Linear Momentum
Nonisolated Systems: Impulse

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Feb 26, 2020

Last time

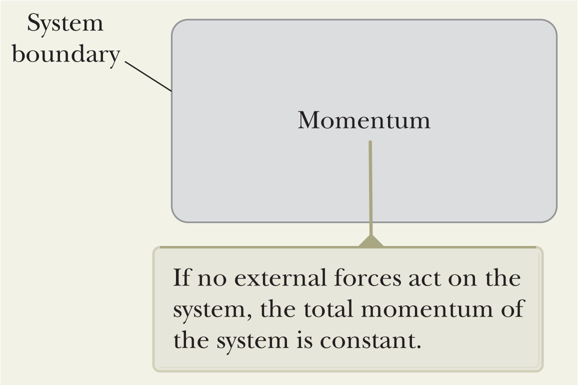
- conservation of momentum
- relation to Newton's third law
- the rocket equation
- applying the rocket equation
- conservation of momentum in isolated systems

Overview

- nonisolated systems
- impulse
- average force
- introduce collisions
- elastic collisions in 1-D

Conservation of Linear Momentum

In the last lecture we considered *isolated* systems, *ie.* systems with no external forces, total linear momentum is *conserved*.



[1]Figures from Serway & Jewett.

Nonisolated Systems

What happens when the system is not isolated: when external forces act?

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What will the change be?

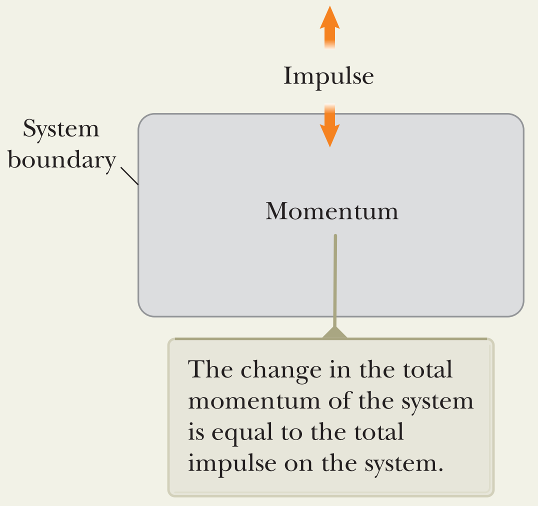
$$\Delta \vec{\mathbf{p}} = \int \vec{\mathbf{F}}_{\text{net}}\, dt$$

This change in momentum is called the *impulse*:

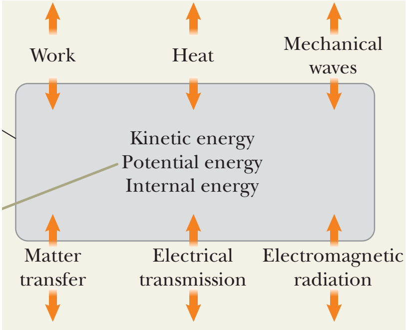
$$\vec{\mathbf{I}} = \Delta \vec{\mathbf{p}} = \int \vec{\mathbf{F}}_{\text{net}}\, dt$$

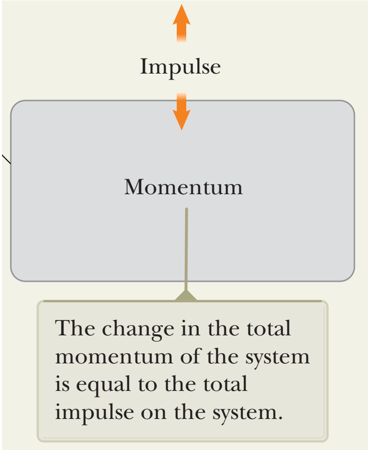
where $\vec{\mathbf{F}}_{\text{net}}$ is the net external force

Nonisolated Systems



Connection to Energy case





Energy transfers cross the boundary

$$W = \int \vec{\mathbf{F}} \cdot d\,\vec{\mathbf{r}}$$

Impulse crosses the boundary

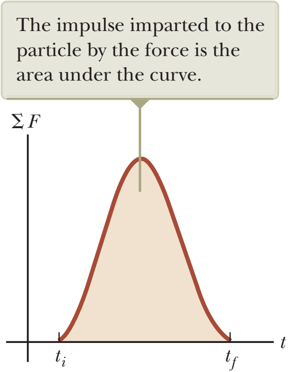
$$\vec{\mathbf{I}} = \int \vec{\mathbf{F}}\,dt = \Delta \vec{\mathbf{p}}$$

Impulse

Since impulse is simply the change in momentum, it has the same units as momentum, kg m s^{-1}.

