Linear Momentum

Lana Sheridan

De Anza College

Feb 24, 2020

Last time

- conservative forces and potential energy
- power

Overview

- wrap up power
- introducing momentum
- Newton's Second Law: more general form
- relation to force
- momentum vs kinetic energy

Power

Power

the *rate* of energy transfer to or from a system.
If the transfer is by means of work, then it is the rate of work done.

The *instantaneous power* is defined as

$$P = \frac{\mathrm{dE}}{\mathrm{dt}}$$

The *average power* is

$$P_{\text{avg}} = \frac{E}{\Delta t}$$

Units? The Watt. $1 \text{ J/s} = 1 \text{ W}$

Power

Most often we are interested in the rate of work done on a system

$$P = \frac{\mathrm{d}W}{\mathrm{d}t}$$

From the definition of work:

$$W = \int \vec{\mathbf{F}} \cdot \mathrm{d}\,\vec{\mathbf{r}}$$

Noticing $\vec{\mathbf{v}} = \frac{\mathrm{d}\,\vec{\mathbf{r}}}{\mathrm{d}t}$

$$\begin{aligned} W &= \int \vec{\mathbf{F}} \cdot (\vec{\mathbf{v}}\,\mathrm{d}t) \\ P = \frac{\mathrm{d}W}{\mathrm{d}t} &= \vec{\mathbf{F}} \cdot \vec{\mathbf{v}} \end{aligned}$$

This gives another expression for instantaneous power

$$P = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}}$$

Power

Page 239, #38

38. A 650-kg elevator starts from rest. It moves upward for 3.00 s with constant acceleration until it reaches its cruising speed of 1.75 m/s. (a) What is the average power of the elevator motor during this time interval? (b) How does this power compare with the motor power when the elevator moves at its cruising speed?

Power

From kinematics:
$a = \frac{v_f}{t}$

$\vec{\mathbf{F}}_T$ is constant:

$$\begin{aligned} P_{\text{avg}} &= \vec{\mathbf{F}}_T \cdot \vec{\mathbf{v}}_{\text{avg}} \\ &= m(g + a)\frac{v_f}{2} \\ &= m\left(g + \frac{v_f}{t}\right)\frac{v_f}{2} \\ &= 5.91 \times 10^3 \text{ W} = 5.91 \text{ kW} \end{aligned}$$

Power

What about moving upward at constant speed?

Power

What about moving upward at constant speed?

$\vec{\mathbf{F}} = mg\,\hat{\mathbf{j}},\ \vec{\mathbf{v}} = 1.75\,\hat{\mathbf{j}}$ m/s

$$\begin{aligned} P &= \vec{\mathbf{F}} \cdot \vec{\mathbf{v}} \\ &= (mg)v \\ &= 1.11 \times 10^4 \text{ W} \\ &= 11.1 \text{ kW} \end{aligned}$$

Linear Momentum

Introduce a new quantity:

For a particle, *linear momentum*, $\vec{\mathbf{p}}$, is product of the particle's mass with its velocity.

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}}$$

Linear Momentum

Introduce a new quantity:

For a particle, *linear momentum*, $\vec{\mathbf{p}}$, is product of the particle's mass with its velocity.

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}}$$

It is a vector.

Linear Momentum

Introduce a new quantity:

For a particle, *linear momentum*, $\vec{\mathbf{p}}$, is product of the particle's mass with its velocity.

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}}$$

It is a vector.

Newton called it the "quantity of motion".

Units:

Linear Momentum

Introduce a new quantity:

For a particle, *linear momentum*, $\vec{\mathbf{p}}$, is product of the particle's mass with its velocity.

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}}$$

It is a vector.

Newton called it the "quantity of motion".

Units: kg m/s

Force and Momentum: Newton's Second Law

Newton's second law for a particle whose mass might change (more general!):

$$\vec{\mathbf{F}}_{net} = \frac{d\vec{\mathbf{p}}}{dt}$$

Force and Momentum: Newton's Second Law

Newton's second law for a particle whose mass might change (more general!):

$$\vec{\mathbf{F}}_{\text{net}} = \frac{d\vec{\mathbf{p}}}{dt}$$

This is how Newton thought of his second law.

Force and Momentum: Newton's Second Law

Newton's second law for a particle whose mass might change (more general!):

$$\vec{\mathbf{F}}_{\text{net}} = \frac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t}$$

This is how Newton thought of his second law.

Also

$$F_{\text{net,avg}} = \frac{\Delta p}{\Delta t}$$

Force and Momentum

This is the definition to use when *mass is changing with time*.

$$\vec{\mathbf{F}}_{net} = \frac{d\vec{\mathbf{p}}}{dt}$$

$$\vec{\mathbf{F}}_{net} = \frac{d(m\vec{\mathbf{v}})}{dt}$$

Force and Momentum

This is the definition to use when *mass is changing with time*.

$$\vec{\mathbf{F}}_{\text{net}} = \frac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{dt}}$$

$$\vec{\mathbf{F}}_{\text{net}} = \frac{\mathrm{d}(\mathrm{m}\vec{\mathbf{v}})}{\mathrm{dt}}$$

$$\vec{\mathbf{F}}_{\text{net}} = m\frac{\mathrm{d}\vec{\mathbf{v}}}{\mathrm{dt}} + \vec{\mathbf{v}}\frac{\mathrm{dm}}{\mathrm{dt}}$$

Force and Momentum

This is the definition to use when *mass is changing with time*.

