Dynamics
Applying Newton's Laws
Accelerated Frames

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De Anza College

Feb 4, 2020

Last time

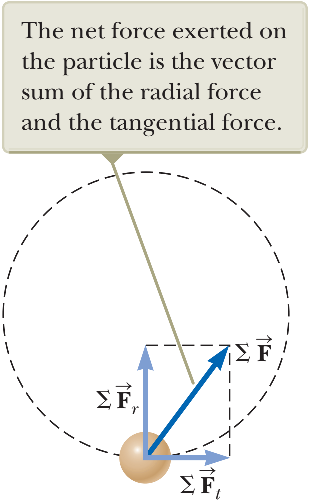
- circular motion and force
- examples

Overview

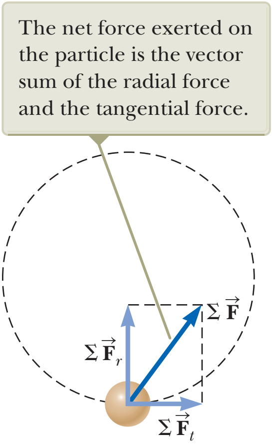
- non-uniform circular motion
- accelerated frames
- fictitious forces

Non-Uniform Circular Motion

If the speed is changing the net force will not be pointed into the center of the circle.



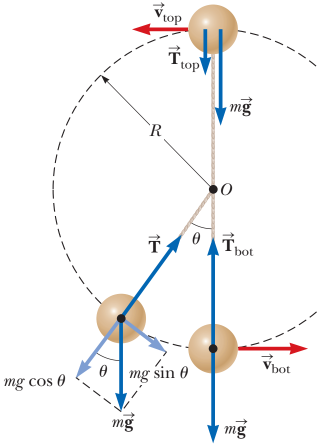
Non-Uniform Circular Motion



The radial/centripetal component of the force will still be given by $\frac{mv^2}{r}$, but now the tangential component is given by $ma_t = m\,\frac{\mathrm{dv}}{\mathrm{dt}}$.

Non-Uniform Circular Motion Example

Consider a ball of mass m swinging in a vertical circle, radius R. What is the tension T when the string makes an angle θ with the vertical if the speed at that instant is v?



Non-Uniform Circular Motion Example

In the radial direction:

$$F_{r,\text{net}} = -\frac{mv^2}{R}$$

Non-Uniform Circular Motion Example

In the radial direction:

$$\begin{aligned} F_{r,\text{net}} &= -\frac{mv^2}{R} \\ mg\cos\theta - T &= -\frac{mv^2}{R} \\ T &= \frac{mv^2}{R} + mg\cos\theta \\ T &= mg\left(\frac{v^2}{Rg} + \cos\theta\right) \end{aligned}$$

Accelerated Frames

Newton's first law and frames of reference...

You are driving a car and push on the accelerator pedal. An object on your dashboard comes flying off toward you, without any force on it. Was Newton's first law violated?

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Newton I (textbook version)

If an object does not interact with other objects, it is possible to identify a reference frame in which the object has zero acceleration.

A zero-acceleration reference frame is called an *inertial reference frame*.

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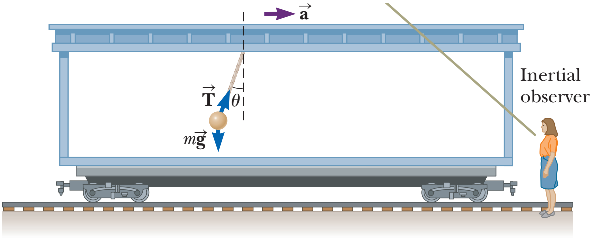
A zero-acceleration reference frame is called an *inertial reference frame*.

In your car, you are not in an inertial frame; you are in an accelerating frame.

