Laws of Motion
Objects Moving Together
Pulleys
Atwood Machines

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Last time

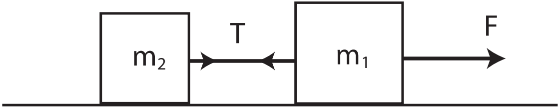
- more problem solving
- inclines
- elevators

Overview

- objects moving together
- pulleys
- Atwood machines

Separate Objects Pulled Along

What is the acceleration of these blocks?



The accelerations of the two blocks are the same.

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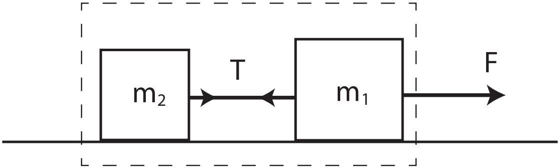


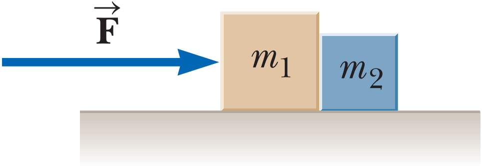
The accelerations of the two blocks are the same.

Imagining them as a single block gives the acceleration straight away:

$$\vec{\mathbf{a}} = \frac{\vec{\mathbf{F}}}{m_1 + m_2}$$

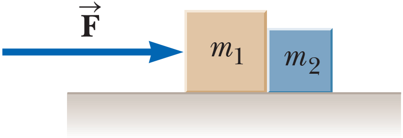
Separate Objects Pushed Together

Consider a force $\vec{\mathbf{F}}$ that acts on two objects, masses m_1 and m_2, free to slide on a frictionless surface:

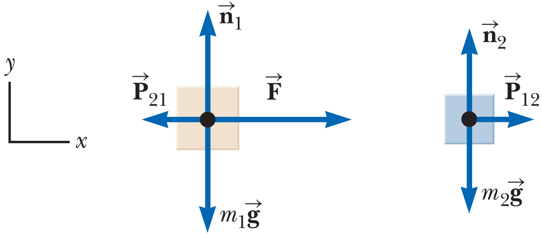


Separate Objects Pushed Together

If objects are pushed or pulled together, then they must all accelerate at the same rate.

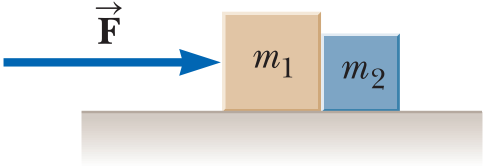


That means that the individual net forces on each must be different:



Separate Objects Pushed Together

Question. What is the acceleration of object m_2?



(A) $\frac{\vec{F}}{m_2}$

(B) $\frac{m_1 \vec{F}}{m_2}$

(C) $\frac{\vec{F}}{m_1 + m_2}$

(D) $\frac{\vec{F}}{m_1 + m_2} + \frac{\vec{F}}{m_2}$

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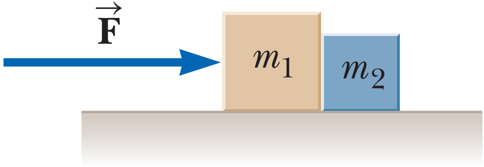

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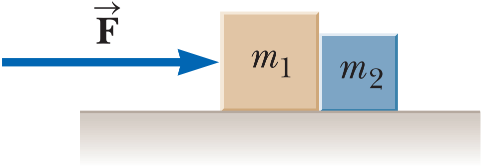

(A) 0

(B) $\frac{m_1 \vec{F}}{m_1+m_2}$

(C) $\frac{m_2 \vec{F}}{m_1+m_2}$

(D) $\vec{F}$

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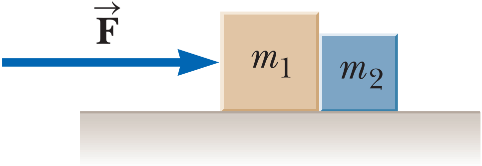

(A) 0

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(C) $\frac{m_2 \vec{\mathbf{F}}}{m_1 + m_2}$ ←

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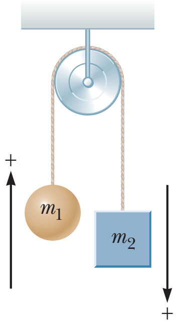
Pulleys

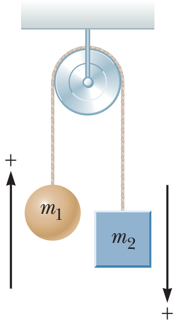
Pulleys "turn tensions around a corner".



