Linear Momentum
Collisions

Lana Sheridan

De Anza College

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Last time

- nonisolated systems
- impulse
- average force
- introduced collisions

Overview

- collisions
- elastic vs. inelastic collisions

Collisions

A major application of momentum conservation is studying collisions.

This is not just useful for mechanics but also for statistical mechanics, subatomic physics, etc.

For our purposes, there are two main kinds of collision:

- elastic
- inelastic

Collisions

If two objects collide and there are no external forces, then the only forces each object experiences are internal forces.

Internal forces obey Newton's third law $\Rightarrow$ **Momentum is conserved.**

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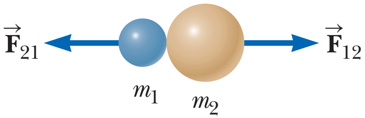
Internal forces obey Newton's third law $\Rightarrow$ **Momentum is conserved.**

This is true for **both** elastic and inelastic collisions. (So long as there is no external net force.)

Collisions

Collisions can occur in macroscopic systems through contact.



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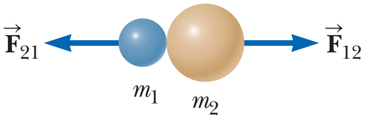
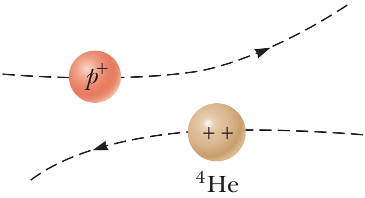


And collisions can occur through purely repulsive forces, even if two particles never make contact.



Collisions

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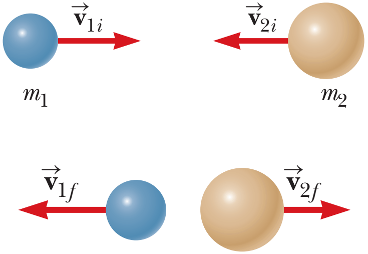
In particular, this is clearly the case for repulsive force collisions.

Let the charge on a proton be q. The the force between a proton and an alpha particle (Helium nucleus) will vary with the separation distance r :

$$F = \frac{k(q)(2q)}{r^2}$$

The force increases as the two particles approach one another, then decreases as they move apart again.

Elastic Collisions



For two particles involved in an elastic collision, we can write two **independent** equations:

$$\vec{\mathbf{p}}_i = \vec{\mathbf{p}}_f \quad \Rightarrow \quad m_1 \vec{\mathbf{v}}_{1i} + m_2 \vec{\mathbf{v}}_{2i} = m_1 \vec{\mathbf{v}}_{1f} + m_2 \vec{\mathbf{v}}_{2f}$$

$$K_i = K_f \quad \Rightarrow \quad \frac{1}{2} m_1 (v_{1i})^2 + \frac{1}{2} m_2 (v_{2i})^2 = \frac{1}{2} m_1 (v_{1f})^2 + \frac{1}{2} m_2 (v_{2f})^2$$

(Assume the masses of the two particles remain unchanged.)

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A convenient trick is to remove the quadratic terms. This equation can be derived using the two equations above:

$$(v_{1i} + v_{1f}) = (v_{2f} + v_{2i})$$

This only applies to 1-dimensional collisions!

(The v's are assumed to lie along a single direction and can be positive or negative)

Example - Elastic Particle Collision

Page 285, #27

27. M A neutron in a nuclear reactor makes an elastic, head-on collision with the nucleus of a carbon atom initially at rest. (a) What fraction of the neutron's kinetic energy is transferred to the carbon nucleus? (b) The initial kinetic energy of the neutron is 1.60×10^{-13} J. Find its final kinetic energy and the kinetic energy of the carbon nucleus after the collision. (The mass of the carbon nucleus is nearly 12.0 times the mass of the neutron.)

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$$-\frac{\Delta K_n}{K_{n,i}} = 0.284$$

(b) Final KE of neutron and carbon, given $K_{n,i} = 1.60 \times 10^{-13}$ J.