Inertial and Non-intertial Observers

Inertial observer, not on accelerating train car:



[1]Figures from Serway & Jewett

Inertial and Non-intertial Observers

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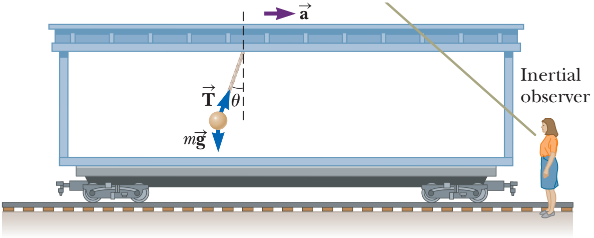
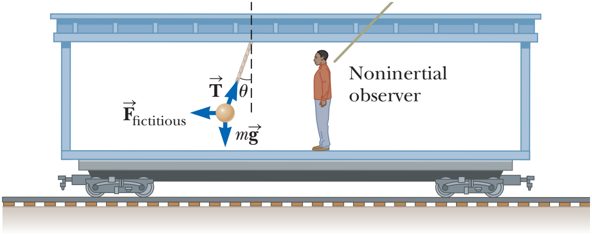


Non-inertial observer, on accelerating train car:



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Inertial and Non-intertial Observers

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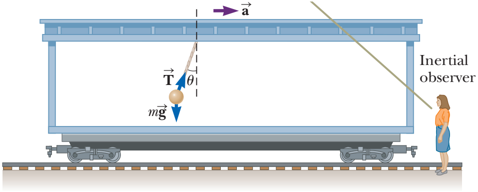
Both observers see the string make the same angle to the vertical direction.

Inertial observer's explanation: the ball is accelerating, $\vec{\mathbf{a}} = a\hat{\mathbf{i}}$.

$$
\begin{aligned}
F_{\text{net},x} &= T\sin\theta = ma \\
F_{\text{net},y} &= T\cos\theta - mg = 0
\end{aligned}
$$



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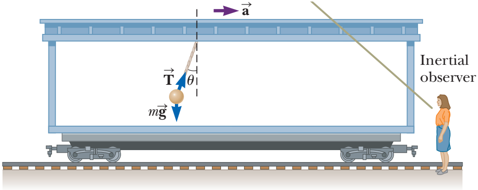

Inertial observer concludes:

$$\theta = \tan^{-1}\left(\frac{a}{g}\right) \quad , \quad T = \frac{ma}{\sin\theta} = m\sqrt{a^2 + g^2}$$

Inertial and Non-intertial Observers

Non-inertial observer's explanation: the ball is experiencing a force, $\vec{\mathbf{F}}_{\text{fictitious}} = -m\vec{\mathbf{a}} = -ma\,\hat{\mathbf{i}}$.

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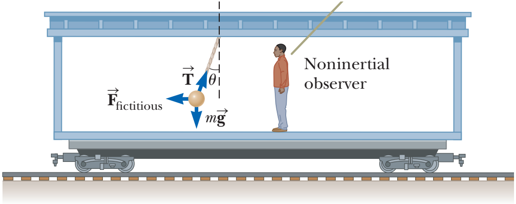

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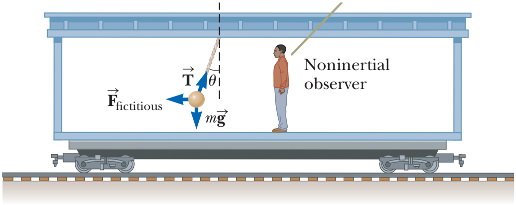

These equations are equivalent to the inertial observer's!

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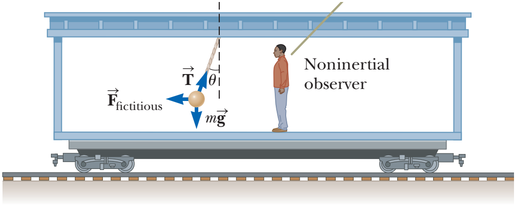

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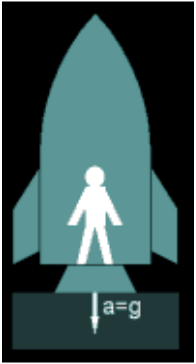
However, this only happened because the accelerating observer included an extra (fictitious) force in his description.

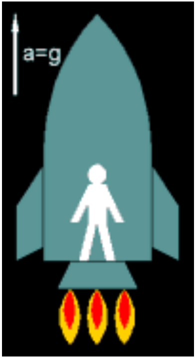
That force needed to be there for him to explain his observations, but he can't find an interaction that caused it.