For the moment, we are just considering *massless, frictionless* pulleys. What does that mean?

Pulleys

Pulleys "turn tensions around a corner".

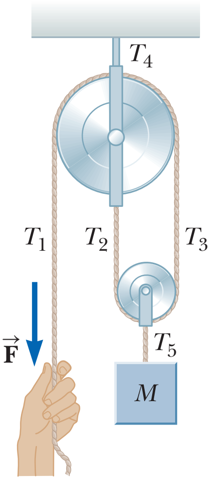


For the moment, we are just considering *massless, frictionless* pulleys. What does that mean?

- Massless: we do not have to worry about force needed to accelerate each atom in the pulley
- Frictionless: the *axle* of the pulley has no friction to resist the wheel turning

Pulley System: #85, page 147

85. An object of mass M is held in place by an applied force $\vec{\mathbf{F}}$ and a pulley system as shown in Figure P5.85. The pulleys are massless and frictionless. (a) Draw diagrams showing the forces on each pulley. Find (b) the tension in each section of rope, T_1, T_2, T_3, T_4, and T_5 and (c) the magnitude of $\vec{\mathbf{F}}$.



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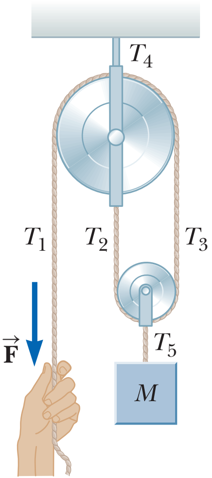
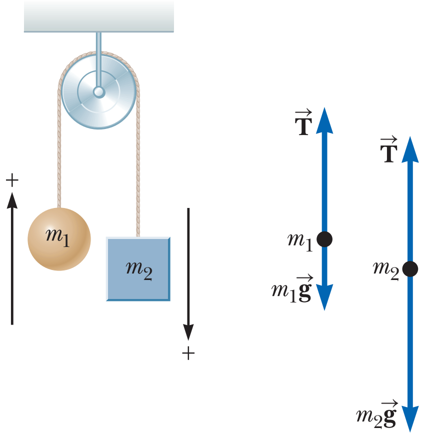


$$T_1 = T_2 = T_3 = F = \frac{Mg}{2}, \; T_4 = \frac{3Mg}{2}, \; T_5 = Mg$$

Pulleys and the Atwood Machine

The Atwood Machine can be used to make careful determinations of g, as well as explore the behavior of forces and accelerations.



[1] http://en.wikipedia.org/wiki/Atwood_machine

Pulleys and the Atwood Machine

Notice that again, like the pushed blocks, the two objects must accelerate together at the same rate, because they are connected through an inextensible rope.

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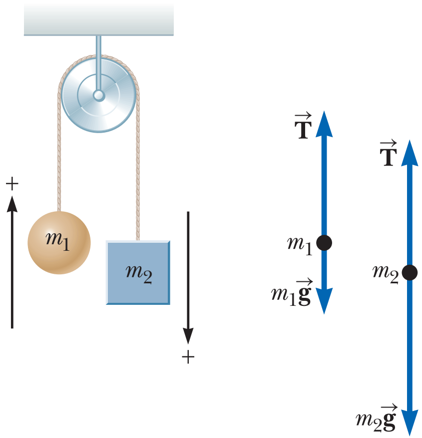
The tension in all parts of the light rope will be the same.
("light" $\Rightarrow$ treat as massless)

Pulleys and the Atwood Machine

Free body diagrams:



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Pulleys and the Atwood Machine

y-direction:

$$\text{up } +\text{ve:} \qquad F_{\text{net},y,1} = T - m_1 g = m_1 a \tag{1}$$

$$\text{down } +\text{ve:} \qquad F_{\text{net},y,2} = m_2 g - T = m_2 a \tag{2}$$

