Energy
Potential Energy and Conservative Forces
Power

Lana Sheridan

De Anza College

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Last time

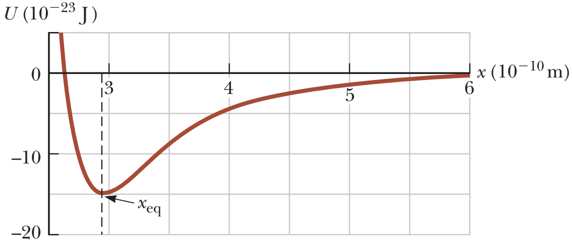
- examples with friction
- relation between conservative forces and potential energy
- potential energy diagrams

Overview

- conservative forces and potential energy
- power
- how to solve problems

Equilibrium Example: The Lennard-Jones Potential

The Lennard-Jones potential energy function describes the force between two neutral atoms in a molecule. x is the atomic separation distance.



$$U(x) = 4\epsilon\left[\left(\frac{\sigma}{x}\right)^{12} - \left(\frac{\sigma}{x}\right)^{6}\right]$$

σ and ϵ are constants: typical values are $\sigma = 0.263$ nm and $\epsilon = 1.51 \times 10^{-22}$ J.

What value does $\frac{dU}{dx}$ give when at equilibrium?

Equilibrium Example: The Lennard-Jones Potential

Find a value for the equilibrium distance of the two atoms, x_{eq}, in terms of σ.

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($x_{eq} = 2.59 \times 10^{-10}$ m $= 2.59$ Å, and 1 Ångström $= 10^{-10}$ m.)

Force and Potential Energy

Page 208, #50

50. A single conservative force acting on a particle within a system varies as $\vec{\mathbf{F}} = (-Ax + Bx^2)\hat{\mathbf{i}}$, where A and B are constants, $\vec{\mathbf{F}}$ is in newtons, and x is in meters. (a) Calculate the potential energy function $U(x)$ associated with this force for the system, taking $U = 0$ at $x = 0$. Find (b) the change in potential energy and (c) the change in kinetic energy of the system as the particle moves from $x = 2.00$ m to $x = 3.00$ m.

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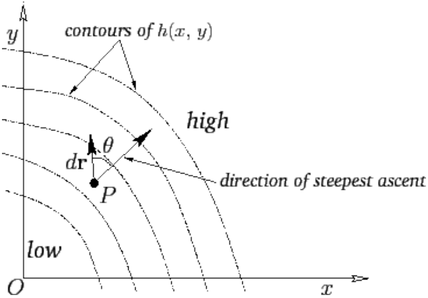
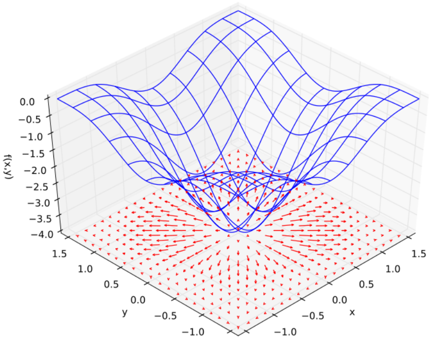
(c) $\Delta K = -\Delta U = -2.5A + 6.33B$

Potential Energy and Force, 2D



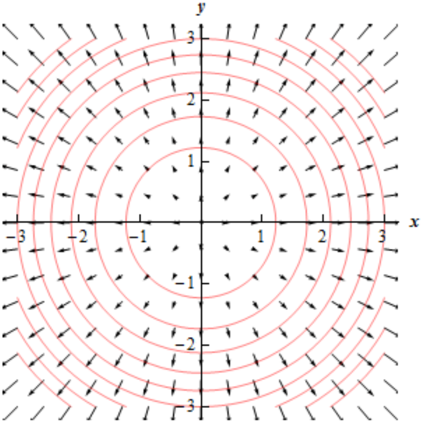


$$F = -\nabla U$$

[1] Figure from http://farside.ph.utexas.edu/teaching (left); Wikipedia by IkamusumeFan (right)

Vector Field of a Conservative Force Example

A force field $\vec{\mathbf{F}}$ that can be expressed as $\vec{\mathbf{F}} = -\nabla U$:



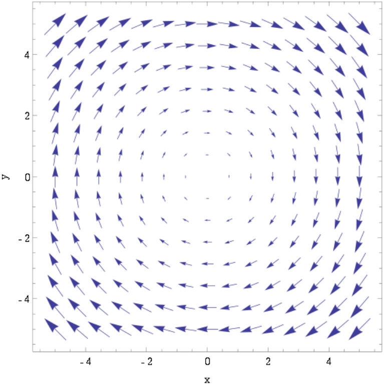
$U(x, y) = -(x^2 + y^2)$.