Equivalence of gravitational acceleration

A rocket on Earth. A person onboard feels a normal force $n = mg$ acting upward from the floor.



A rocket accelerating in space. A person onboard feels a normal force $n = ma = mg$ (if $a = g$) acting upward from the floor.



[1]Figures from http://www.ex-astris-scientia.org

Example, accelerating frame

Page 171, # 20

20. An object of mass $m = 5.00$ kg, attached to a spring scale, rests on a frictionless, horizontal surface as shown in Figure P6.20. The spring scale, attached to the front end of a boxcar, reads zero when the car is at rest. (a) Determine the acceleration of the car if the spring scale has a constant reading of 18.0 N when the car is in motion. (b) What constant reading will the spring scale show if the car moves with constant velocity? Describe the forces on the object as observed (c) by someone in the car and (d) by someone at rest outside the car.

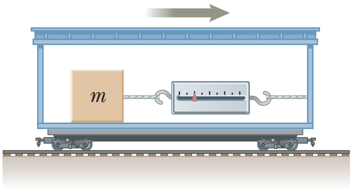
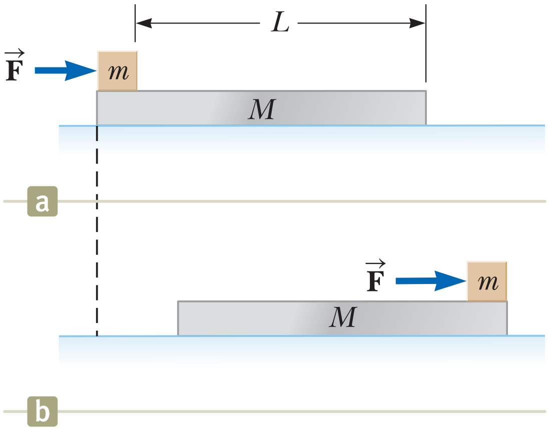


Figure P6.20

Sliding Blocks with Friction: #103, page 149

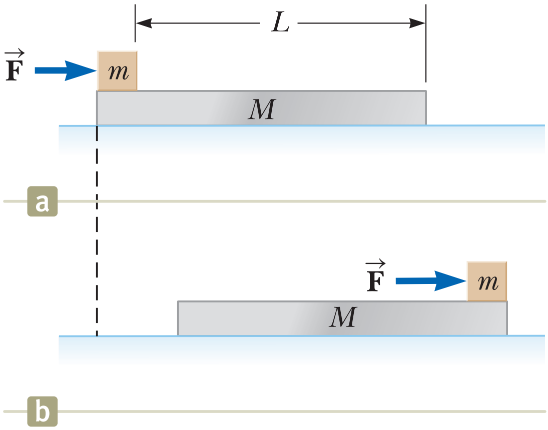


Between m and M is friction, coefficient μ_k.

How long does it take for the little block to reach the end of the long block?

How far did the long block move?

Sliding Blocks with Friction: #103, page 149



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Sliding Blocks with Friction: #103, page 149

103. A block of mass $m = 2.00$ kg rests on the left edge of a block of mass $M = 8.00$ kg. The coefficient of kinetic friction between the two blocks is 0.300, and the surface on which the 8.00-kg block rests is frictionless. A constant horizontal force of magnitude $F = 10.0$ N is applied to the 2.00-kg block, setting it in motion as shown in Figure P5.103a. If the distance L that the leading edge of the smaller block travels on the larger block is 3.00 m, (a) in what time interval will the smaller block make it to the right side of the 8.00-kg block as shown in Figure P5.103b? (*Note:* Both blocks are set into motion when $\vec{\mathbf{F}}$ is applied.) (b) How far does the 8.00-kg block move in the process?

Sliding Blocks with Friction: #103, page 149

Notice that this problem could be solved using a frame that moves with the long block of mass M.

In that frame, the small block moves a distance L.

Summary

- non-uniform circular motion
- accelerating frames
- fictitious forces

(Uncollected) Homework Serway & Jewett,

- Read ahead in Chapter 6 about resistive forces.
- Ch 6, onward from page 171. Probs: 45 (circ motion)
- Ch 6, Probs: 21, 23, 24, 51 (accel frames)