Take eq (1) + eq (2):

$$m_2 g - m_1 g \;=\; m_1 a + m_2 a$$

$$\boxed{a \;=\; \frac{(m_2 - m_1)g}{m_1 + m_2}}$$

Pulleys and the Atwood Machine

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Take eq (1) + eq (2):

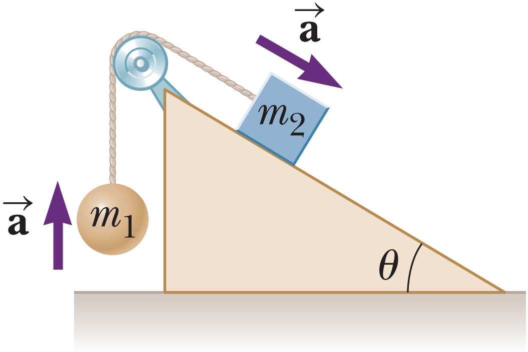
$$m_2 g - m_1 g = m_1 a + m_2 a$$

$$\boxed{a = \frac{(m_2 - m_1)g}{m_1 + m_2}}$$

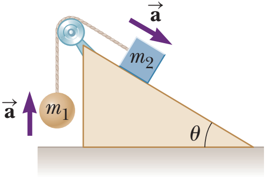
$$\boxed{T = \frac{2 m_1 m_2 g}{m_1 + m_2}}$$

Pulley with an Incline

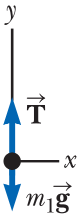
What if we change up our Atwood machine apparatus so that one of the masses is on an incline with no friction?

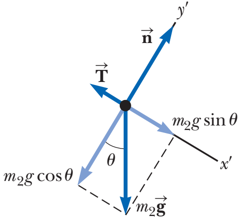


Pulley with an Incline



We can still consider each object separately, with separate axes:





Acceleration? Tension?

Pulley with an Incline

Object 1:
x-direction: $F_{\text{net},x} = 0 \;\Rightarrow\; a_x = 0.$
y-direction:

$$\begin{aligned} F_{\text{net,y}} &= m_1 a_y \\ T - m_1 g &= m_1 a_y \end{aligned}$$

Pulley with an Incline

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Object 2:
x'-direction:

$$\begin{aligned} F_{\text{net},x'} &= m_2 a_{x'} \\ m_2 g \sin\theta - T &= m_2 a_{x'} \end{aligned}$$

y'-direction: $a_{y'} = 0.$

Pulley with an Incline

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y'-direction: $a_{y'} = 0$.

We must also have $a_y = a_{x'} = a$.

Pulley with an Incline

$$m_1 a = T - m_1 g \quad (3)$$

$$m_2 a = m_2 g \sin\theta - T \quad (4)$$

Add eq (3) and (4):

$$(m_1 + m_2)a = m_2 g \sin\theta - m_1 g$$

$$\boxed{a = \frac{(m_2 \sin\theta - m_1)g}{m_1 + m_2}}$$

Pulley with an Incline

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m_2 a &= m_2 g \sin\theta - T \quad (4)
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$$

Putting a into (3):

$$
m_1 \frac{(m_2 \sin\theta - m_1)g}{m_1 + m_2} = T - m_1 g
$$

$$
\boxed{T = \frac{m_1 m_2 (\sin\theta + 1)g}{m_1 + m_2}}
$$

Pulley with an Incline

$$m_1 a = T - m_1 g \tag{3}$$
$$m_2 a = m_2 g \sin\theta - T \tag{4}$$

Add eq (3) and (4):

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$$m_1 \frac{(m_2 \sin\theta - m_1)g}{m_1 + m_2} = T - m_1 g$$
$$\boxed{T = \frac{m_1 m_2 (\sin\theta + 1)g}{m_1 + m_2}}$$

Does this agree with what we had for the Atwood machine when $\theta = 90^\circ$?

Summary

- objects moving together
- pulleys
- Atwood machines

Quiz tomorrow.

(Uncollected) Homework Serway & Jewett,

- Ch 5, onward from page 136. Problems: 29, 83, 40, 45, 49, 93, 101