Kinematics
Motion in 1 Dimension and Graphs

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Jan 8, 2020

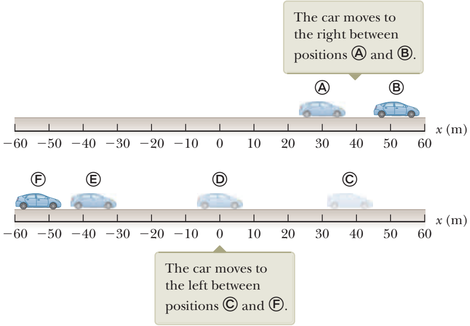
Last time

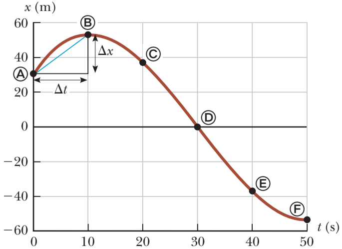
- motion in 1-dimension
- some kinematic quantities
- graphs

Overview

- velocity and speed
- acceleration
- more graphs

Position vs. Time Graphs

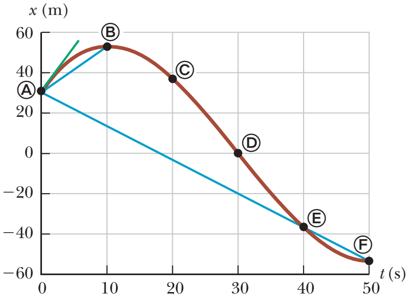


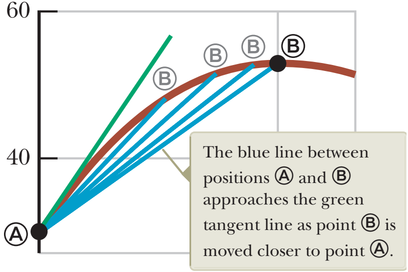


[1] Figures from Serway & Jewett

Velocity from Position vs. Time Graphs

The slope of the position vs. time graph is the velocity at that point.





$$v_x = \lim_{\Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{t + \Delta t - t} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{\mathrm{dx}}{\mathrm{dt}}$$

Kinematics Part I: Motion in 1 Dimension

Velocity

How position changes with time.

(instantaneous) velocity	$\vec{\mathbf{v}} = \frac{\mathrm{d}\,\vec{\mathbf{r}}}{\mathrm{d}t}$	speed and direction
average velocity	$\vec{\mathbf{v}}_{\text{avg}} = \frac{\overrightarrow{\Delta \mathbf{r}}}{\Delta t}$	
instantaneous speed	v or $\lvert\vec{\mathbf{v}}\rvert$	"speedometer speed"
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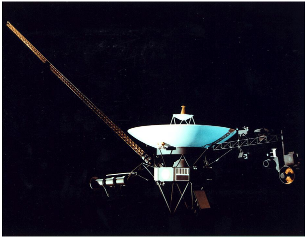
Units: meters per second, m/s

Some Examples

Traveling with constant velocity:

- a car doing exactly the speed limit on a straight road
- Voyager I (nearly)



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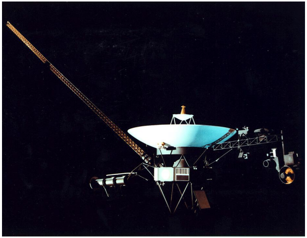

Traveling with constant speed:

- a car doing exactly the speed limit on a road with curves
- a planet traveling in a perfectly circular orbit

Conceptual Question

$\boxed{\textbf{1.}}$ If the average velocity of an object is zero in some time interval, what can you say about the displacement of the object for that interval?

[1] Serway & Jewett, page 50.

Question

Quick Quiz 2.1[1] Under which of the following conditions is the magnitude of the average velocity of a particle moving in one dimension smaller than the average speed over some time interval?

A A particle moves in the $+x$ direction without reversing.

B A particle moves in the $-x$ direction without reversing.

C A particle moves in the $+x$ direction and then reverses the direction of its motion.

D There are no conditions for which this is true.

[1] Serway & Jewett, page 24.

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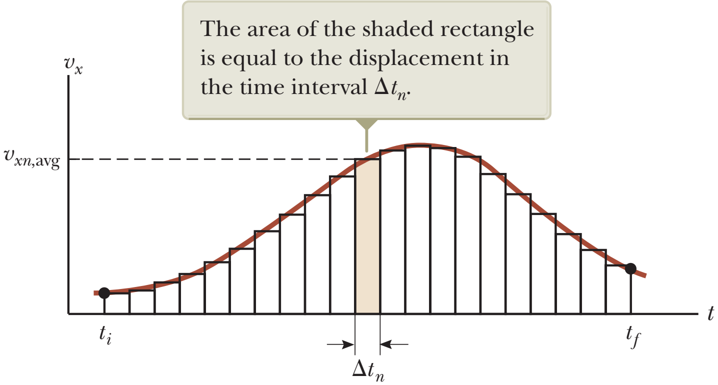
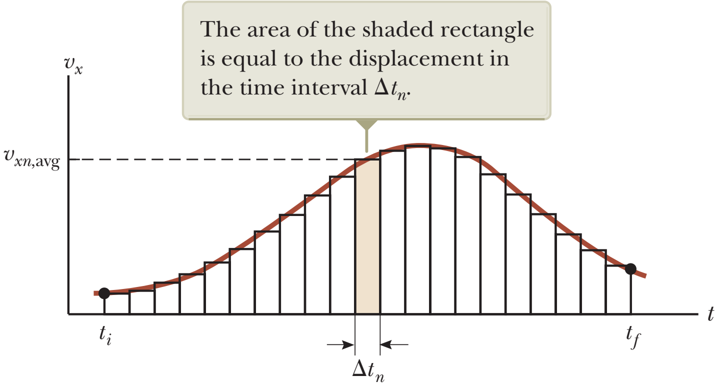
Velocity vs. Time Graphs