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(b) Final KE of neutron and carbon, given $K_{n,i} = 1.60 \times 10^{-13}$ J.

$$K_{n,f} = 1.15 \times 10^{-13} \text{ J}$$

$$K_{c,f} = 4.54 \times 10^{-14} \text{ J}$$

Question

Quick Quiz 9.6 A table-tennis ball is thrown at a stationary bowling ball. The table-tennis ball makes a one-dimensional elastic collision and bounces back along the same line. Compared with the bowling ball after the collision, the table-tennis ball has which of the following?

(A) a larger magnitude of momentum and more kinetic energy

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Types of Collision

Elastic collisions are collisions in which kinetic energy is conserved.

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- all macroscopic collisions are somewhat inelastic
- when the colliding objects stick together afterwards the collision is *perfectly inelastic*

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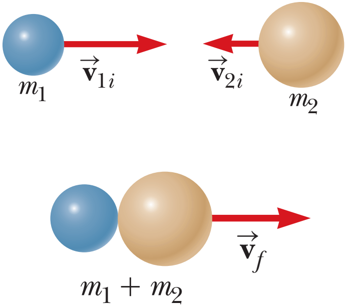
This means we can only solve for 1 unknown.
(Or 2 unknowns if we have a 2-D collision, 2 component equations.)

However, there is a special case where we have more information: *perfectly* inelastic collisions.

Perfectly Inelastic Collisions



Now the two particles stick together after colliding $\Rightarrow$ same final velocity!

$$\vec{\mathbf{p}}_i = \vec{\mathbf{p}}_f \quad \Rightarrow \quad m_1 \vec{\mathbf{v}}_{1i} + m_2 \vec{\mathbf{v}}_{2i} = (m_1 + m_2) \vec{\mathbf{v}}_f$$

Perfectly Inelastic Collisions

In this case it is straightforward to find an expression for the final velocity:

$$m_1 \vec{\mathbf{v}}_{1i} + m_2 \vec{\mathbf{v}}_{2i} = (m_1 + m_2)\vec{\mathbf{v}}_f$$

So,

$$\vec{\mathbf{v}}_f = \frac{m_1 \vec{\mathbf{v}}_{1i} + m_2 \vec{\mathbf{v}}_{2i}}{m_1 + m_2}$$

Question

Quick Quiz 9.5 In a perfectly inelastic one-dimensional collision between two moving objects, what condition alone is necessary so that the final kinetic energy of the system is zero after the collision?[1]

(A) The objects must have initial momenta with the same magnitude but opposite directions.

(B) The objects must have the same mass.

(C) The objects must have the same initial velocity.

(D) The objects must have the same initial speed, with velocity vectors in opposite directions.

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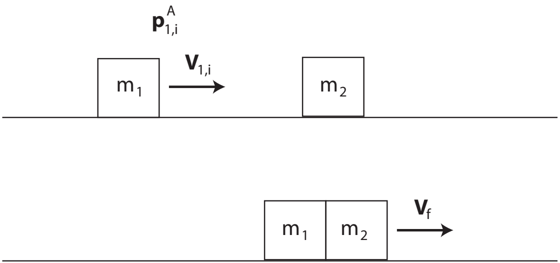
Let's consider why.

Perfectly Inelastic Collisions

Consider the same collision, viewed from different inertial frames.

Suppose Alice sees:



In her frame, block 2 is at rest, and block 1 moves with velocity $\mathbf{v}_{1,i}$. After the collision, both blocks move with velocity $\mathbf{v}_f$.

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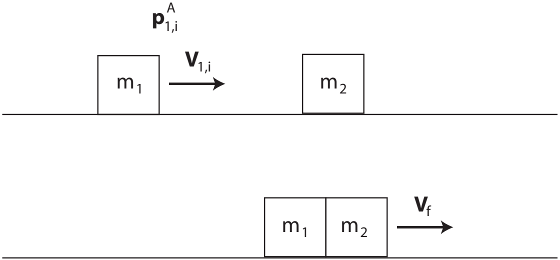
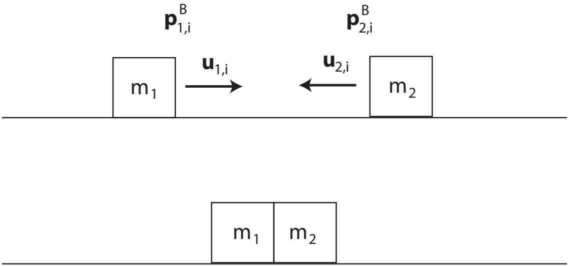


In her frame, block 2 is at rest, and block 1 moves with velocity $\mathbf{v}_{1,i}$. After the collision, both blocks move with velocity $\mathbf{v}_f$.

