Dynamics
Applying Newton's Laws
Circular Motion

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Last time

- friction

Overview

- Circular motion and force
- Examples
- Banked turns

Circular Motion - Now with Force

If an object moves in a **uniform circle**, its velocity must always be changing. $\Rightarrow$ It is accelerating.

[1] Figures from Serway & Jewett.

Circular Motion - Now with Force

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$$\vec{\mathbf{F}}_{\text{net}} = m\vec{\mathbf{a}} \;\Rightarrow\; \vec{\mathbf{F}}_{\text{net}} \neq 0$$

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Circular Motion - Now with Force

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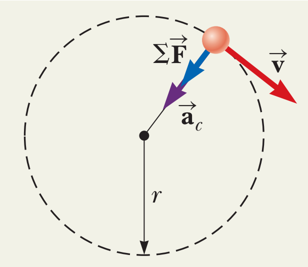
Any object moving in a circular (or curved) path must be experiencing a force.

The net force on an object that moves in a uniform circle is directed to the center of the turn and is called a *centripetal force*.



[1]Figures from Serway & Jewett.

Uniform Circular Motion

For an object moving in a **uniform circle**, $a = a_c = \frac{v^2}{r}$.

This gives the expression for the net force required:

$$\vec{\mathbf{F}}_{\text{net}} = m\vec{\mathbf{a}}$$

so,

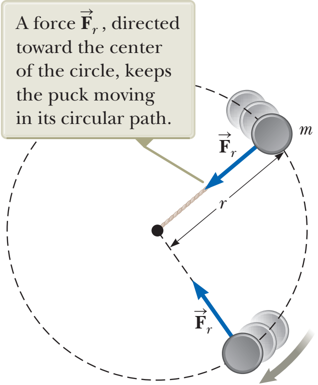
$$F_{\text{net}} = \frac{mv^2}{r}$$

As a vector:

$$\vec{\mathbf{F}}_{\text{net}} = -\frac{mv^2}{r}\hat{\mathbf{r}}$$

Centripetal Force

Something must provide this force, which means at least one component of at least on force must point towards the center of the circle:



It could be tension in a rope.

Centripetal Force

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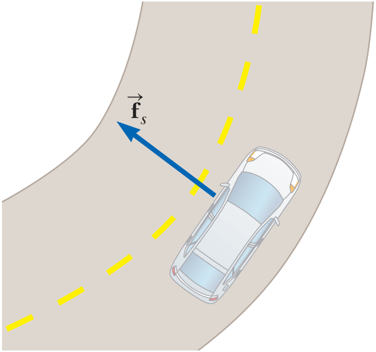
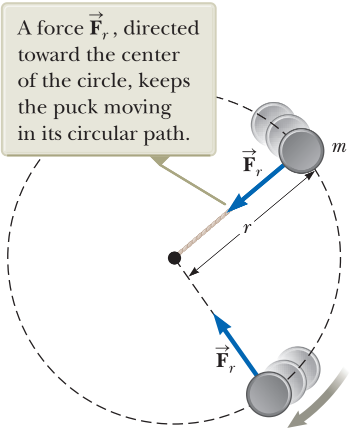


It could be friction.

Centripetal Force

Consider the example of a string constraining the motion of a puck:



Centripetal Force

Question. What will the puck do if the string breaks?

(A) Fly radially outward.

(B) Continue along the circle.

(C) Move tangentially to the circle.

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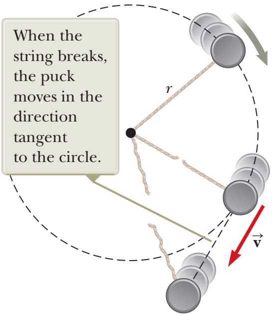

UCM and Force Example

Page 169, # 4

4. A curve in a road forms part of a horizontal circle. As a car goes around it at constant speed 14.0 m/s, the total horizontal force on the driver has magnitude 130 N. What is the total horizontal force on the driver if the speed on the same curve is 18.0 m/s instead?

