Dynamics
Applying Newton's Laws
Rotating Frames

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Last time

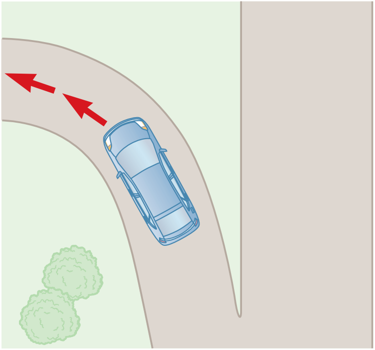
- accelerated frames: linear motion

Overview

- accelerated frames: rotating frames

Rotating Frames

Consider a car making a turn to the left.



We know that there is a force (the centripetal force) towards the center of the circle.

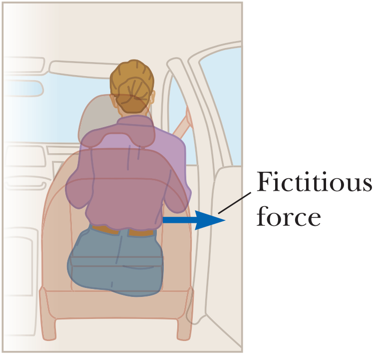
Rotating Frames

What does a passenger in the car feel?

Rotating Frames

What does a passenger in the car feel?

She feels pushed toward the right against the side of the car.



She is a non-inertial observer.

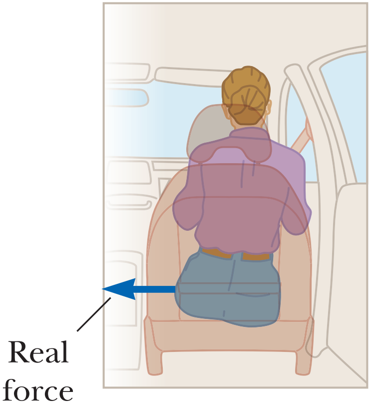
Rotating Frames

What is an inertial observer's explanation?

Rotating Frames

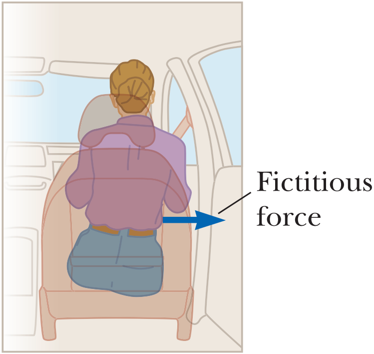
What is an inertial observer's explanation?

The car must exert a force on the passenger to give her an acceleration toward the center of the circular turn.



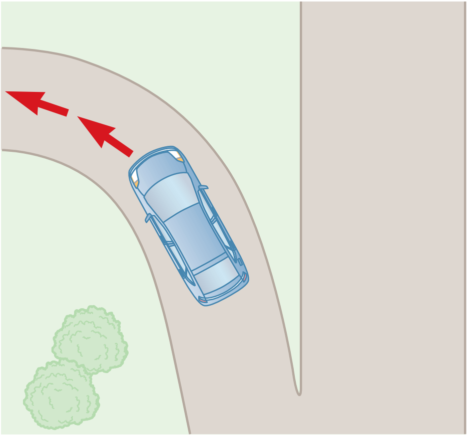
Rotating Frames: Centrifugal "Force"

We call the non-inertial observer's fictitious outward force the centrifugal "force".



Rotating Frames

Car making a turn to the left.



Question

Quick Quiz 6.3[1] Consider the passenger in the car making a left turn. Which of the following is correct about forces in the horizontal direction if she is making contact with the right-hand door?

The passenger is ...

(A) in equilibrium between real forces acting to the right and real forces acting to the left.

(B) subject only to real forces acting to the right.

(C) subject only to real forces acting to the left.

(D) None of those statements is true.

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[2]Serway & Jewett, page 160.

Question

Quick Quiz 6.3[1] Consider the passenger in the car making a left turn. Which of the following is correct about forces in the horizontal direction if she is making contact with the right-hand door?

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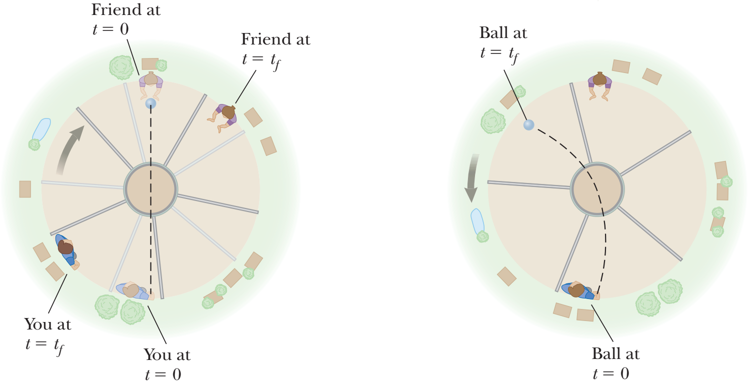
(C) subject only to real forces acting to the left. $\longleftarrow$

(D) None of those statements is true.

[2]Serway & Jewett, page 160.

Rotating Frames: Coriolis "Force"

There is another fictitious force that non-inertial observers see in a rotating frame.

