline{A}}$$

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Relative Motion

In summary, when comparing two frames moving relative to each other:

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One more useful fact:

$$\vec{\mathbf{v}}_{AB} = -\vec{\mathbf{v}}_{BA}$$
$$\vec{\mathbf{a}}_{AB} = -\vec{\mathbf{a}}_{BA}$$

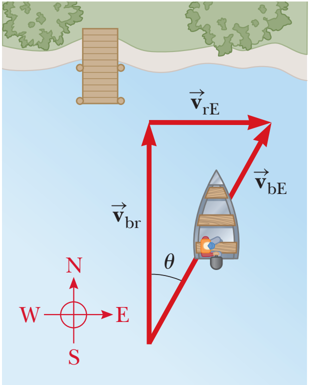
Swapping subscripts flips the sign.

Relative Motion Conceptual Idea in 2 Dimensions

The boat's heading is due North, but it is pulled East by the river's current.

Relative to a stick floating in the river, the boat moves due North.

Relative to the bank of the river, the boat moves at an angle θ East of North.



(Also, you could check out the simple example 4.8 on Page 97 of Serway & Jewett.)

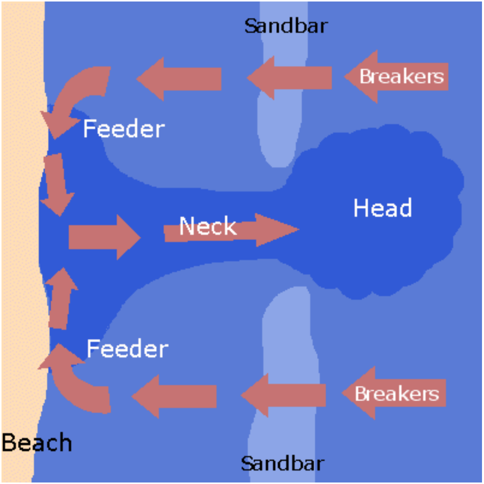
Relative Motion and Rip Currents

Life and death application: rip currents.

In shallow ocean water, a rip current is a strong flow of water away from the shore.

Relative Motion and Rip Currents

If you are caught in one, which way should you swim?



[2] Diagram from Wikipedia.

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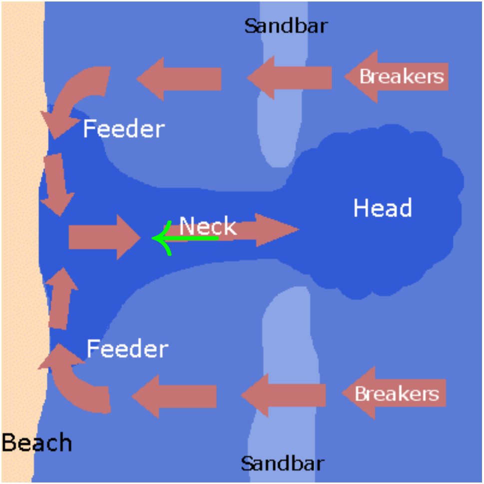

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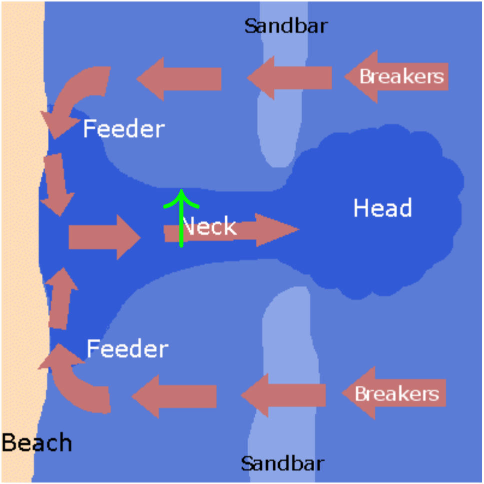

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A Relative Motion Example

A light plane attains an airspeed of 500 km/h. The pilot sets out for a destination 800 km due north but discovers that the plane must be headed 20.0° east of due north to fly there directly. The plane arrives in 2.00 h. What were the (a) magnitude and (b) direction of the wind velocity?[1]

Let p refer to the plane, a refer to the air, E refers to the Earth. The wind velocity is $\vec{\mathbf{v}}_{aE}$.

[1] Halliday, Resnick, Walker, 10th ed, page 83, #76.

Summary

- relative motion

(Uncollected) Homework Serway & Jewett,

- Ch 4, onward from page 104. Problems: 45, 51, 47, 79, 83 (relative motion probs)