Dynamics
Applying Newton's Laws
Air Resistance

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Last Time

- accelerated frames and rotation

Overview

- resistive forces
- two models for resistive forces
- terminal velocities
- Stokes' model ($\propto v$)

Resistive Forces

Galileo predicted (correctly) that all objects at the Earth's surface accelerate at the same rate, g.

This was a revolutionary idea because it seems obvious that less massive objects should fall more slowly: consider a feather and a bowling ball.

What is happening there?

Resistive Forces

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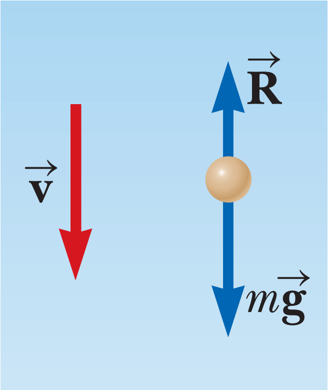
Air resistance can play a big role in determining an object's motion.

Resistive Forces

Resistive forces act on an object when it moves through a fluid medium, like a liquid or gas.

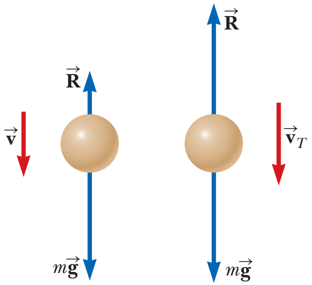
Is this object accelerating?



Resistive Forces

Air resistance increases with speed.

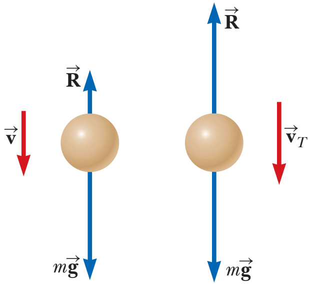
Will the object continue to increase it's velocity without bound?



Resistive Forces

Air resistance increases with speed.

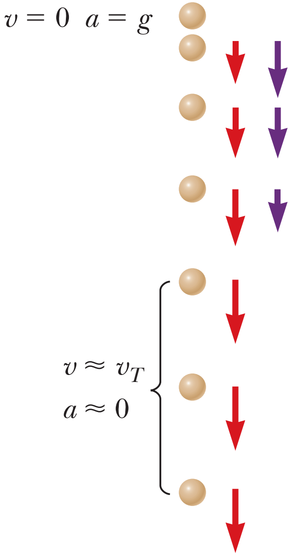
Will the object continue to increase it's velocity without bound?
No.



The velocity will not exceed some terminal value.

Resistive Forces

What is happening to the acceleration vector?



Resistive Forces Question

A feather and a bowling ball are dropped at the same time from the top of an 100 story building. Over the course of the fall, which experiences the largest force of air resistance?

(A) the feather

(B) the bowling ball

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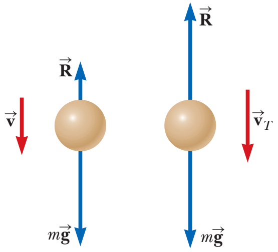

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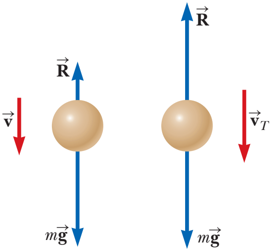

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Resistive Forces resist motion.

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There are two main models for how this happens. Either the resistive force $\vec{\mathbf{R}}$

- is proportional to $\vec{\mathbf{v}}$, or
- is proportional to v^2

Either way, this leads to a force in the opposite direction to the object's velocity, and that depends on the object's speed through the fluid.

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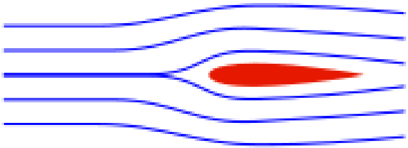
This model applies when the *Reynolds Number* of the fluid is very low and/or the object is moving very slowly.

Reynolds Number, Re $\sim \frac{\text{inertial forces}}{\text{viscous forces}}$.

More viscous fluids have lower Reynolds Numbers.

Model 1: Stokes Drag

Low Reynolds Number and/or low speed means there will be no turbulence. Flow will be *laminar*, or close to laminar. (*ie.* smooth flow lines, no vorticies)



Model 1: Stokes Drag

To summarize, cases where the resistive force will be proportional to v:

- slow moving objects
- very viscous fluids
- laminar (smooth) flow in the fluid

Model 1: Stokes Drag

If $\vec{\mathbf{R}} = -b\vec{\mathbf{v}}$, what is the terminal velocity?

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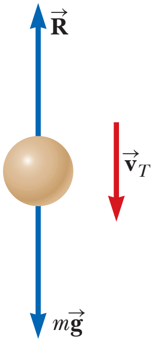
Remember, at terminal velocity, the object is in equilibrium.



Model 1: Stokes Drag

Equilibrium $\Rightarrow$ $F_{\text{net}} = 0$.

$$F_{\text{net}} = bv_T - mg \quad = \quad 0$$

$$v_T = \frac{mg}{b}$$

Example

Page 171, #30

30. W A small piece of Styrofoam packing material is dropped from a height of 2.00 m above the ground. Until it reaches terminal speed, the magnitude of its acceleration is given by $a = g - Bv$. After falling 0.500 m, the Styrofoam effectively reaches terminal speed and then takes 5.00 s more to reach the ground. (a) What is the value of the constant B? (b) What is the acceleration at $t = 0$? (c) What is the acceleration when the speed is 0.150 m/s?

