2D Kinematics
Relative Motion
Uniform Circular Motion

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Last Time

- relative motion

Overview

- relative motion examples
- uniform circular motion

A Relative Motion Example

A light plane attains an airspeed of 500 km/h. The pilot sets out for a destination 800 km due north but discovers that the plane must be headed 20.0° east of due north to fly there directly. The plane arrives in 2.00 h. What were the (a) magnitude and (b) direction of the wind velocity?[1]

[1] Halliday, Resnick, Walker, 10th ed, page 83, #76.

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Let p refer to the plane, a refer to the air, E refers to the Earth. The wind velocity is $\vec{\mathbf{v}}_{aE}$.

$$\vec{\mathbf{v}}_{pE} = \vec{\mathbf{v}}_{pa} + \vec{\mathbf{v}}_{aE}$$

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rearranging:

$$\begin{aligned}
\vec{\mathbf{v}}_{aE} &= \vec{\mathbf{v}}_{pE} - \vec{\mathbf{v}}_{pa} \\
\vec{\mathbf{v}}_{aE} &= \left(\frac{800}{2}\,\hat{\mathbf{j}}\right) \text{ km/h} - (500 \sin 20^\circ\,\hat{\mathbf{i}} + 500 \cos 20^\circ\,\hat{\mathbf{j}}) \text{ km/h}
\end{aligned}$$

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$$\begin{aligned} \vec{\mathbf{v}}_{aE} &= \left(\frac{800}{2}\,\hat{\mathbf{j}}\right)\ \text{km/h} - (500\sin 20^\circ\,\hat{\mathbf{i}} + 500\cos 20^\circ\,\hat{\mathbf{j}})\ \text{km/h} \\ \vec{\mathbf{v}}_{aE} &= (-171\,\hat{\mathbf{i}} - 70.0\,\hat{\mathbf{j}})\ \text{km/h} \end{aligned}$$

magnitude:

$$\begin{aligned} v_{aE} &= \sqrt{v_{aE,x}^2 + v_{aE,y}^2} \\ &= \underline{185\ \text{km/h}} \end{aligned}$$

direction:

$$\theta = \tan^{-1}\left(\frac{70}{171}\right) = \underline{22.3^\circ\ \text{South of West}}$$

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Relative Motion Example, #49, pg 206

A bolt drops from the ceiling of a moving train car that is accelerating northward at a rate of 2.50 m/s^2.

(a) What is the acceleration of the bolt relative to the train car?

(b) What is the acceleration of the bolt relative to the Earth?

(c) Describe the trajectory of the bolt as seen by an observer inside the train car.

(d) Describe the trajectory of the bolt as seen by an observer fixed on the Earth.

Let the x-axis point north, and the y-axis point up.

Relative Motion Example, #49, pg 206

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(a) What is the acceleration of the <u>b</u>olt relative to the train <u>c</u>ar?

As the bolt falls it is no longer attached to the train car. It does not move with the car. Its acceleration is down relative to the <u>E</u>arth, but the car keeps accelerating northward.

$$\vec{\mathbf{a}}_{bE} = \vec{\mathbf{a}}_{bc} + \vec{\mathbf{a}}_{cE}$$

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$$\begin{aligned}\vec{\mathbf{a}}_{bE} &= \vec{\mathbf{a}}_{bc} + \vec{\mathbf{a}}_{cE} \\ \vec{\mathbf{a}}_{bc} &= \vec{\mathbf{a}}_{bE} - \vec{\mathbf{a}}_{cE} \\ &= (-9.8\,\hat{\mathbf{j}} - 2.50\,\hat{\mathbf{i}})\ \text{m/s}^2\end{aligned}$$

$$a_{bc} = \sqrt{2.50^2 + 9.8^2}\ \text{m/s}^2\ , \quad \theta = \tan^{-1}\left(\frac{9.8}{2.50}\right)$$

$\vec{\mathbf{a}}_{bc} = 10.1$ m/s^2, at 75.7°below the horizontal, southward.

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A bolt drops from the ceiling of a moving train car that is accelerating northward at a rate of 2.50 m/s^2.

(b) What is the acceleration of the bolt relative to the Earth?

$$\vec{\mathbf{a}}_{bE} = \vec{\mathbf{g}} = -9.8\,\hat{\mathbf{j}}\ \text{m/s}^2$$

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$$\vec{\mathbf{a}}_{bE} = \vec{\mathbf{g}} = -9.8\,\hat{\mathbf{j}}\ \mathrm{m/s}^2$$

(c) Describe the trajectory of the bolt as seen by an observer inside the train car.

