Energy
Work & Kinetic Energy
Potential Energy
Conservative & Nonconservative Forces

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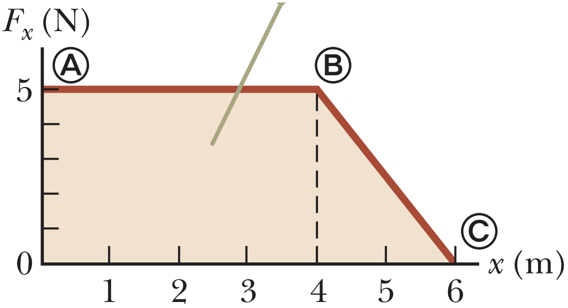
Feb 12, 2020

Last time

- energy
- kinetic energy
- work

Warm Up Question

What is the work done by the force indicated in the graph as the particle moves from $x = 0$ to $x = 6$ m?



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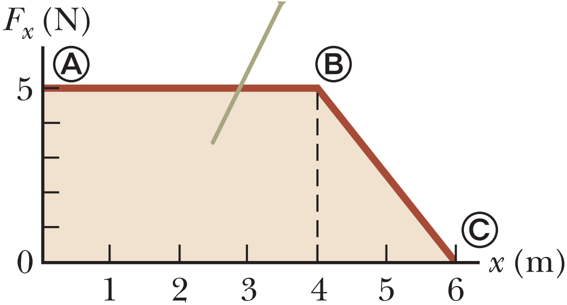


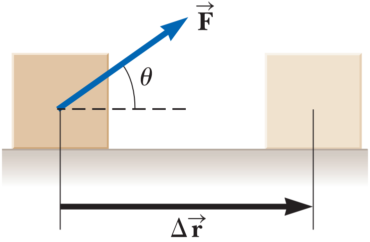
$W = 25$ J.

Overview

- work as an integral and springs
- Work-Kinetic Energy theorem
- potential energy
- conservative forces and non-conservative forces

Work: more general definition

What if F is not constant?



Work, W

For an applied force, $\vec{\mathbf{F}}(\vec{\mathbf{r}})$:

$$W = \int \vec{\mathbf{F}}(\vec{\mathbf{r}}) \cdot \mathrm{d}\,\vec{\mathbf{r}}$$

Work is the amount of energy transferred to a system by an interaction with the environment.

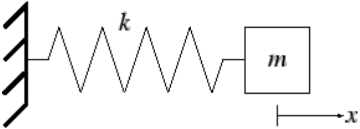
Springs and Work

The force exerted by many types of springs is governed by Hooke's Law.

$$\vec{\mathbf{F}}_{\text{spring}} = -k\,\vec{\mathbf{x}}$$

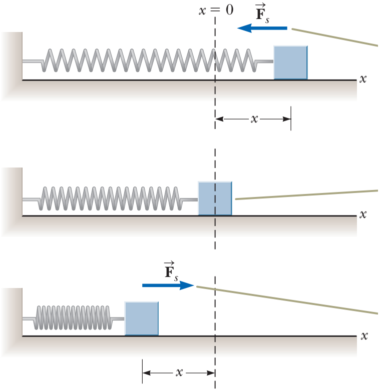
where

- $\vec{\mathbf{x}}$ is the amount of displacement of one end of a spring from its natural length. (The amount of compression or extension.
- k is the *force constant* or *spring constant*.



[1]Figure from CCRMA Stanford Univ.

Spring Force depends on position

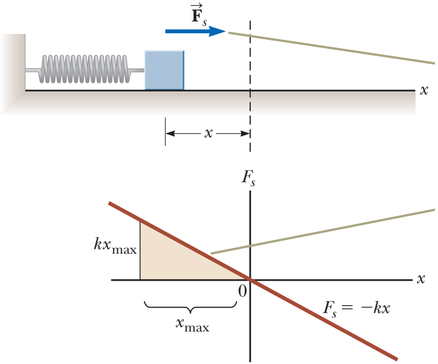


$$\vec{\mathbf{F}}_s = -kx\,\hat{\mathbf{i}}$$

where x can take a positive or negative sign.