There is still some KE after the collision, but there must be at least some, since the momentum after cannot be zero.

Perfectly Inelastic Collisions

Now consider what another observer, Bob, who is in the **center-of-momentum frame**, would see in the same collision:



In his frame, both blocks are in motion, and $\mathbf{p}_{1,i} = -\mathbf{p}_{2,i}$. After the collision, $\mathbf{p}_{1,f} + \mathbf{p}_{2,f} = 0$.

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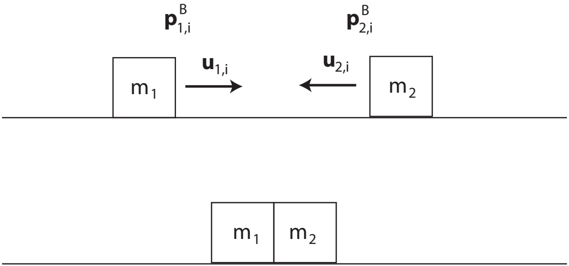

In his frame, both blocks are in motion, and $\mathbf{p}_{1,i} = -\mathbf{p}_{2,i}$. After the collision, $\mathbf{p}_{1,f} + \mathbf{p}_{2,f} = 0$.

The final KE in this case is 0. ($\mathbf{p}_{1,f} = \mathbf{p}_{2,f} = 0$)

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Observers in all inertial frames will see momentum conserved in any collision (with no external forces).

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If the objects stick together, the final kinetic energy in this frame is zero.

No observer in another frame can assign less final kinetic energy than $K = \frac{1}{2}mv_{\text{rel}}^2$ to the objects, where v_{rel} is the relative speed of the other frame to the center-of-momentum frame.

Perfectly Inelastic Collisions

Observers in different inertial frames will see different kinetic energies of the system.

However, all inertial observers will see the same *change* in kinetic energy.

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Since the kinetic energy cannot be negative, we there is a limit on how much can be lost:

The loss cannot be more than the initial KE in the center-of-momentum frame.

Example

Page 285, #24

24. A car of mass m moving at a speed v_1 collides and couples with the back of a truck of mass $2m$ moving initially in the same direction as the car at a lower speed v_2. (a) What is the speed v_f of the two vehicles immediately after the collision? (b) What is the change in kinetic energy of the car–truck system in the collision?

Example

(a) Speed v_f?

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$$\mathbf{v}_f = \frac{m_1 \mathbf{v}_{1i} + m_2 \mathbf{v}_{2i}}{m_1 + m_2}$$

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$$\mathbf{v}_f = \frac{m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i}}{m_1 + m_2}$$

$$v_f = \frac{mv_1 + 2mv_2}{m + 2m}$$

$$v_f = \frac{v_1 + 2v_2}{3}$$

Example

(b) Change in kinetic energy?

$$\Delta K = K_f - K_i$$

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$$\Delta K = \frac{1}{2}3m\left(\frac{v_1 + 2v_2}{3}\right)^2 - \left(\frac{1}{2}mv_1^2 + \frac{1}{2}2mv_2^2\right)$$

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\Delta K &= K_f - K_i \\
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\Delta K &= \frac{m}{6}\left(\left(v_1^2 + 4v_2^2 + 4v_1v_2\right) - 3\left(v_1^2 + 2v_2^2\right)\right)
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\Delta K &= \frac{m}{6}\left(4v_1v_2 - 2v_2^2 - 2v_1^2\right) \\
\Delta K &= \frac{m}{3}\left(2v_1v_2 - v_2^2 - v_1^2\right)
\end{aligned}$$

Summary

- elastic collision example
- inelastic collisions

Test Mon.

(Uncollected) Homework Serway & Jewett,

- Ch 9, onward from page 285. Probs: 23, 25, 27, 29, 31