UCM and Force Example

Page 169, # 4

$$F_{\text{net}} = \frac{mv^2}{r}$$

and

$$F'_{\text{net}} = \frac{m(v')^2}{r}$$

UCM and Force Example

Page 169, # 4

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and

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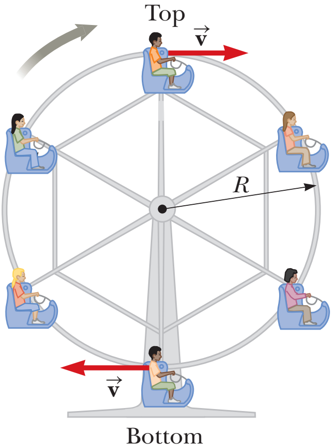
$$\begin{aligned}
\frac{m}{r} &= \frac{F_{\text{net}}}{v^2} = \frac{F_{\text{net}'}}{(v')^2} \\
F_{\text{net}'} &= \frac{F_{\text{net}}(v')^2}{v^2} \\
&= \frac{130 \text{ N}(18.0\text{m/s})^2}{(14.0\text{m/s})^2} \\
&= 215 \text{ N}
\end{aligned}$$

Ferris Wheel Forces

A Ferris wheel is a ride you tend to see at fairs and theme parks.



During the ride the speed, v, is constant.

Ferris Wheel Forces

Quick Quiz 6.1[1] You are riding on a Ferris wheel that is rotating with constant speed. The car in which you are riding always maintains its correct upward orientation; it does not invert.

(i) What is the direction of the normal force on you from the seat when you are at the top of the wheel?

(A) upward

(B) downward

(C) impossible to determine

[1] Page 153, Serway & Jewett

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Ferris Wheel Forces

Quick Quiz 6.1[1] You are riding on a Ferris wheel that is rotating with constant speed. The car in which you are riding always maintains its correct upward orientation; it does not invert.

(ii) From the same choices, what is the direction of the net force on you when you are at the top of the wheel?

(A) upward

(B) downward

(C) impossible to determine

[1]Page 153, Serway & Jewett

Ferris Wheel Forces

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(C) impossible to determine

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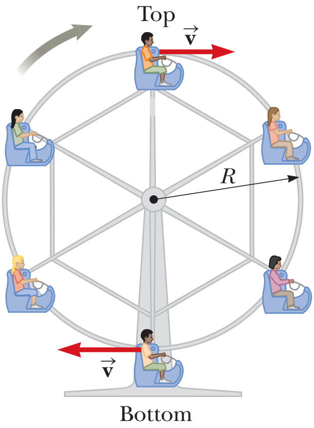
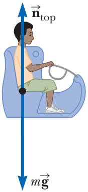
Ferris Wheel

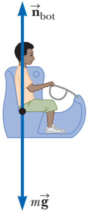
Assume the speed, v, is constant.



$n_{\text{top}} < mg$: $\vec{\mathbf{F}}_{\text{net}}$ points down

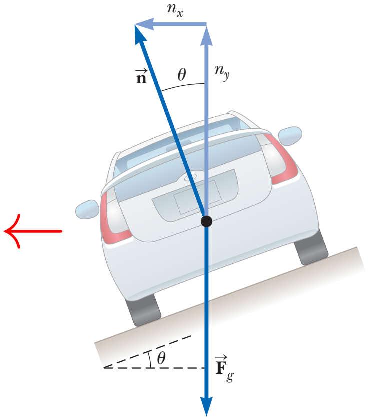


$n_{\text{bot}} > mg$: $\vec{\mathbf{F}}_{\text{net}}$ points up



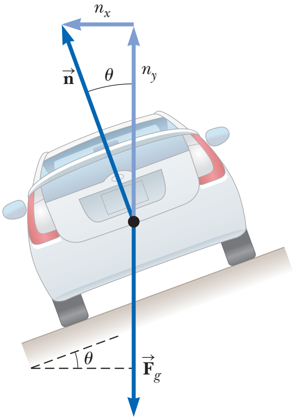
A Banked Turn

Sharp turns in roads are often banked inwards to assist cars in making the turn: the centripetal force comes from the normal force, not friction.



