Kinematics: Circular Motion

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Last Time

- relative motion example
- uniform circular motion

Overview

- uniform circular motion, angular speed
- nonuniform circular motion

Uniform Circular Motion

The magnitude of the acceleration is given by

$$a_c = \frac{v^2}{r}$$

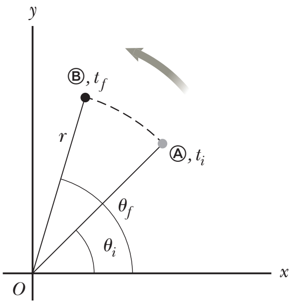
If the constant speed is v, then the time period for one complete orbit is

$$T = \frac{2\pi r}{v}$$

($f = 1/T$ is the *frequency*, or the rate of revolutions in time.)

Uniform Circular Motion

We can also consider the rate at which the angular coordinate is changing:



$$\Delta\theta = \theta_f - \theta_i$$

Then we can define the **angular speed**, ω, as

$$\omega = \frac{d\theta}{dt}$$

Uniform Circular Motion

ω gives the amount by which the angle θ advances in radians, per unit time. Therefore,

$$\omega = \frac{2\pi}{T}$$

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$$\begin{aligned} \omega &= 2\pi \frac{v}{2\pi r} \\ \omega &= \frac{v}{r} \end{aligned}$$

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$$\omega = 2\pi \frac{v}{2\pi r}$$

$$\omega = \frac{v}{r}$$

This gives us another expression for the centripetal acceleration:

$$a_c = \omega^2 r$$

Radial and Tangential Accelerations

Quick Quiz 4.4[1] A particle moves in a circular path of radius r with speed v. It then increases its speed to $2v$ while traveling along the same circular path.

(i) The centripetal acceleration of the particle has changed by what factor?

A 0.25

B 0.5

C 2

D 4

[1]Page 93, Serway & Jewett

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[1] Page 93, Serway & Jewett

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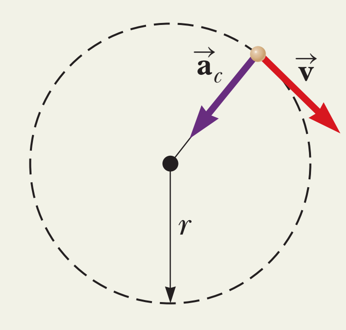
[1]Page 93, Serway & Jewett

Uniform Circular Motion Summary

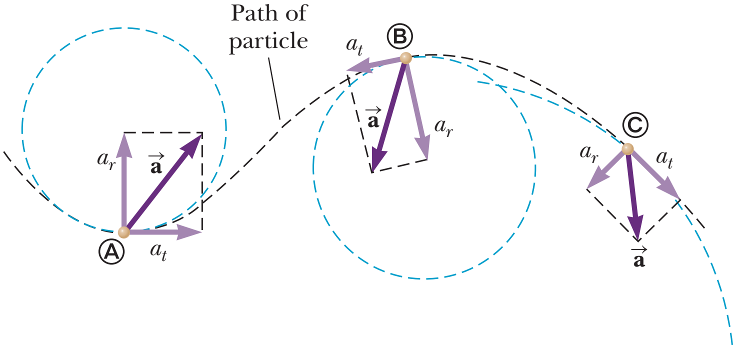
The velocity vector points along a tangent to the circle



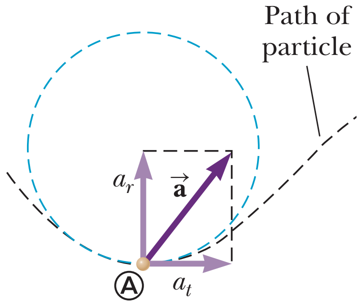
For uniform circular motion:

- the radius is constant
- the speed is constant
- the *magnitude* of the acceleration is constant, $a_c = \frac{v^2}{r} = \omega^2 r$

Non-Uniform Circular Motion



Radial and Tangential Accelerations



$$\vec{\mathbf{a}} = \vec{\mathbf{a}}_t + \vec{\mathbf{a}}_r$$

$$\vec{\mathbf{a}} = a_t \hat{\theta} - a_c \hat{\mathbf{r}} \quad \text{(defining } a_c \text{ +ve)}$$

Let $\hat{\theta}(t)$ be a unit vector in the direction of the velocity. Note that its direction changes with time!

$$\vec{\mathbf{v}}(t) = v(t)\,\hat{\theta}(t)$$

Radial and Tangential Accelerations

$$\vec{\mathbf{a}} = \frac{\mathrm{d}\,\vec{\mathbf{v}}}{\mathrm{dt}} \quad ; \quad \vec{\mathbf{v}}(t) = v(t)\,\hat{\theta}(t)$$

Find the acceleration using the product rule:

$$\vec{\mathbf{a}} \;=\; \frac{\mathrm{dv}}{\mathrm{dt}}\,\hat{\theta} + v\,\frac{\mathrm{d}\hat{\theta}}{\mathrm{dt}}$$

Radial and Tangential Accelerations

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$$\vec{\mathbf{a}} \;=\; \frac{dv}{dt}\,\hat{\theta} + v\,\frac{d\hat{\theta}}{dt}$$

The term $\left(\frac{dv}{dt}\,\hat{\theta}\right)$ is all in the tangential component of the acceleration.