$$W = \int \vec{\mathbf{F}}_s(x) \cdot \mathrm{d}\,\vec{\mathbf{x}}$$

Work done by a spring on a block

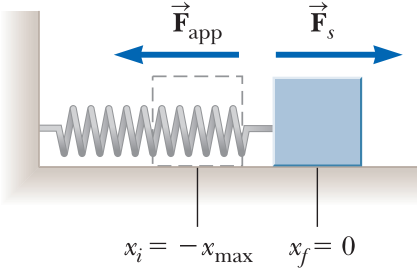


The work done by the spring on the block as the spring moves the block from $(-x_{\max}) \to 0$:

$$
\begin{aligned}
W_s &= \int_{-x_{\max}}^{0} \vec{\mathbf{F}}_s(x) \cdot \mathrm{d}\vec{\mathbf{x}} \\
&= \int_{-x_{\max}}^{0} (-kx)\hat{\mathbf{i}} \cdot \mathrm{d}\vec{\mathbf{x}} \\
&= -\int_{-x_{\max}}^{0} k\, x\, \mathrm{d}x \\
&= \frac{1}{2}k(x_{\max})^2
\end{aligned}
$$

Work done by an applied force on block / spring

Why was the spring compressed to begin with? Suppose there is an applied force. Suppose $\vec{\mathbf{F}}_{\text{app}}$ does not allow the block to accelerate: block's $\vec{\mathbf{v}}$ is constant.



$$W_{\text{app}} = \int_{-x_{\max}}^{0} (kx)\mathrm{d}x \Rightarrow W_{\text{app}} = -W_s$$

Question

Quick Quiz 7.4[1] A dart is inserted into a spring-loaded dart gun by pushing the spring in by a distance x. For the next loading, the spring is compressed a distance $2x$. How much work is required to load the second dart compared with that required to load the first?

(A) four times as much

(B) two times as much

(C) the same

(D) half as much

[2]Serway & Jewett.

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Work and Kinetic Energy

We have already said:

Kinetic energy, K

the energy that a system has as a result of its motion, or the motion of its constituent parts.

$$K = \frac{1}{2}mv^2$$

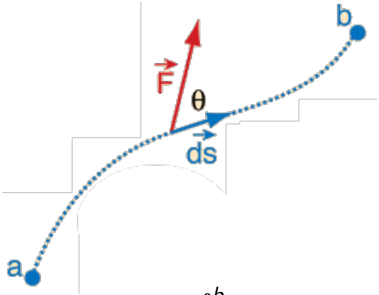
but where did this expression come from? Why is it an energy?

Work and Kinetic Energy: The Work-KE Theorem

We can relate work to kinetic energy.

Reminder: In general, work is a path integral.

$$W = \int \vec{\mathbf{F}} \cdot \mathrm{d}\,\vec{\mathbf{s}}$$



$$W_{a \to b} = \int_a^b F \cos\theta \, \mathrm{ds}$$

Work and Kinetic Energy: The Work-KE Theorem

The net work done on a (particle) system is the total energy that is transferred to the system from the environment.

Consider a particle of mass m with forces on it. How much work do we do as we change its speed?

For a particle,

$$\begin{aligned} W_{\text{net}} &= \int \vec{\mathbf{F}}_{\text{net}} \cdot \mathrm{d}\,\vec{\mathbf{r}} \\ &= \int m\vec{\mathbf{a}} \cdot \mathrm{d}\,\vec{\mathbf{r}} \\ &= m\int \frac{\mathrm{d}\,\vec{\mathbf{v}}}{\mathrm{dt}} \cdot \mathrm{d}\,\vec{\mathbf{r}} \end{aligned}$$

Work and Kinetic Energy: The Work-KE Theorem

$\mathrm{d}\vec{\mathbf{r}} = \vec{\mathbf{v}}\,\mathrm{dt}$, so

$$\begin{aligned} W_{\mathrm{net}} &= m \int \frac{\mathrm{d}\vec{\mathbf{v}}}{\mathrm{dt}} \cdot \mathrm{d}\vec{\mathbf{r}} \\ &= m \int \frac{\mathrm{d}\vec{\mathbf{v}}}{\mathrm{dt}} \cdot \vec{\mathbf{v}}\,\mathrm{dt} \end{aligned}$$

Now, we need to evaluate that dot product $\frac{\mathrm{d}\vec{\mathbf{v}}}{\mathrm{dt}} \cdot \vec{\mathbf{v}}$.

