Energy
Energy Diagrams and Equilibrium

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Last time

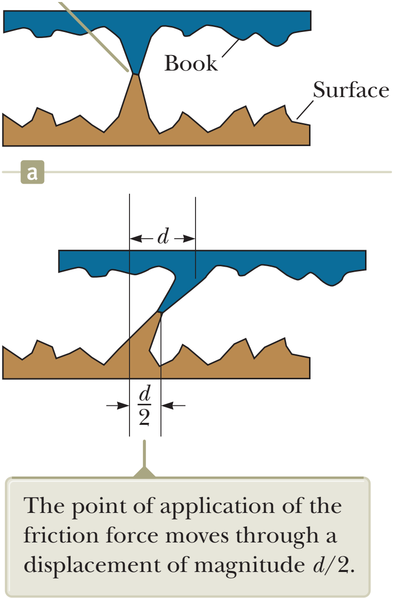
- energy conservation in isolated systems
- kinetic friction and energy

Overview

- examples with friction
- relation between conservative forces and potential energy
- potential energy diagrams

Kinetic Friction

The textbook does not ever refer to W_{f_k}.

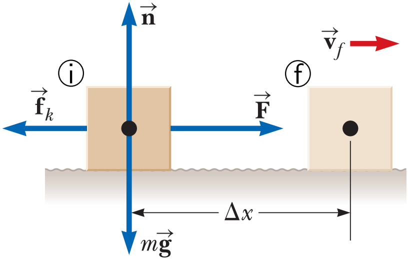


Example: Block pulled across surface

Example 8.4, Page 224

A 6.0 kg block initially at rest is pulled to the right along a horizontal surface by a constant horizontal force of 12 N.

Find the speed of the block after it has moved 3.0 m if the surfaces in contact have a coefficient of kinetic friction of $\mu_k = 0.15$.



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K - 0 &= W - \Delta E_{\text{int}} \\
K &= \int \mathbf{F} \cdot \mathrm{d}\mathbf{r} - f_k s \\
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\frac{1}{2} m v^2 &= Fs - \mu_k (mg) s \\
v &= \sqrt{\frac{2(Fs - \mu_k (mg) s)}{m}}
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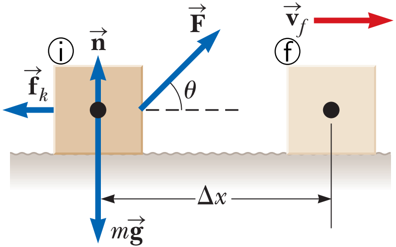
v &= \sqrt{\frac{2(Fs - \mu_k (mg) s)}{m}} \\
&= \sqrt{\frac{2\big((12\ \mathrm{N})(3\ \mathrm{m}) - (0.15)(6\ \mathrm{kg})g(3\ \mathrm{m})\big)}{6\ \mathrm{kg}}} \\
&= \underline{1.8\ \mathrm{m\ s^{-1}}}
\end{aligned}$$

Example 8.4

Suppose the force **F** is applied at an angle θ. At what angle should the force be applied to achieve the largest possible speed after the block has moved 3.0 m to the right?



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Again, we'd like an expression for K:

$$\begin{aligned} K &= W - \Delta E_{\text{int}} \\ &= \int \vec{\mathbf{F}} \cdot \mathrm{d}\,\vec{\mathbf{r}} - f_k s \\ &= Fs\cos\theta - \mu_k(mg - F\sin\theta)s \end{aligned}$$

where we noticed $n = mg - F\sin\theta$.

Example 8.4

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$$Fs(\mu_k\cos\theta - \sin\theta) = 0 \Rightarrow \mu_k\cos\theta = \sin\theta$$

$$\theta = \tan^{-1}\mu_k = 8.5^\circ$$

Question

Example 8.5[1] A car traveling at an initial speed v slides a distance d to a halt after its brakes lock. (This means the car is in a skid.) If the car's initial speed is instead $2v$ at the moment the brakes lock, what is the distance it slides?

(A) d
(B) $2d$
(C) $4d$
(D) $8d$

[1] Drawn from Serway and Jewett, page 225.

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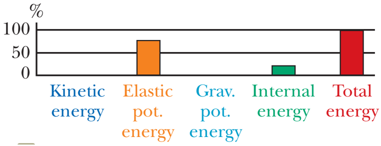
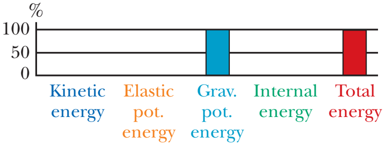
Energy Distribution

Question. Would you expect to see an evolution of an isolated system in a mechanics problem to go from a state with this energy distribution:



to this one?