A Banked Turn

A turn has a radius r. What should the angle θ be so that a car traveling at speed v can turn the corner without relying on friction?



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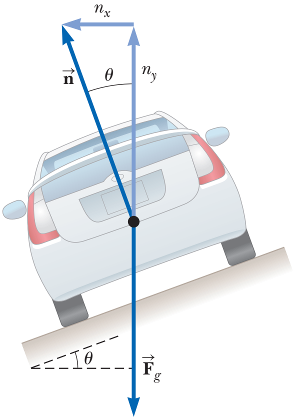
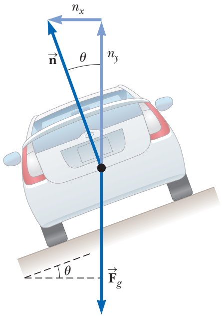


Hint: consider what the net force vector must be in this case.

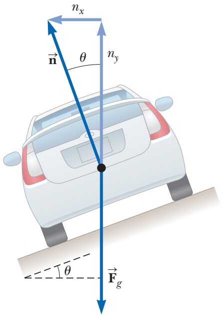
A Banked Turn



y-direction (vertical):

$$\begin{aligned} F_{y,\text{net}} &= 0 \\ n_y - mg &= 0 \end{aligned}$$

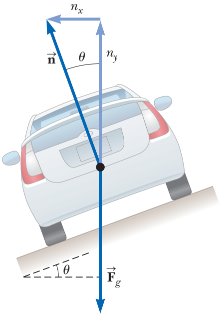
A Banked Turn



y-direction (vertical):

$$\begin{aligned} F_{y,\text{net}} &= 0 \\ n_y - mg &= 0 \\ n\cos\theta &= mg \\ n &= \frac{mg}{\cos\theta} \end{aligned}$$

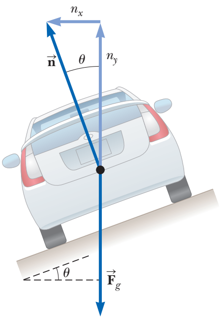
A Banked Turn



x-direction (horizontal):

$$F_{x,\text{net}} = m\,a_c$$
$$n_x = \frac{mv^2}{r}$$

A Banked Turn

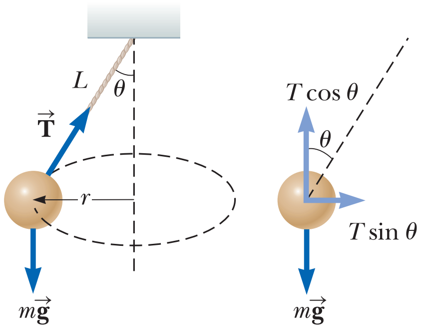


x-direction (horizontal):

$$
\begin{aligned}
F_{x,\text{net}} &= m\,a_c \\
n_x &= \frac{mv^2}{r} \\
n \sin\theta &= \frac{mv^2}{r} \\
\frac{mg}{\cos\theta}\sin\theta &= \frac{mv^2}{r} \\
\tan\theta &= \frac{v^2}{rg} \Rightarrow \theta = \tan^{-1}\left(\frac{v^2}{rg}\right)
\end{aligned}
$$

Conical pendulum

In a "conical pendulum" the bob moves in a horizontal circle at the end of a string. The string traces out a cone shape.



Look at the force diagram, and think about which way the acceleration vector points.

Does this situation look familiar?

[1]Serway & Jewett, page 152.

Summary

- Uniform circular motion with forces
- Banked turns

First Test Monday, 10 Feb.

(Uncollected) Homework Serway & Jewett,

- Ch 6, onward from page 169. OQ: 4; Probs: 1, 5, 9, 15, 17, 18, 63, 61