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Now, we need to evaluate that dot product $\frac{\mathrm{d}\vec{\mathbf{v}}}{\mathrm{d}t}\cdot\vec{\mathbf{v}}$.

Product rule: $\frac{\mathrm{d}(v^2)}{\mathrm{d}t} = \frac{\mathrm{d}\,\vec{\mathbf{v}}\cdot\vec{\mathbf{v}}}{\mathrm{d}t} = 2\,\frac{\mathrm{d}\vec{\mathbf{v}}}{\mathrm{d}t}\cdot\vec{\mathbf{v}}$.

Work-KE Theorem

Therefore $\frac{d\vec{\mathbf{v}}}{dt} \cdot \vec{\mathbf{v}} = \frac{1}{2}\,\frac{d(v^2)}{dt}$,

$$\begin{aligned}
W_{\text{net}} &= m \int \left(\frac{1}{2}\,\frac{dv^2}{dt} \right) dt \\
&= \frac{1}{2} m \int_{v_i^2}^{v_f^2} d(v^2) \\
&= \frac{1}{2} m (v_f^2 - v_i^2) \\
&= K_f - K_i \\
&= \Delta K
\end{aligned}$$

Work-Kinetic Energy Theorem

So,

$$W_{\text{net}} = \Delta K$$

This is the **Work-Kinetic Energy Theorem**[2], which could also be stated as:

"When the environment does work on a system and the only change in a system is in its speed, the net work done on the system equals the change in kinetic energy of the system."

[2]Note: we have assumed the system consists of *only a particle*. There is no way to define a potential energy in this case.

Question

Quick Quiz 7.5[3] A dart is inserted into a spring-loaded dart gun by pushing the spring in by a distance x. For the next loading, the spring is compressed a distance $2x$. How much faster does the second dart leave the gun compared with the first?

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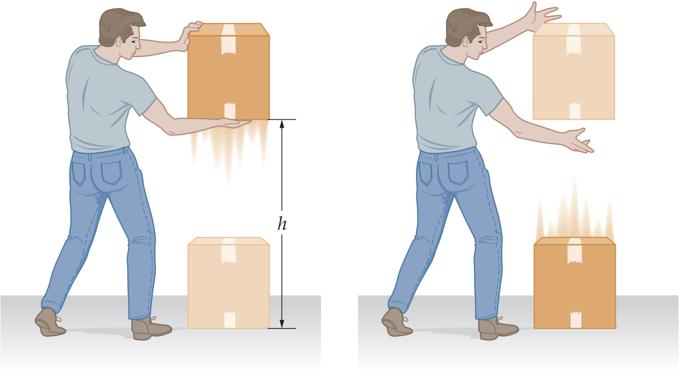
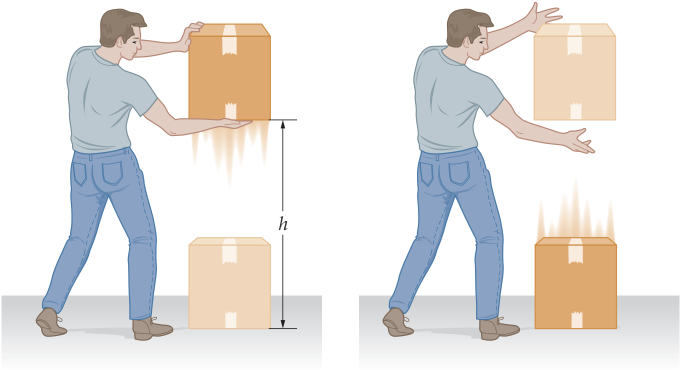
Work Done Lifting a Box

Work done by person (applied force) $W = F(\Delta y)\cos(0^\circ) = mgh$.



When box falls, this energy becomes kinetic energy.
$W_{\text{net}} = mgh = \Delta K$.

Potential Energy



When the box is in the air, it has the “potential” to have kinetic energy.

The man put in work lifting it, as long as the box is held in the air, this energy is stored.

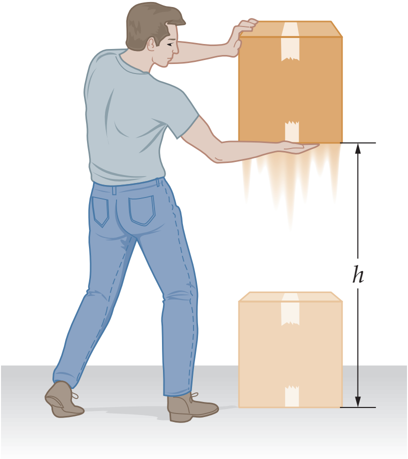
Potential Energy

This illustrates that there is another type of energy that it makes intuitive sense to assign in some systems.

