Linear Momentum
Collisions and Energy
Collisions in 2 Dimensions

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Last time

- collisions
- elastic collision example
- inelastic collisions

Overview

- the ballistic pendulum
- 2 dimensional collisions

Perfectly Inelastic Collisions

Perfectly inelastic collisions are the special case of inelastic collisions where the two colliding objects stick together.

In this case the maximum amount of kinetic energy is lost. (The loss must be consistent with the conservation of momentum.

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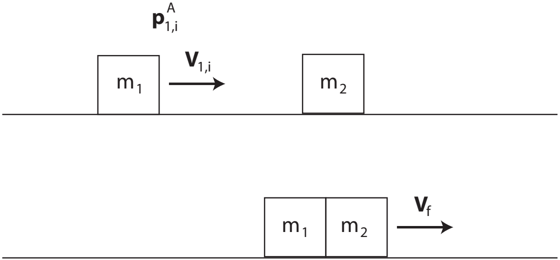
Let's consider why.

Perfectly Inelastic Collisions

Consider the same collision, viewed from different inertial frames.

Suppose Alice sees:



In her frame, block 2 is at rest, and block 1 moves with velocity $\mathbf{v}_{1,i}$. After the collision, both blocks move with velocity $\mathbf{v}_f$.

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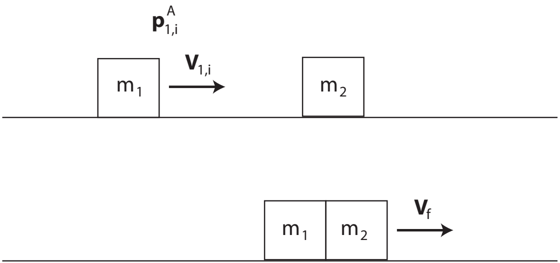
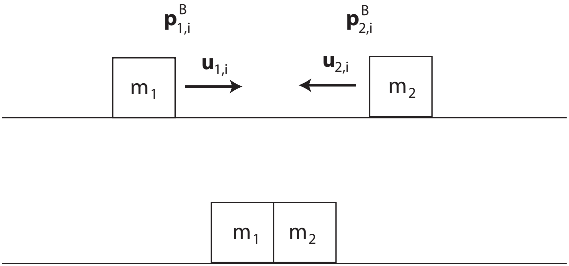


In her frame, block 2 is at rest, and block 1 moves with velocity $\mathbf{v}_{1,i}$. After the collision, both blocks move with velocity $\mathbf{v}_f$.

There is still some KE after the collision, but there must be at least some, since the momentum after cannot be zero.

Perfectly Inelastic Collisions

Now consider what another observer, Bob, who is in the **center-of-momentum frame**, would see in the same collision:



In his frame, both blocks are in motion, and $\mathbf{p}_{1,i} = -\mathbf{p}_{2,i}$. After the collision, $\mathbf{p}_{1,f} + \mathbf{p}_{2,f} = 0$.

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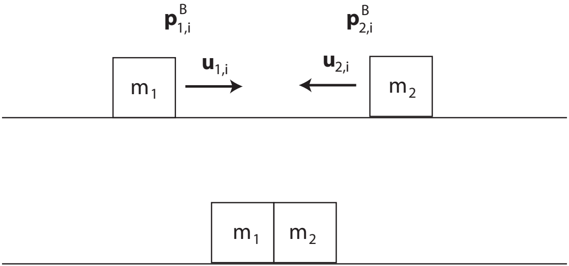

In his frame, both blocks are in motion, and $\mathbf{p}_{1,i} = -\mathbf{p}_{2,i}$. After the collision, $\mathbf{p}_{1,f} + \mathbf{p}_{2,f} = 0$.

The final KE in this case is 0. ($\mathbf{p}_{1,f} = \mathbf{p}_{2,f} = 0$)

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If the objects stick together, the final kinetic energy in this frame is zero.

No observer in another frame can assign less final kinetic energy than $K = \frac{1}{2}mv_{\mathrm{rel}}^2$ to the objects, where v_{rel} is the relative speed of the other frame to the center-of-momentum frame.

