Dynamics
Applying Laws of Motion
Inclines, Elevators

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Last time

- static equilibrium example
- normal force

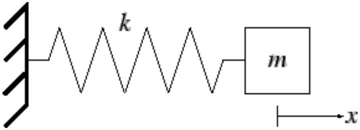
Overview

- elastic forces
- non-equilibruim
- Problem solving with more complex scenarios
    - inclines
    - elevators

Some types of forces

Elastic Forces

Springs exert forces as they are being compressed or extended. They have a natural length, at which they remain if there are no external forces acting.



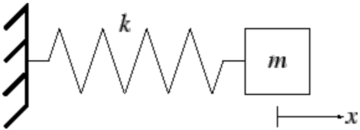
Hooke's Law gives

$$\vec{\mathbf{F}}_{\text{spring}} = -k\,\vec{\mathbf{x}}$$

where k is a constant. $\vec{\mathbf{x}}$ is the amount of displacement of one end of a spring from it's natural length. (The amount of compression or extension.

[1]Figure from CCRMA Stanford Univ.

Springs

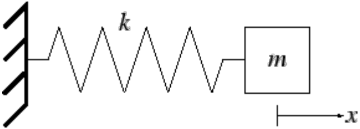


Question. A mass is attached to the end of a spring which obeys Hooke's Law. The spring is compressed and then released. Is the acceleration of the mass constant?

(A) Yes.

(B) No.

Springs



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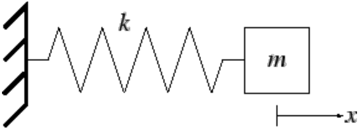
Equilibrium Implications



Question. A mass is attached to the end of a spring which obeys Hooke's Law ($\vec{\mathbf{F}}_{\text{spring}} = -k\vec{\mathbf{x}}$). A force $\vec{\mathbf{F}}_{\text{app}}$ is applied to the object so that the spring is compressed through a distance x and the object moves at a constant velocity $\vec{\mathbf{v}}$. Is $\vec{\mathbf{F}}_{\text{app}}$ constant over the object's motion?

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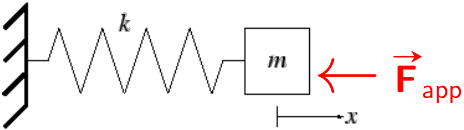

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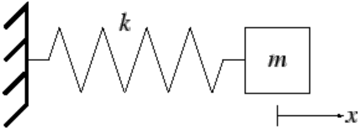

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(A) Yes.

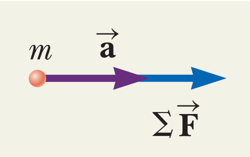
(B) No. $\longleftarrow$

Not in Equilibrium: A Net Force

When the net force is *not* zero, there will be an acceleration of the object.

The object might speed up or slow down, or the direction of its motion might change.

Likewise, if an object is accelerating, it must be experiencing a net force (*ie.* it must be experiencing at least one force).



(Have a look at Example 5.1 on page 116 of the textbook and make sure you could solve that.)

Solving Problems Using Forces

- Either consider every force on the object and then find the net force and acceleration.
- Or start from the motion and deduce what the magnitudes or directions of some forces must be.

Object on an Incline

Problems with an object placed on an incline often require us to find the net force on the object or its acceleration.

[1] Figures from Serway & Jewett

Object on an Incline

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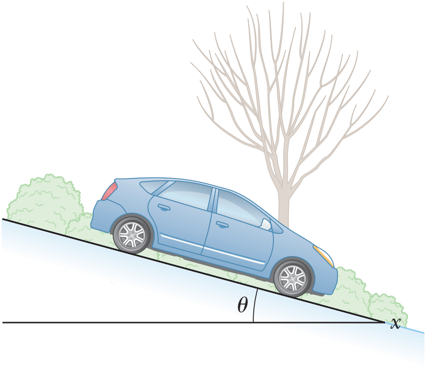
Consider a car on a frictionless driveway.[1] (Or free to roll, with frictionless, massless wheels.)