Example

Page 171, #30

(a) Terminal velocity $v_T = \frac{\Delta x}{\Delta t} = \frac{2.00 - 0.500 \text{ m}}{5.00 \text{ s}} = 0.300 \text{ m/s}$.

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(b) at $t = 0$, $a = g$, directed downward.

(c) at $v = 0.150$ m/s, $a = g - Bv = 4.9 \text{ ms}^{-2}$, directed downward.

Model 1: Stokes Drag

Can we find an expression for how the velocity changes with time before reaching v_T?

Model 1: Stokes Drag

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Yes! In general, (calling down positive)

$$\begin{aligned} F_{\text{net}} &= mg - bv \\ ma &= mg - bv \\ m\frac{\mathrm{dv}}{\mathrm{dt}} &= mg - bv \end{aligned}$$

A differential equation for v.

Model 1: Stokes Drag

Solving[1] $m \frac{\mathrm{dv}}{\mathrm{dt}} = mg - bv$:

[1]See the end of the slides for another way to solve.

Model 1: Stokes Drag

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Separate the variables:

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expression in v: expression in t (potentially):

$$\left(\frac{m}{mg - bv}\right)\frac{\mathrm{dv}}{\mathrm{dt}} = 1$$

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Integrating with respect to t, we can set $v = 0$ at $t = 0$:

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We have an expression for v.

$$v(t) = \frac{mg}{b}(1 - e^{-bt/m})$$

Model 1: Stokes Drag

Exercise: Check that the solution $v(t) = \frac{mg}{b}(1 - e^{-bt/m})$ satisfies the equation

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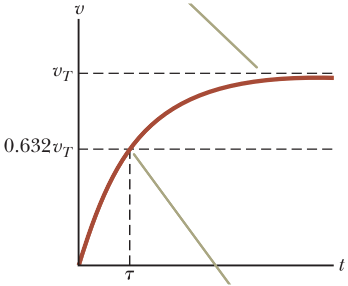

Time constant, $\tau = \frac{m}{b}$. The solution can also be written $v(t) = \frac{mg}{b}(1 - e^{-t/\tau})$.

Model 1: Stokes Drag, Question

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Suppose two falling objects had different masses, but similar shapes and fall through the same fluid, and so have the same constant b.

Which one reaches 99% of its terminal velocity first?

(A) the one with more mass

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Summary

- Stokes drag
- terminal velocity

Test Monday.

(Uncollected) Homework Serway & Jewett,

- Read Chapter 6 if you haven't already.
- Ch 6, onward from page 171. Probs: 29, 31, 35, 43

Appendix: Stokes Drag - Alternate Way to Solve

Another way of solving $\frac{\mathrm{dv}}{\mathrm{dt}} + \frac{b}{m} v = g$:

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Another way of solving $\frac{\mathrm{dv}}{\mathrm{dt}} + \frac{b}{m} v = g$:

Use an *integrating factor* $\mu(t)$. This is just some unknown function of t that we multiply through the equation.

$$\mu(t) \frac{\mathrm{dv}}{\mathrm{dt}} + \mu(t) \frac{b}{m} v = \mu(t) g$$

But $\mu(t)$ was arbitrary, so let it have the property $\mu'(t) = \mu(t) \frac{b}{m}$.
$\Rightarrow \mu(t) = e^{bt/m}$

Our equation becomes

$$\mu(t) \frac{\mathrm{dv}}{\mathrm{dt}} + \mu'(t) v = \mu(t) g$$

Now we can integrate both sides! (Product rule...)

Appendix: Stokes Drag - Alternate Way to Solve

$$\begin{aligned}
\mu(t)\,\frac{\mathrm{dv}}{\mathrm{dt}} + \mu'(t)v &= \mu(t)g \\
\frac{\mathrm{d}}{\mathrm{dt}}(\mu(t)v) &= \mu(t)g
\end{aligned}$$

Integrating with respect to t, and choosing $v = 0$ at $t = 0$ (falls from rest):

$$\begin{aligned}
\mu(t)v &= \int_0^t \mu(t')g\mathrm{dt}' \\
v &= \frac{\int_0^t \mu g\mathrm{dt}'}{\mu}
\end{aligned}$$

We have an expression for v. All we need to do is substitute back for $\mu(t)$.

Appendix: Stokes Drag - Alternate Way to Solve

$\mu(t) = e^{bt/m}$:

$$
\begin{aligned}
v &= \frac{\int_0^t e^{bt'/m} g \mathrm{dt}'}{e^{bt/m}} \\
&= e^{-bt/m}\left(\frac{mg}{b}e^{bt/m} - \frac{mg}{b}\right) \\
&= \frac{mg}{b} - \frac{mg}{b}e^{-bt/m}
\end{aligned}
$$

$$v(t) = \frac{mg}{b}(1 - e^{-bt/m})$$

Summary (Again, to prevent confusion)

- Stokes drag
- terminal velocity

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