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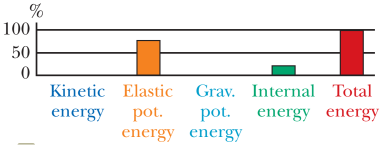

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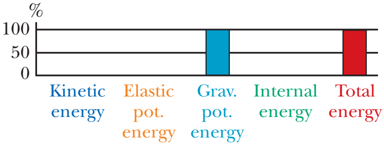

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Mechanical Energy Decreasing due to Nonconservative Forces

In the problems you will encounter in this course ΔE_{int} is always positive or zero. (E_{int} increases with time!)

In a simple isolated system, there is no way to recover energy used overcoming friction or drag forces.

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In a simple isolated system, there is no way to recover energy used overcoming friction or drag forces.

A system's mechanical energy can increase only if work is done on it by an external force.

If no work is done (isolated system) the system's mechanical energy decreases (or stays the same) over time.

Conservative and Nonconservative Forces

Conservative force

A force that has the property that the work done by the force when on a particle that moves between any give initial and final points is independent of the path taken by the particle.

Equivalently, the work done by the force as the particle moves through a *closed path* is zero.

examples:

- gravity
- spring force

Nonconservative force

Any force that is not a conservative force.

examples:

- friction
- air resistance

Conservative Forces and Potential Energy

We now return to **conservative** forces.

Potential energy

energy that system has as a result of its configuration. Is always the result of the effect of a conservative force.

$$\Delta U = -W_{\text{cons}}$$

Conservative Forces and Potential Energy

In general, a conservative force **F** can be related to its potential energy:

$$\Delta U = -W_{\text{cons}} = -\int_i^f \vec{\mathbf{F}} \cdot \mathrm{d}\,\vec{\mathbf{r}}$$

Conservative Forces and Potential Energy

For conservative forces, when the particle moves along the x-axis,

$$\Delta U = -\int_{x_i}^{x_f} F_x \, \mathrm{dx}$$

and

$$F_x = -\frac{\mathrm{dU}}{\mathrm{dx}}$$

Conservative Forces and Potential Energy

In general, for a conservative force $\vec{\mathbf{F}}$ the particle might move along an arbitrary path s:

$$\Delta U = -\int_s \vec{\mathbf{F}} \cdot \mathrm{d}\,\vec{\mathbf{r}}$$

and[2]

$$\vec{\mathbf{F}} = -\nabla U$$

where:

$$\boldsymbol{\nabla} U = \frac{\partial}{\partial x} U\ \hat{\mathbf{i}} + \frac{\partial}{\partial y} U\ \hat{\mathbf{j}} + \frac{\partial}{\partial z} U\ \hat{\mathbf{k}}$$

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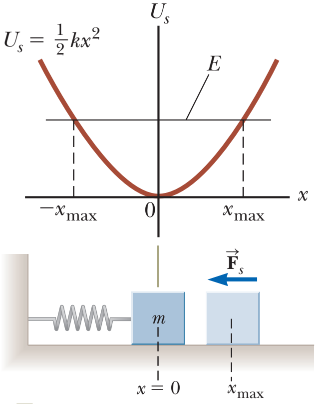
[1] If you are not yet familiar with this vector calculus notation, you will not need it for this course.

Energy Diagrams

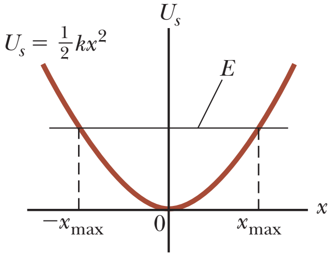
Potential energy can be plotted as a function of position. *eg.* potential energy of a spring:



$$F_{s,x} = -\frac{\mathrm{d}U_s}{\mathrm{d}x} = -kx$$

Energy Diagrams and Equilibrium

System is in equilibrium when $F_{\text{net}} = F_s = 0$. This happens when the slope of $U(x)$ is zero.



In this case, the force is always back toward the $x = 0$ point, so this is a **stable equilibrium**.

Examples:

- spring force
- ball inside a bowl

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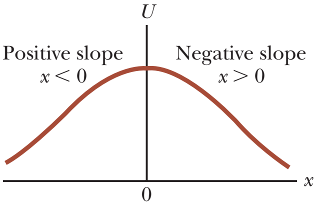

In this case, the force is always away from the $x = 0$ point, so this is a **unstable equilibrium**.
Examples:

- the L1 Lagrange point between the Sun and Earth
- ball on upside-down a bowl

Neutral Equilibrium

A system can also be in **neutral equilibrium**.

In this case, no forces act, even when the system is displaced left or right.

Example:

- ball on a flat surface

Summary

- examples with friction
- relation between conservative forces and potential energy
- potential energy diagrams

Quiz tomorrow.

(Uncollected) Homework Serway & Jewett,

- Ch 7, onward from page 207. Probs: 49, 51, 52
- Ch 8, onward from page 236. Probs: 21, 23, 27