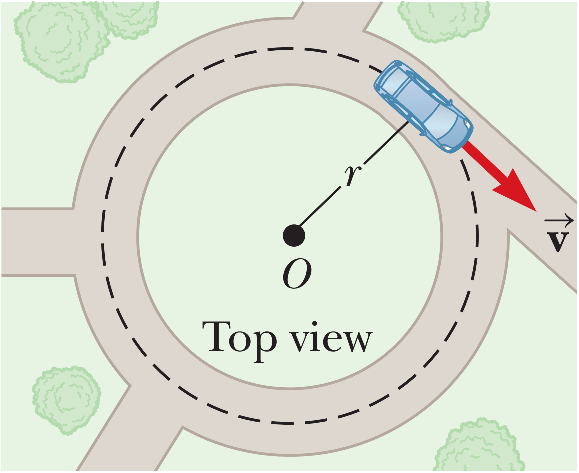
It accelerates from rest, relative to the car. It must follow the direction of the acceleration. A straight line, 75.7° below the horizontal, southward.

(d) Describe the trajectory of the bolt as seen by an observer fixed on the Earth.

It falls as a projectile with an initial horizontal velocity equal to the velocity of the car relative to the ground at the moment the screw drops.

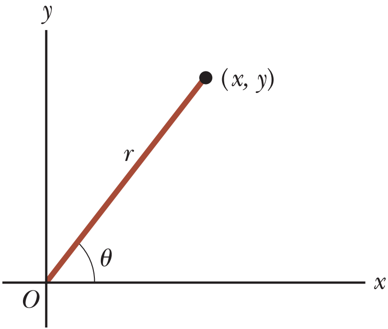
Circular Motion

Frequently in physics, we encounter situations where an object moves in a circle.



[1]Figure from Serway and Jewett, page 91.

Circular Motion: Radial Coordinates

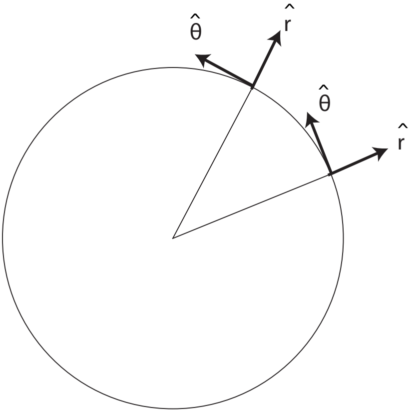


It is sometimes convenient to give position coordinates in terms of r and θ. To transform from r, θ to x, y:

$$x = r\cos\theta \qquad y = r\sin\theta$$

It is typical to speak of **radial** and **tangential** directions.

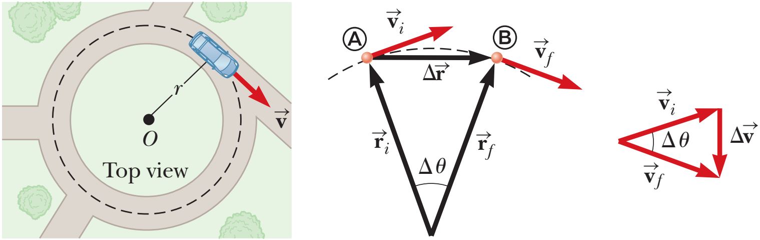
Radial Coordinates



We can define perpendicular radial and tangential unit vectors, but their direction changes with the motion of a particle around a circle.

Circular Motion

As a car moves around a circular path its velocity changes, if not in magnitude, then in direction.

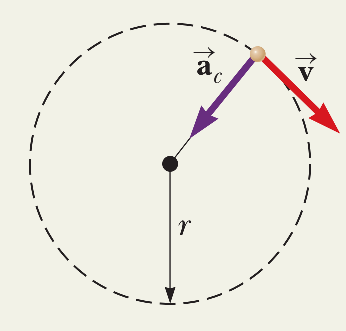


If the radius remains constant and the speed of the car does as well, then $\overrightarrow{\Delta v}$ points toward the center of the circle.

That means the acceleration vector does, too.

Uniform Circular Motion

The velocity vector points along a tangent to the circle



For uniform circular motion:

- the radius is constant
- the speed is constant
- the *magnitude* of the acceleration is constant

Uniform Circular Motion

The magnitude of the acceleration is given by

$$a_c = \frac{v^2}{r}$$

If the constant speed is v, then the time period for one complete orbit is

$$T = \frac{2\pi r}{v}$$

Summary

- relative motion examples
- uniform circular motion

(Uncollected) Homework Serway & Jewett,

- Ch 4, onward from page 104. OQ: 9; Problems: 53, 61 (relative motion)
- Ch 4, Problems: 35, 37, 39 (uniform circular motion)