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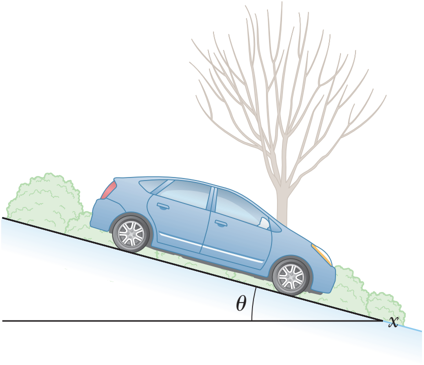
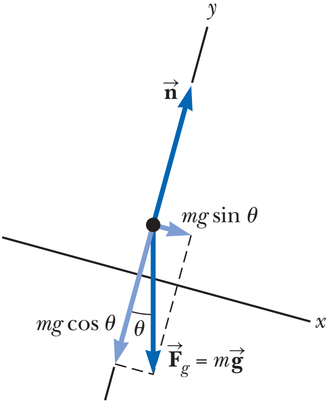
Object on an Incline

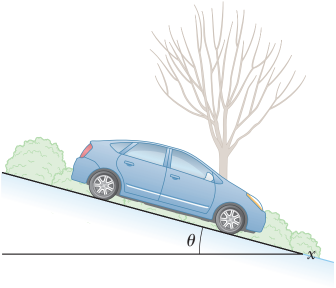


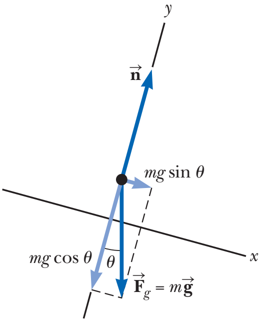


The forces acting on the car: weight and normal force.

In this case, it is useful to pick a coordinate system that is rotated: the *x* axis points along slope, the *y* axis perpendicular to the slope.

Object on an Incline

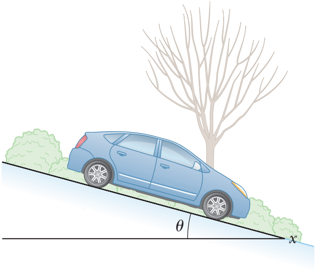


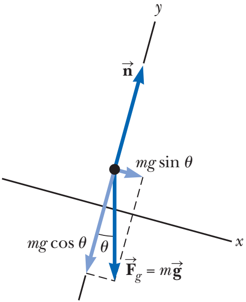


The forces acting on the car: weight and normal force.

Imagine the car starts from rest. If it were to accelerate off the surface, the normal force would go to zero immediately. The car also cannot sink (accelerate) into the surface. $\Rightarrow a_y = 0$.

Object on an Incline

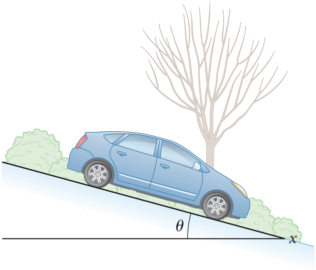


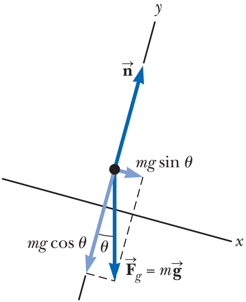


So, the forces in the (tilted) y-direction cancel:

$$\begin{aligned} F_{\text{net},y} &= m\cancel{a_y}^{\,0} \\ n - mg\cos\theta &= 0 \end{aligned}$$

Object on an Incline





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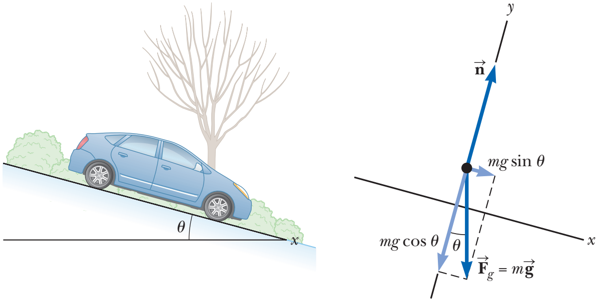
Rearranging:

$$n = mg\cos\theta$$

If $\theta > 0$ the normal force will be less than the weight, mg.