But how to find what $\left(v\,\frac{d\hat{\theta}}{dt}\right)$ is? We need to find how $\hat{\theta}$ changes with time. (It rotates, but at what rate?)

Radial and Tangential Accelerations: How do the perpendicular axes change?

Let's find out!

$\hat{\theta}$ is changing, so let us say that $\hat{\theta}_i$ is the initial tangential unit vector and $\hat{\theta}_f$ is the final tangential unit vector.

Radial and Tangential Accelerations: How do the perpendicular axes change?

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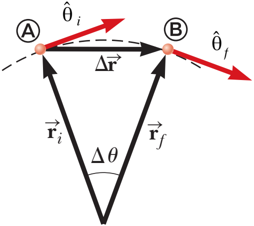
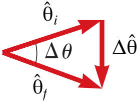
$\hat{\theta}$ changes at the same rate as θ itself.

$$ds = r\,d\theta$$





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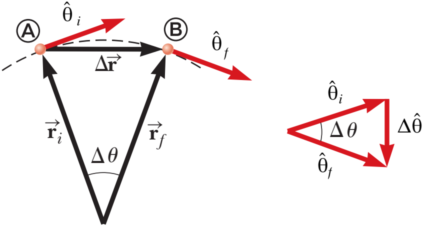

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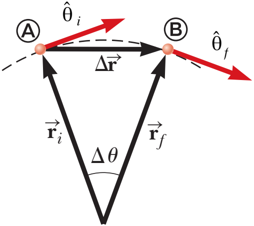

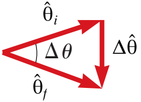

$$ds = r\,d\theta$$

$$\frac{ds}{dt} = r\frac{d\theta}{dt}$$

$$\frac{d\theta}{dt} = \frac{1}{r}\frac{ds}{dt}$$

$$\boxed{\frac{d\theta}{dt} = \frac{v}{r}}$$

This tells us how fast the tangential unit vector changes in direction.

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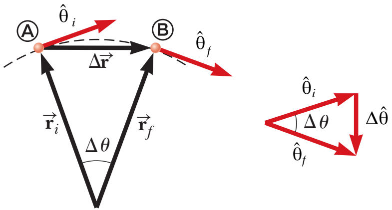
Now consider that the direction of change must be radial!

$$\frac{\mathrm{d}}{\mathrm{dt}}\hat{\theta} = -\frac{v}{r}\hat{\mathbf{r}}$$



Radial and Tangential Accelerations

$$\vec{\mathbf{a}} = \frac{d\vec{\mathbf{v}}}{dt} = \frac{d}{dt}(v\hat{\theta})$$

Find the acceleration using the product rule:

$$\begin{aligned}\vec{\mathbf{a}} &= \frac{dv}{dt}\,\hat{\theta} + v\,\frac{d\hat{\theta}}{dt} \\ &= \frac{dv}{dt}\,\hat{\theta} + v\left(-\frac{v}{r}\hat{\mathbf{r}}\right) \\ &= \underbrace{\frac{dv}{dt}\,\hat{\theta}}_{\text{tangen.}} - \underbrace{\frac{v^2}{r}\hat{\mathbf{r}}}_{\text{radial}}\end{aligned}$$

We said $\vec{\mathbf{a}} = a_t\hat{\theta} - a_c\hat{\mathbf{r}}$ so,

$$a_c = \frac{v^2}{r}$$

Radial and Tangential Accelerations

pg 105, #41

41. M A train slows down as it rounds a sharp horizontal turn, going from 90.0 km/h to 50.0 km/h in the 15.0 s it takes to round the bend. The radius of the curve is 150 m. Compute the acceleration at the moment the train speed reaches 50.0 km/h. Assume the train continues to slow down at this time at the same rate.

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Radial and Tangential Accelerations

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$a_t = -0.741 \text{ m/s}^2$; $a_r = -1.29 \text{ m/s}^2$ (calling outward positive)

$\vec{\mathbf{a}} = 1.48 \text{ m/s}^2$ inward at an angle 29.9°

backward from the direction of travel

[1]Page 93, Serway & Jewett

Summary

- uniform circular motion
- nonuniform circular motion

(Uncollected) Homework Serway & Jewett,

- Ch 4, Problems: 40, 43, 70 (nonuniform circular motion)