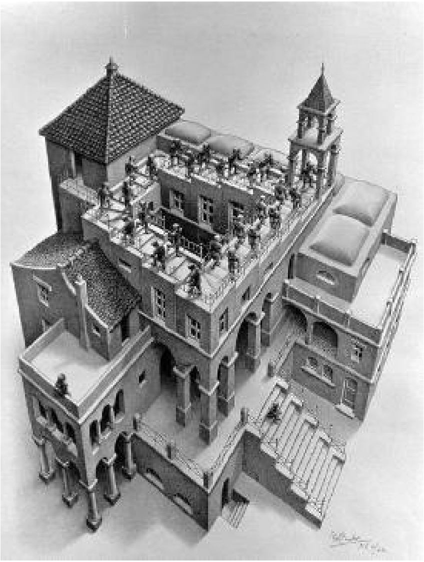
The potential function U is constant along each red line.

Non-Conservative Forces

Non-conservative vector fields cannot be represented with a topological map.

No way to define potential energy.





[1]Lithograph in the mathematically-inspired impossible reality style, by M.C. Escher.

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If the transfer is by means of work, then it is the rate of work done.

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Units? The Watt. $1 \text{ J/s} = 1 \text{ W}$

Power

Most often we are interested in the rate of work done on a system

$$P = \frac{\mathrm{d}W}{\mathrm{d}t}$$

From the definition of work:

$$W = \int \vec{\mathbf{F}} \cdot \mathrm{d}\,\vec{\mathbf{r}}$$

Noticing $\vec{\mathbf{v}} = \frac{\mathrm{d}\,\vec{\mathbf{r}}}{\mathrm{d}t}$

$$\begin{aligned} W &= \int \vec{\mathbf{F}} \cdot (\vec{\mathbf{v}}\,\mathrm{d}t) \\ P = \frac{\mathrm{d}W}{\mathrm{d}t} &= \vec{\mathbf{F}} \cdot \vec{\mathbf{v}} \end{aligned}$$

This gives another expression for instantaneous power

$$P = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}}$$

Power

Page 239, #38

38. A 650-kg elevator starts from rest. It moves upward for 3.00 s with constant acceleration until it reaches its cruising speed of 1.75 m/s. (a) What is the average power of the elevator motor during this time interval? (b) How does this power compare with the motor power when the elevator moves at its cruising speed?

Power

Average power, $P_{\text{avg}} = \frac{W}{\Delta t}$.

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What is the work done by the motor lifting the elevator?

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$$
\begin{aligned}
W &= \int \vec{\mathbf{F}}_T \cdot \mathrm{d}\vec{\mathbf{r}} \\
&= \int (mg + ma)\mathrm{dy}
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From kinematics: $v_i = 0$, $v_f = 1.75$ m/s, so
$\Delta y = v_{\text{avg}} t = \frac{v_f t}{2}$ and $a = \frac{v_f}{t}$

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$\Delta y = v_{\text{avg}} t = \frac{v_f t}{2}$ and $a = \frac{v_f}{t}$

$$W = m\left(g + \frac{v_f}{t}\right)\left(\frac{v_f t}{2}\right)$$

Power

$$W = \frac{mv_f}{2}\left(gt + v_f\right)$$

$$\begin{aligned} P_{\text{avg}} &= \frac{W}{t} \\ &= \frac{mv_f}{2}\left(g + \frac{v_f}{t}\right) \\ &= 5.91 \times 10^3 \text{ W} = 5.91 \text{ kW} \end{aligned}$$

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OR since $\vec{\mathbf{F}}_T$ is constant:

$$\begin{aligned} P_{\text{avg}} &= \vec{\mathbf{F}}_T \cdot \vec{\mathbf{v}}_{\text{avg}} \\ &= m(g + a)\frac{v_f}{2} \\ &= 5.91 \times 10^3 \text{ W} = 5.91 \text{ kW} \end{aligned}$$

Power

What about moving upward at constant speed?