Object on an Incline



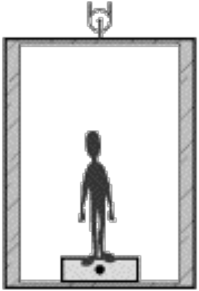
In the (tilted) x-direction:

$$\begin{aligned} F_{net,x} &= ma_x \\ \cancel{m}g \sin\theta &= \cancel{m}a_x \end{aligned}$$

$$\Rightarrow \quad \vec{\mathbf{F}}_{net} = (mg \sin\theta)\hat{\mathbf{i}}$$

$$\Rightarrow \quad \vec{\mathbf{a}} = (g \sin\theta)\hat{\mathbf{i}}$$

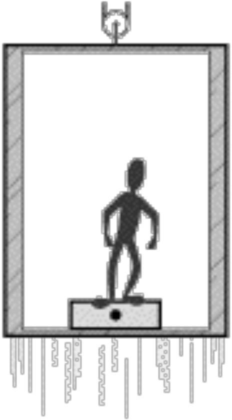
Elevator Problems



$$\vec{\mathbf{a}} = 0$$

Elevator is at rest or moving with constant velocity. You feel the same as you normally do. Your weight and normal force are both of magnitude mg.

Elevator Problems

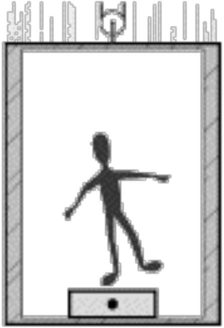


$\vec{\mathbf{a}} = +a\,\hat{\mathbf{j}}$ (a is a positive number)

Elevator could be moving upward increasing speed **or** downward decreasing speed. You feel as if your weight has increased.

Your weight is $-mg\,\hat{\mathbf{j}}$, but the normal force is $\vec{\mathbf{n}} = m(g + a)\,\hat{\mathbf{j}}$.

Elevator Problems



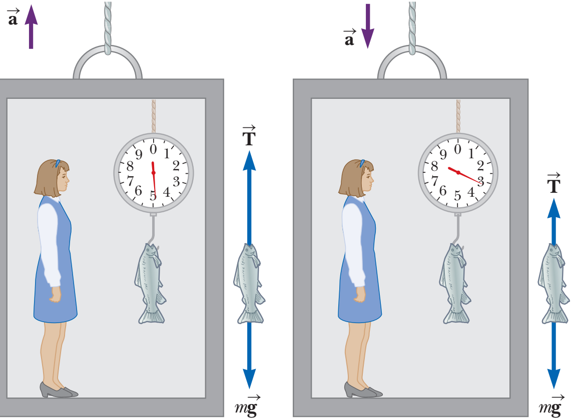
$\vec{\mathbf{a}} = -a\,\hat{\mathbf{j}}$ (a is a positive number)

Elevator could be moving upward and slowing down **or** moving downward increasing speed. You feel as if your weight has decreased.

Your weight is $-mg\,\hat{\mathbf{j}}$, but the normal force is $\vec{\mathbf{n}} = m(g-a)\,\hat{\mathbf{j}}$.

Elevator Problems

An example with the normal force replaced by a tension instead:



Whether the reaction of the floor of the tension in the cable cause the upward force, the *net force* is still calculated in the same way.

Elevator Problems

Question. You are hired to design an elevator that can lift people up to the top of a 70 story building in a short amount of time. The weight of the elevator car is 3500 N and the max load the elevator should be rated to carry is 2000 N and the weight of the entire cable used to lift the elevator car is 3000 N. You add these numbers together and decide that you can choose a low-cost cable rated to carry a load of 9000 N without breaking as the elevator cable. Will you end up fired or with a commendation?

(A) Commendation!

(B) Fired!

(C) I'm not sure.

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Summary

- elastic forces
- non-equilibrium
- more problem solving
  - inclines
  - elevators

Quiz this Friday.

(Uncollected) Homework

Serway & Jewett,

- prev: Ch 5, onward from page 136. Probs: 39
- 'new': Ch 5, CQ 13; Probs: 28, 47, 95, 43, 29, 83 (can wait)