That is a kind of energy that results from the configuration of the system, the **potential energy**.

Work Done Lifting a Box

Work done by person (applied force)
$W_{\text{app}} = F(\Delta y)\cos(0^\circ) = mgh.$



Work done by gravity $W_g = F(\Delta y)\cos(180^\circ) = -mgh.$

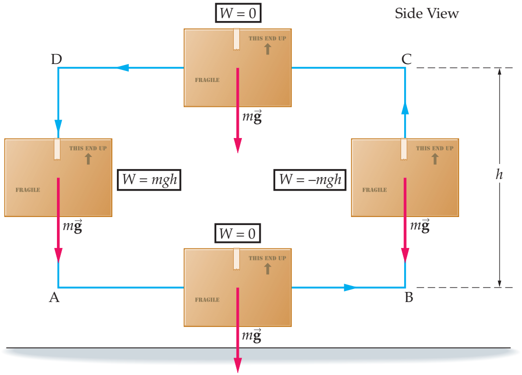
Potential Energy

When could we define a potential energy for a system and have it be meaningful?

When there is an **internal, conservative force** acting within our system.

Conservative Forces

The work done by gravity when raising and lowering an object around a closed path is zero.



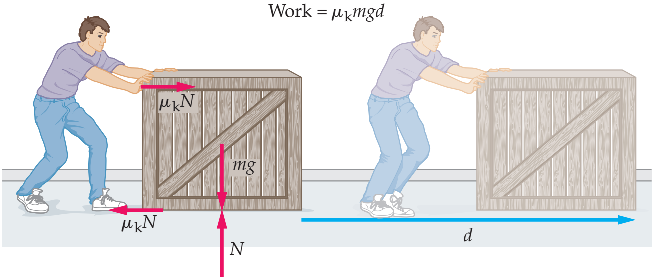
The path taken doesn't matter; if it comes back to the start, the work done is zero.

Forces (like gravity) that behave this way are called **conservative forces**.

Nonconservative Forces: Friction

The work done by kinetic friction is always negative.

Kinetic friction points in the opposite direction to the velocity / instantaneous displacement.

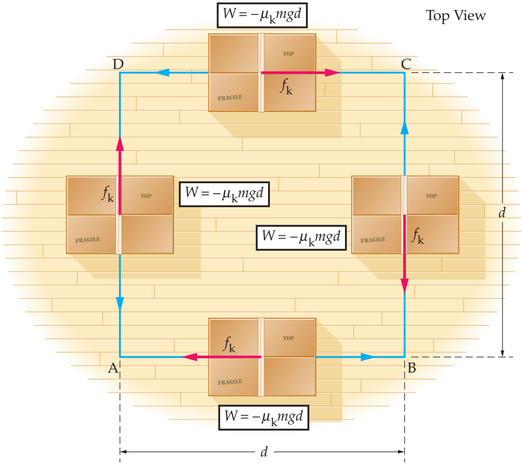


$$W_{f_k} = -f_k d = -\mu_k N d$$

where d is the distance the object moves along the surface.

Nonconservative Forces

The work done by friction when pushing an object around a closed path is **not** zero.



Forces (like friction) where the work done over a closed path is not zero are called **nonconservative forces**.

Conservative and Nonconservative Forces

Conservative force

A force that has the property that the work done by the force on a particle that moves between any given initial and final points is independent of the path taken by the particle.

Equivalently, the work done by the force as the particle moves through a *closed path* is zero.

examples:

- gravity
- spring force

Nonconservative force

Any force that is not a conservative force.

examples:

- friction
- air resistance

Summary

- work and springs
- kinetic energy
- Work-Kinetic Energy Theorem
- potential energy
- conservative and non-conservative forces

Assignment 2 has been posted, due Wednesday, Feb 19.

Presidents' day weekend no classes Friday, 14th - Monday, 17.

(Uncollected) Homework Serway & Jewett,

- Read Chapter 7.
- Ch 7, onward from page 205. Probs: 25, 35, 39, 43, 45