Dynamics
Energy and Work

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Last Time

- resistive forces: Drag Equation

Overview

- Energy (systems and environments)
- Work

Energy

Energy is almost impossible to clearly define, yet everyone has a good intuitive notion of what it is.

Energy is a property of physical systems. It tells us something about the states or configurations the system can be in. In fact, it is possible to find the dynamics of a system purely from understanding the distribution of energy in the system.

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Energy is a property of physical systems. It tells us something about the states or configurations the system can be in. In fact, it is possible to find the dynamics of a system purely from understanding the distribution of energy in the system.

Importantly, it can neither be created or destroyed, but it can be transferred between systems, and take different forms.

Types of Energy

- motional energy (kinetic)
- energy as a result of object's configurations or stored energy (potential)
- heat, light, sound - can carry away energy from a mechanical system

Kinetic Energy

Kinetic energy, K

the energy that a system has as a result of its motion, or the motion of its constituent parts.

$$K = \frac{1}{2}mv^2$$

The larger the speed of the system, or its parts, the higher the kinetic energy.

Systems and Environments

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Outside influences on the system (*eg.* forces or energies) can be included in the description, but the source of these effects is not described.

Everything outside the system is the system's *environment*.

Systems and Environments

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Kinetic Energy

Systems and Environments

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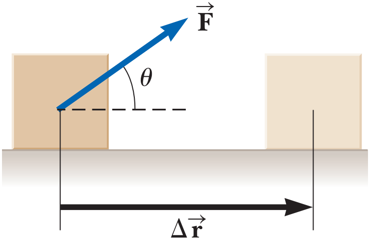
$$W \longrightarrow \boxed{\begin{array}{c}\text{System}\\ \text{Kinetic Energy}\end{array}} \longleftarrow W$$

Work is an energy transfer from the environment to the system, or vice versa.

Work

Let's consider how the environment can affect the system by exchanging energy with it.

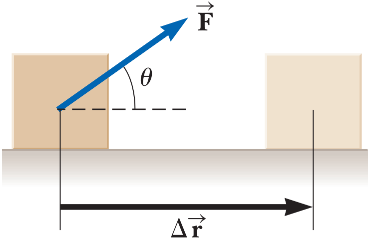
Take the *system* to be a block. An external force (from the *environment*) acts on it.



This force effects the block: it can accelerate it.

It can also change the energy of the block.

Work



For a constant applied force *Work* is defined as:

$$W = \vec{\mathbf{F}} \cdot \overrightarrow{\Delta r}$$

Work is the amount of energy transferred to a system by an interaction with the environment.

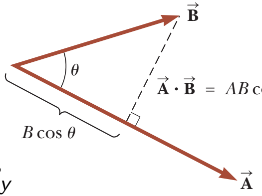
Units: Joules, J.

$$1 \text{ J} = 1 \text{ Nm}$$

Vectors Properties and Operations

Multiplication by a vector:

The Dot Product



Let $\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}$

$\vec{\mathbf{B}} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}}$,

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = A_x B_x + A_y B_y$$

The output of this operation is a scalar.

Equivalently,

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = AB\cos\theta$$

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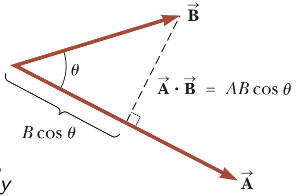

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Properties

- The dot product is commutative: $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}}$
- If $\vec{\mathbf{A}} \parallel \vec{\mathbf{B}}$, $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = AB$.
- If $\vec{\mathbf{A}} \perp \vec{\mathbf{B}}$, $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = 0$.

Vectors Properties and Operations

Multiplication by a vector: The Dot Product

Try it! Find $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}$ when $\vec{\mathbf{A}}$ is a vector of magnitude 6 N directed at 60° above the x-axis and B is a vector of magnitude 2 m pointed along the x-axis.

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$$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = (6\ \text{N})(2\ \text{m})\cos(60^\circ) = 6\ \text{J}$$

(1 J = 1 Nm)

Vectors Properties and Operations

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$(1 \text{ J} = 1 \text{ Nm})$

Now find $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}$ when:

$$\vec{\mathbf{A}} = 1\,\hat{\mathbf{i}} + 2\,\hat{\mathbf{j}}\ ; \quad \vec{\mathbf{B}} = -1\,\hat{\mathbf{i}} - 4\,\hat{\mathbf{j}}$$

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$$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = (1)(-1) + (2)(-4) = -9$$

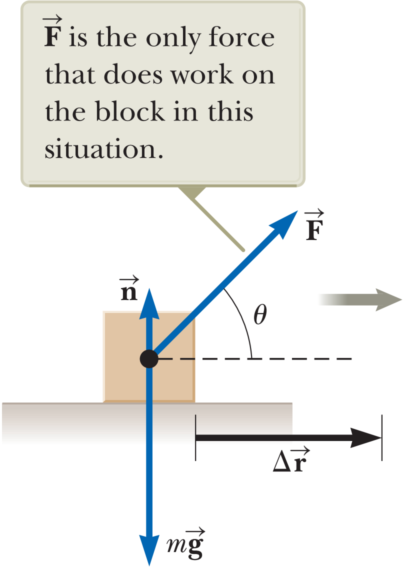
Work

If there are several forces acting on a system, each one can have an associated work.

In other words, we can ask what is the work done on the system by each force separately.

$$W_i = \vec{\mathbf{F}}_i \cdot (\overrightarrow{\boldsymbol{\Delta r}})$$

Work



[1] Figure from Serway & Jewett.