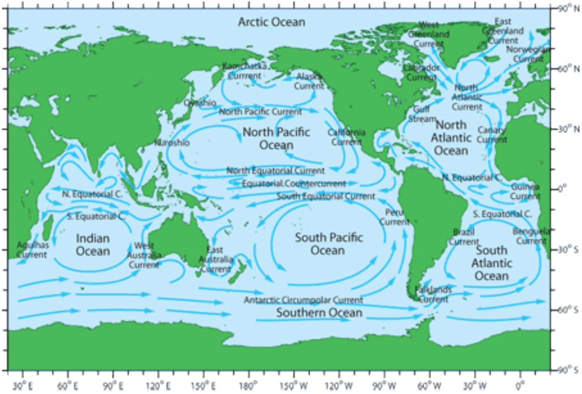


The Coriolis "force" appears as a fictitious sideways force to a non-inertial observer.

Rotating Frames: Coriolis “Force”

We can detect the effect of Earth’s rotation: they manifest as Coriolis effects.

eg. ocean surface currents:



[1] Figure from http://www.seos-project.eu/

Example, Force on rotating object

Page 171, #25

25. A small container of water is placed on a turntable inside a microwave oven, at a radius of 12.0 cm from the center. The turntable rotates steadily, turning one revolution in each 7.25 s. What angle does the water surface make with the horizontal?

Example, Force on rotating object

Hint: consider the forces that act on a small blob of water in the container.

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This situation is similar to the banked turn problem from the previous lecture.

$$v = \frac{2\pi r}{T}\ , \quad r = 0.12\ \text{m}\ , \quad T = 7.25\ \text{s}$$

Example, Force on rotating object

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This situation is similar to the banked turn problem from the previous lecture.

$$v = \frac{2\pi r}{T} \ , \quad r = 0.12 \text{ m} \ , \quad T = 7.25 \text{ s}$$

$$\begin{aligned} v &= \frac{2\pi r}{T} \\ v &= \frac{2\pi(0.12)}{(7.25)} \\ v &= 0.1040 \text{ m/s} \end{aligned}$$

Example, Force on rotating object

y-direction:

$$\begin{aligned} F_{\text{net},y} &= 0 \\ \implies n_y &= mg \\ n\cos\theta &= mg \\ n &= \frac{mg}{\cos\theta} \end{aligned}$$

x-direction:

$$\begin{aligned} F_{\text{net},x} &= \frac{mv^2}{r} \\ \implies n_x &= \frac{mv^2}{r} \\ n\sin\theta &= \frac{mv^2}{r} \\ \frac{mg}{\cos\theta}\sin\theta &= \frac{mv^2}{r} \\ \theta &= \tan^{-1}\left(\frac{v^2}{rg}\right) \end{aligned}$$

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Rotating Frame Problem

Page 176, #70

70. Because of the Earth's rotation, a plumb bob does not hang exactly along a line directed to the center of the Earth. How much does the plumb bob deviate from a radial line at 35.0° north latitude? Assume the Earth is spherical.

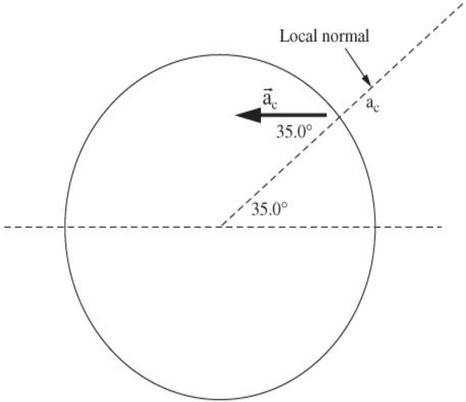
Rotating Frame Problem

Page 176, #70

70. Because of the Earth's rotation, a plumb bob does not hang exactly along a line directed to the center of the Earth. How much does the plumb bob deviate from a radial line at 35.0° north latitude? Assume the Earth is spherical.

$R_{\mathsf{Earth}} = 6.37 \times 10^6 \text{ m},\ T_{\mathsf{rev}} = 24 \text{ h} \times 3600 \text{ s h}^{-1} = 86,400 \text{ s}$

Rotating Frame Problem



[1]Figure from Serway & Jewett Instructor materials.

Rotating Frame Problem

$$v = \frac{\Delta x}{\Delta t} = \frac{2\pi r}{T_{\text{rev}}}$$

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$$\begin{aligned}
F_{\text{net},y} &= -\frac{mv^2}{r}\cos(35) \\
T\cos\phi - mg &= -\frac{mv^2}{r}\cos(35) \\
T\cos\phi &= mg - \frac{mv^2}{r}\cos(35)
\end{aligned}$$

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Dividing:

$$\tan\phi = \frac{\frac{v^2}{r}\sin(35)}{g - \frac{v^2}{r}\cos(35)}$$

Rotating Frame Problem

$$\phi = \tan^{-1}\left(\frac{\frac{v^2}{r}\sin(35)}{g - \frac{v^2}{r}\cos(35)}\right)$$

$R_{\mathrm{Earth}} = 6.37 \times 10^6\ \mathrm{m},\ T_{\mathrm{rev}} = 24\ \mathrm{h} \times 3600\ \mathrm{s}\ \mathrm{h}^{-1} = 86,400\ \mathrm{s}$

$\frac{v^2}{r} = \frac{1}{r}\left(\frac{2\pi r}{T}\right)^2 = \frac{4\pi^2 (R_{\mathrm{Earth}}\cos(35))}{T_{\mathrm{rev}}^2}$

$$\phi = 0.0928^\circ$$

Summary

- accelerated frames: rotating frames
- practice with rotating frames

Test Monday, Feb 10.

(Uncollected) Homework

Serway & Jewett,

- Read Chapter 6 if you haven't already.
- Ch 6, onward from page 171. Probs: 7, 59, 67(don't spend too long on it)