For a force applied over a time interval $\Delta t = t_f - t_i$:

$$\vec{\mathbf{I}} = \int_{t_i}^{t_f} \vec{\mathbf{F}}_{\text{net}} \, \mathrm{dt}$$



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We can also relate the impulse to the *average* net force applied on this time interval.

$$\vec{\mathbf{I}} = \vec{\mathbf{F}}_{\text{net,avg}} \Delta t$$

the definition of the average value of a function:

$$\vec{\mathbf{F}}_{\text{net,avg}} = \frac{1}{\Delta t} \int_{t_i}^{t_f} \vec{\mathbf{F}}_{\text{net}}\, \mathrm{dt}$$

Impulse

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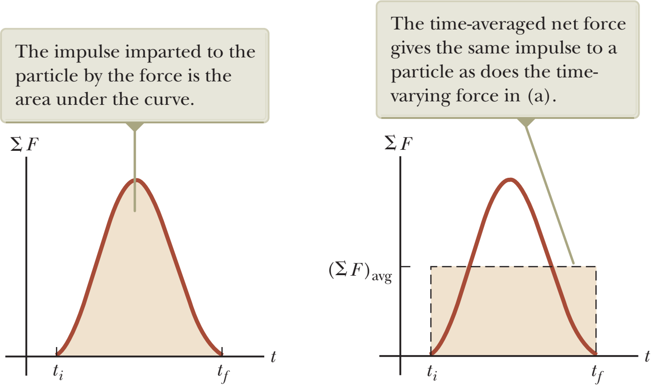


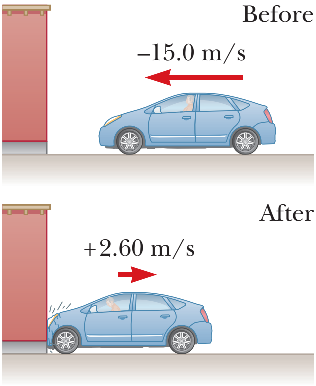
$$\int_{t_i}^{t_f} \vec{\mathbf{F}}_{\text{net}}\, \mathrm{dt} = \vec{\mathbf{I}} = \vec{\mathbf{F}}_{\text{net,avg}}\Delta t$$

Example 9.3

In a particular crash test, a car of mass 1500 kg collides with a wall. The initial and final velocities of the car are:

$$\vec{\mathbf{v}}_i = -15.0\ \hat{\mathbf{i}}\ \text{m/s}$$
$$\text{and}\ \ \vec{\mathbf{v}}_f = 2.60\ \hat{\mathbf{i}}\ \text{m/s}.$$

If the collision lasts 0.150 s, find the impulse caused by the collision and the average net force exerted on the car.



[1] Serway & Jewett, page 255.

Example 9.3

Impulse?

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Example 9.3

Impulse?

$$\begin{aligned}
\vec{\mathbf{I}} &= \Delta\vec{\mathbf{p}} \\
&= m(\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_f) \\
&= (1500\ \mathrm{kg})(2.60 - (-15.0)\ \mathrm{m/s})\ \hat{\mathbf{i}} \\
&= 2.64 \times 10^4\ \hat{\mathbf{i}}\ \mathrm{kg\ m/s}
\end{aligned}$$

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Average net force?

$$\begin{aligned} \vec{\mathbf{F}}_{\text{net,avg}} &= \frac{\vec{\mathbf{I}}}{\Delta t} \\ &= \frac{2.46 \times 10^4\ \hat{\mathbf{i}} \text{ kg m/s}}{0.150 \text{ s}} \\ &= 1.76 \times 10^5\ \hat{\mathbf{i}} \text{ N} \end{aligned}$$

Question

12. A man claims that he can hold onto a 12.0-kg child in a head-on collision as long as he has his seat belt on. Consider this man in a collision in which he is in one of two identical cars each traveling toward the other at 60.0 mi/h relative to the ground. The car in which he rides is brought to rest in 0.10 s. (a) Find the magnitude of the average force needed to hold onto the child. (b) Based on your result to part (a), is the man's claim valid? (c) What does the answer to this problem say about laws requiring the use of proper safety devices such as seat belts and special toddler seats?

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(b) man's claim?
It seems unlikely that he will be able to exert 3200 N of force on the child.

(c) Secure your toddler with a child safety seat!

Collisions

A major application of momentum conservation is studying collisions.

This is not just useful for mechanics but also for statistical mechanics, subatomic physics, etc.

For our purposes, there are two main kinds of collision:

- elastic
- inelastic

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- energy is lost as sound, heat, or in deformations
- all macroscopic collisions are somewhat inelastic
- when the colliding objects stick together afterwards the collision is *perfectly inelastic*

Collisions

If two objects collide and there are no external forces, then the only forces each object experiences are internal forces.

Internal forces obey Newton's third law $\Rightarrow$ **Momentum is conserved.**

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Internal forces obey Newton's third law $\Rightarrow$ **Momentum is conserved.**

This is true for **both** elastic and inelastic collisions. (So long as there is no external net force.)

Summary

- nonisolated systems
- impulse
- average force
- introducing collisions
- elastic collisions in 1-D

Test Monday.

(Uncollected) Homework Serway & Jewett,

- Read along in Chapter 9.
- Ch 9, onward from page 284. Probs: 13, 15, 17, 19