Net Work

Work done on the system by each force separately:

$$W_i = \vec{\mathbf{F}}_i \cdot (\overrightarrow{\Delta r})$$

Net Work

$$W_{\text{net}} = \sum_i W_i$$

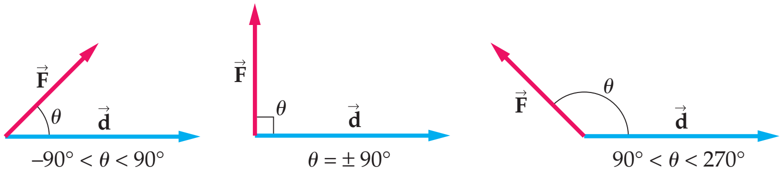
where the sum includes the work of all forces acting on the system.

If the system is treated as a particle:

$$W_{\text{net}} = \vec{\mathbf{F}}_{\text{net}} \cdot (\overrightarrow{\Delta r})$$

Units of Work

Work can be positive or negative!



$W = Fd\cos\theta > 0$ | $W = Fd\cos\theta = 0$ | $W = Fd\cos\theta < 0$

positive work | zero work | negative work

For work done *on* a system:

- Positive $\Rightarrow$ energy is transferred *to* the system.
- Negative $\Rightarrow$ energy is transferred *from* the system.

Question

Quick Quiz 7.1[1] The gravitational force exerted by the Sun on the Earth holds the Earth in an orbit around the Sun. Let us assume that the orbit is perfectly circular. The work done by this gravitational force during a short time interval in which the Earth moves through a displacement in its orbital path is

(A) zero

(B) positive

(C) negative

(D) impossible to determine

[1] Serway and Jewett, page 180.

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[1] Serway and Jewett, page 180.

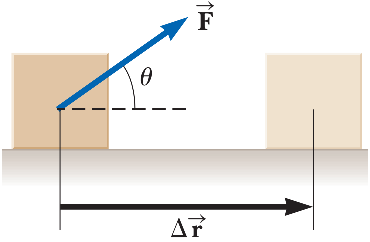
Work example

Page 204, #1

1. A shopper in a supermarket pushes a cart with a force of 35.0 N directed at an angle of 25.0° below the horizontal. The force is just sufficient to balance various friction forces, so the cart moves at constant speed. (a) Find the work done by the shopper on the cart as she moves down a 50.0-m-long aisle. (b) The shopper goes down the next aisle, pushing horizontally and maintaining the same speed as before. If the friction force doesn't change, would the shopper's applied force be larger, smaller, or the same? (c) What about the work done on the cart by the shopper?

Work: more general definition

What if F is not constant?



Work, W

For an applied force, $\vec{\mathbf{F}}(\vec{\mathbf{r}})$:

$$W = \int \vec{\mathbf{F}}(\vec{\mathbf{r}}) \cdot \mathrm{d}\vec{\mathbf{r}}$$

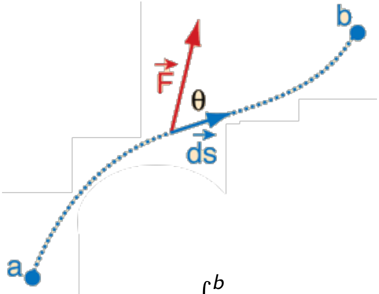
Work is the amount of energy transferred to a system by an interaction with the environment.

Work: more general definition

Consider moving an object across a surface, not necessarily in a straight line. The force may also vary in magnitude and direction during the motion.

Work is a path integral

$$W = \int \vec{\mathbf{F}} \cdot \mathrm{d}\,\vec{\mathbf{s}}$$

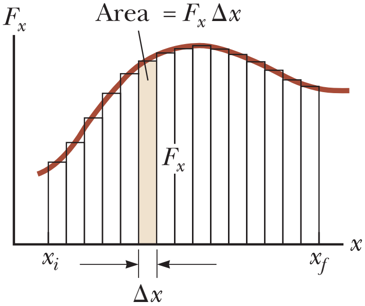


$$W_{a \to b} = \int_a^b F \cos\theta \,\mathrm{ds}$$

Work: more general definition

F_x is the x-component of $\vec{\mathbf{F}}$.

Break up our x-displacement into little slices; F_x can be different in each slice. The work for each little slice is $W_i = F(x)\Delta x$



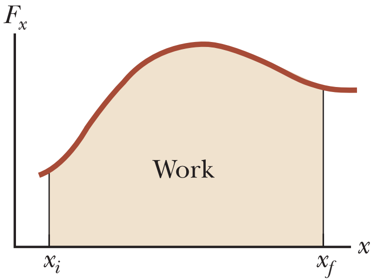
Then add them together: $W = \sum_x F_x \Delta x$ (work done by x-comp)

As the length of the little slices goes to zero:

$$\lim_{\Delta x \to 0} \sum_x F_x \Delta x = \int F(x)\,\mathrm{d}x$$

Work: more general definition

Work done is the area under a force-displacement curve.



If there are y and z components, the work of those components also contributes:

$$W = \int \vec{\mathbf{F}}(r) \cdot \mathrm{d}\,\vec{\mathbf{r}} = \int F_x \,\mathrm{dx} + \int F_y \,\mathrm{dy} + \int F_z \,\mathrm{dz}$$

[1]Figures from Serway & Jewett.

Summary

- Energy, systems, environments
- Work

(Uncollected) Homework Serway & Jewett,

- Read Chapter 7.
- Ch 7, onward from page 204. Probs 3, 5, 7, 11, 9, 15, 17