$$\Delta x = \lim_{\Delta t \to 0} \sum_n v_{xn} \, \Delta t = \int_{t_i}^{t_f} v_x \, dt$$

where Δx represents the change in position (displacement) in the time interval t_i to t_f.

Velocity vs. Time Graphs



Or we can write

$$x(t) = \int_{t_i}^{t} v_x \, dt'$$

if the object starts at position $x = 0$ when $t = t_i$.

t' is called a "dummy variable".

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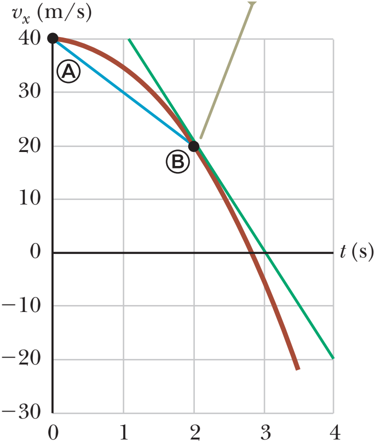

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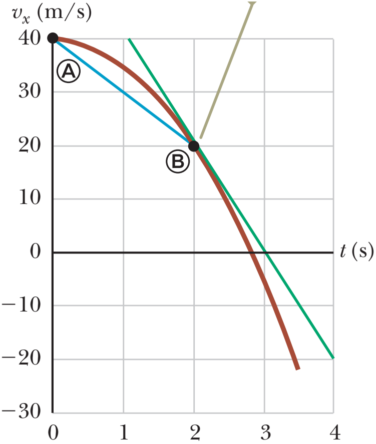
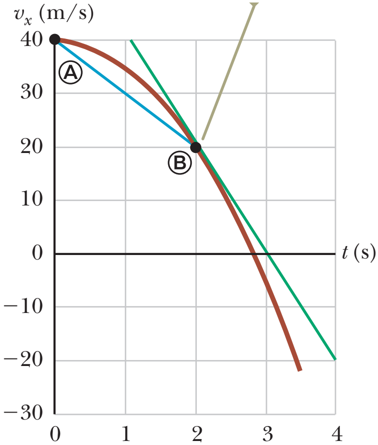


What does the slope represent?

Velocity vs. Time Graphs



The slope at any point of the velocity-time curve is the **acceleration** at that time.

Acceleration

$$\text{acceleration} \qquad \vec{\mathbf{a}} = \frac{\mathrm{d}\,\vec{\mathbf{v}}}{\mathrm{dt}} = \frac{\mathrm{d}^2\vec{\mathbf{r}}}{\mathrm{d}t^2}$$

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In general, acceleration can be a function of time $\vec{\mathbf{a}}(t)$.

Acceleration and Velocity-Time Graphs

If the acceleration vector is pointed in the **same** direction as the velocity vector (*ie.* both are positive or both negative), the particle's **speed is increasing**.

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If the acceleration vector is pointed in the **same** direction as the velocity vector (*ie.* both are positive or both negative), the particle's **speed is increasing**.

If the acceleration vector is pointed in the **opposite** direction as the velocity vector (*ie.* one is positive the other is negative), the particle's **speed is decreasing**. (It is "decelerating".)

Example

Suppose a particle has a velocity described by:

$$\vec{\mathbf{v}} = (3 + 4t)\,\hat{\mathbf{i}} \;\; \text{m/s}$$

What is the acceleration of this particle?

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$$\overrightarrow{\boldsymbol{\Delta r}} = \int_0^3 \vec{\mathbf{v}}\,dt = 27\,\hat{\mathbf{i}} \ \text{ m}$$

Summary

- velocity and acceleration
- graphs
- kinematic quantities are related by derivatives / antiderivatives

Assignment Posted today. Due in class Thursday, Jan 16.

Quiz Start of class Friday, Jan 10.

(Uncollected) Homework

Serway & Jewett,

- Set yesterday: Ch 2, onward from page 49. Obj. Q: 1; CQ: Concep. Q: 1; Probs: 1, 3, 7, 11
- New: Ch 2, onward from page 49. Conceptual Q: 4, 5; Probs: 17, 19, 62

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*Ans for 62: (a) 0, (b) 6 m/s^2, (c) -3.6 m/s^2, (d) $t = 6$ s and $t = 18$ s, (e) $t = 18$ s, (f) $x = 84$ m, (g) $d = 204$ m.