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$\vec{\mathbf{F}} = mg\,\hat{\mathbf{j}}$, $\vec{\mathbf{v}} = 1.75\,\hat{\mathbf{j}}$ m/s

$$\begin{aligned} P &= \vec{\mathbf{F}} \cdot \vec{\mathbf{v}} \\ &= (mg)v \\ &= 1.11 \times 10^4 \text{ W} \\ &= 11.1 \text{ kW} \end{aligned}$$

Energy

That pretty much concludes everything covered in chapters 7 & 8.

The rest of this material is just a few more examples with worked solutions for you to look at.

Drag and Power (page 241)

54. As it plows a parking lot, a snowplow pushes an ever-growing pile of snow in front of it. Suppose a car moving through the air is similarly modeled as a cylinder of area A pushing a growing disk of air in front of it. The originally stationary air is set into motion at the constant speed v of the cylinder as shown in Figure P8.54. In a time interval Δt, a new disk of air of mass Δm must be moved a distance $v\,\Delta t$ and hence must be given a kinetic energy $\frac{1}{2}(\Delta m)v^2$. Using this model, show that the car's power loss owing to air resistance is $\frac{1}{2}\rho A v^3$ and that the resistive force acting on the car is $\frac{1}{2}\rho A v^2$, where ρ is the density of air. Compare this result with the empirical expression $\frac{1}{2}D\rho A v^2$ for the resistive force.

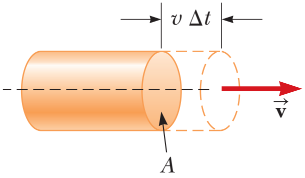


Figure P8.54

Drag and Power

What is the volume of air being accelerated in time Δt?

$$\Delta V = A v \Delta t$$

The density of air, $\rho = \frac{m}{V}$, so

$$\Delta m = \rho A v \Delta t$$

Then,

$$\Delta K = \frac{1}{2}(\Delta m)v^2 = \frac{1}{2}\rho A v^3 \Delta t$$

Power is the rate of transfer of energy. The car is *losing* energy ΔK in time Δt, so

$$P = \frac{\mathrm{dK}}{\mathrm{dt}} = -\frac{1}{2}\rho A v^3$$

Drag and Power

Force?

$$\begin{aligned} P &= \vec{\mathbf{F}} \cdot \vec{\mathbf{v}} \\ -\frac{1}{2}\rho A v^3 &= -Fv \end{aligned}$$

Giving a force,

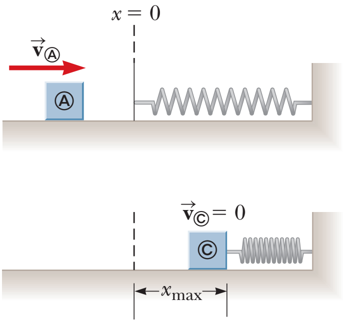
$$F = \frac{1}{2}\rho A v^2$$

c.f. the Drag Equation: $F = \frac{1}{2} D \rho A v^2$.

The only missing piece is the drag coefficient. This model is not refined enough to deal with different shapes of object, so that's not really surprising. (Basically, we have $D = 1$, about right for a flat-fronted object.)

Example 8.8: Spring Collisions and Friction

A block having a mass of 0.80 kg is given an initial velocity $v = 1.2$ m/s to the right, just as it collides with a spring whose mass is negligible and whose force constant is $k = 50$ N/m.



A constant force of kinetic friction acts between the block and the surface, $\mu_k = 0.50$. What is the maximum compression x in the spring?

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$$\Delta K + \Delta U + \Delta E_{\text{int}} = 0$$

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$$\begin{aligned}
(0 - \frac{1}{2}mv_i^2) + (\frac{1}{2}kx_f^2 - 0) + f_k x_f &= 0 \\
\frac{1}{2}kx_f^2 + \mu_k mgx - \frac{1}{2}mv_i^2 &= 0
\end{aligned}$$

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\frac{1}{2}kx_f^2 + \mu_k mgx - \frac{1}{2}mv_i^2 &= 0
\end{aligned}$$

Quadratic expression. Solution:

$$x_f = 0.092 \text{ m} = \underline{9.2 \text{ cm}}$$

(The other solution is $x_f = -0.25$ m, what does that correspond to?)

Summary

- potential energy and force
- power

Next Test Mon, Mar 2.

(Uncollected) Homework

- Go back and look at assignment 1, question 2. Is the acceleration given the acceleration you would get from a Lennard-Jones potential energy function? Could you solve that problem using energy instead?

Serway & Jewett,

- Ch 7, Probs: 47, 54.
- Ch 8, onward from page 236. Probs: 29, 31, 41, 43, 57, 67