$$\vec{\mathbf{F}}_{\text{net}} = \frac{d\vec{\mathbf{p}}}{dt}$$

$$\vec{\mathbf{F}}_{\text{net}} = \frac{d(m\vec{\mathbf{v}})}{dt}$$

$$\vec{\mathbf{F}}_{\text{net}} = m\frac{d\vec{\mathbf{v}}}{dt} + \vec{\mathbf{v}}\frac{dm}{dt}$$

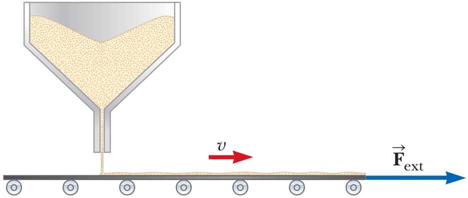
If m is constant $\frac{dm}{dt} = 0$, and this reduces to:

$$\vec{\mathbf{F}}_{\text{net}} = m\,\vec{\mathbf{a}}$$

Generalized Newton's second law example

#94, page 292

Sand from a stationary hopper falls onto a moving conveyor belt at the rate of 5.00 kg/s as shown. The conveyor belt is supported by frictionless rollers and moves at a constant speed of $v = 0.750$ m/s under the action of a constant horizontal external force $\vec{\mathbf{F}}_{\text{ext}}$ supplied by the motor that drives the belt. Find (a) the sand's rate of change of momentum in the horizontal direction, (b) the force of friction exerted by the belt on the sand, and (c) the external force $\vec{\mathbf{F}}_{\text{ext}}$.



Generalized Newton's second law example

Let the x-direction point to the right in the diagram.

(a) the sand's rate of change of momentum in the horizontal direction <u>system: sand on belt</u>

$$\begin{aligned}
\frac{d\vec{\mathbf{p}}}{dt} &= \frac{d(m\vec{\mathbf{v}})}{dt} \\
&= m\cancelto{0}{\frac{d\vec{\mathbf{v}}}{dt}} + \vec{\mathbf{v}}\frac{dm}{dt} \\
&= \vec{\mathbf{v}}\frac{dm}{dt}
\end{aligned}$$

Generalized Newton's second law example

Let the x-direction point to the right in the diagram.

(a) the sand's rate of change of momentum in the horizontal direction <u>system: sand on belt</u>

$$\begin{aligned}
\frac{d\vec{\mathbf{p}}}{dt} &= \frac{d(m\vec{\mathbf{v}})}{dt} \\
&= m\cancelto{0}{\frac{d\vec{\mathbf{v}}}{dt}} + \vec{\mathbf{v}}\frac{dm}{dt} \\
&= \vec{\mathbf{v}}\frac{dm}{dt} \\
&= (0.750 \text{ m/s})(5.00 \text{ kg/s})\,\hat{\mathbf{i}} \\
&= 3.75 \text{ N}\,\hat{\mathbf{i}}
\end{aligned}$$

Momentum vs. Kinetic energy

Momentum: $\vec{\mathbf{p}} = m\vec{\mathbf{v}}$

Kinetic energy: $K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$

Momentum vs. Kinetic energy

Momentum: $\vec{\mathbf{p}} = m\vec{\mathbf{v}}$

Kinetic energy: $K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$

- Both depend on m and $\vec{\mathbf{v}}$ only.
- K is a scalar.
- $\vec{\mathbf{p}}$ is a vector.
- In a system with many particles, can have $\sum_i \vec{\mathbf{p}}_i = 0$ even if $\sum_i K_i \neq 0$.

Question

Quick Quiz 9.1[1] Two objects have equal kinetic energies. How do the magnitudes of their momenta compare?

(A) $p_1 < p_2$

(B) $p1 = p2$

(C) $p1 > p2$

(D) not enough information to tell

[1] From Serway & Jewett, page 250.

Question

Quick Quiz 9.1[1] Two objects have equal kinetic energies. How do the magnitudes of their momenta compare?

(A) $p_1 < p_2$

(B) $p1 = p2$

(C) $p1 > p2$

(D) not enough information to tell $\leftarrow$

[1]From Serway & Jewett, page 250.

Question

Quick Quiz 9.2[2] Your physical education teacher throws a baseball to you at a certain speed and you catch it. The teacher is next going to throw you a medicine ball whose mass is ten times the mass of the baseball. You are given the following choices: You can have the medicine ball thrown with

(i) the same speed as the baseball
(ii) the same momentum, or
(iii) the same kinetic energy.

Rank these choices from easiest to hardest to catch, in terms of work done to catch the ball.

(A) (i), (ii), (iii)

(B) (iii), (ii), (i)

(C) (ii), (iii), (i)

(D) all are equivalent

[2] From Serway & Jewett, page 250.

Question

Quick Quiz 9.2[2] Your physical education teacher throws a baseball to you at a certain speed and you catch it. The teacher is next going to throw you a medicine ball whose mass is ten times the mass of the baseball. You are given the following choices: You can have the medicine ball thrown with

(i) the same speed as the baseball
(ii) the same momentum, or
(iii) the same kinetic energy.

Rank these choices from easiest to hardest to catch, in terms of work done to catch the ball.

(A) (i), (ii), (iii)

(B) (iii), (ii), (i)

(C) (ii), (iii), (i) ←

(D) all are equivalent

---

[2] From Serway & Jewett, page 250.

Summary

- finished power
- linear momentum
- momentum in an isolated system

Test Monday, Mar 2, energy and linear momentum.

(Uncollected) Homework Serway & Jewett,

- prev: Ch 8, onward from page 236. Probs: 29, 31, 41, 43, 57, 67 (power)
- new: Ch 9, onward from page 283. Probs: 1, 3, 5
- Read the first sections of Chapter 9, read ahead about non-isolated systems.