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However, **all inertial observers will see the same <u>change</u> in kinetic energy.**

The most KE that can be "lost" is the amount lost in a perfectly inelastic collision.

Example

Page 285, #24

24. A car of mass m moving at a speed v_1 collides and couples with the back of a truck of mass $2m$ moving initially in the same direction as the car at a lower speed v_2. (a) What is the speed v_f of the two vehicles immediately after the collision? (b) What is the change in kinetic energy of the car–truck system in the collision?

Example

(a) Speed v_f?

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System: car and truck

$$\begin{aligned}\mathbf{p}_i &= \mathbf{p}_f \\ \mathbf{v}_f &= \frac{m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i}}{m_1 + m_2}\end{aligned}$$

Example

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System: car and truck

$$
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\mathbf{p}_i &= \mathbf{p}_f \\
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v_f &= \frac{mv_1 + 2mv_2}{m + 2m} \\
v_f &= \frac{v_1 + 2v_2}{3}
\end{aligned}
$$

Example

(b) Change in kinetic energy?

$$\Delta K = K_f - K_i$$

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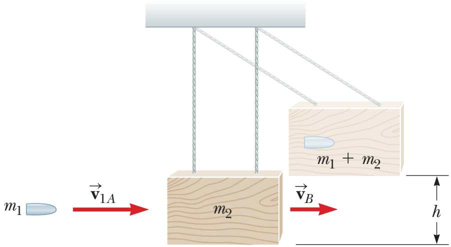
\begin{aligned}
\Delta K &= K_f - K_i \\
\Delta K &= \frac{1}{2}3mv_f^2 - \left(\frac{1}{2}mv_1^2 + \frac{1}{2}2mv_2^2\right) \\
\Delta K &= \frac{1}{2}3m\left(\frac{v_1 + 2v_2}{3}\right)^2 - \left(\frac{1}{2}mv_1^2 + \frac{1}{2}2mv_2^2\right) \\
\Delta K &= \frac{m}{6}\left(\left(v_1^2 + 4v_2^2 + 4v_1v_2\right) - 3\left(v_1^2 + 2v_2^2\right)\right) \\
\Delta K &= \frac{m}{6}\left(4v_1v_2 - 2v_2^2 - 2v_1^2\right) \\
\Delta K &= \frac{m}{3}\left(2v_1v_2 - v_2^2 - v_1^2\right)
\end{aligned}
$$

Collisions and Energy

The Ballistic Pendulum (Example 9.6)



The ballistic pendulum is an apparatus used to measure the speed of a fast-moving projectile such as a bullet. A projectile of mass m_1 is fired into a large block of wood of mass m_2 suspended from some light wires. The projectile embeds in the block, and the entire system swings through a height h. How can we **determine the speed of the projectile** from a measurement of h?

[1] Serway & Jewett, page 262.

The Ballistic Pendulum

We know m_1, m_2, and h. We want to know the speed of the bullet, v_1.

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Step 1: how does the speed of the block v_b depend on the bullet speed? Conservation of momentum, perfectly inelastic collision:

$$\begin{aligned} (m_1 + m_2)v_b &= m_1 v_1 + m_2(0) \\ v_b &= \frac{m_1 v_1}{m_1 + m_2} \end{aligned}$$

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\Delta K + \Delta U_g &= 0 \\
(0 - \frac{1}{2}(m_1 + m_2)v_b^2) + ((m_1 + m_2)gh - 0) &= 0 \\
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Replace $v_b = \frac{m_1 v_1}{m_1 + m_2}$:

$$\begin{aligned}
\frac{1}{2}(m_1 + m_2)\left(\frac{m_1 v_1}{m_1 + m_2}\right)^2 &= (m_1 + m_2)gh \\
\left(\frac{m_1^2 v_1^2}{m_1 + m_2}\right) &= 2(m_1 + m_2)gh \\
v_1 &= \left(\frac{m_1 + m_2}{m_1}\right)\sqrt{2gh}
\end{aligned}$$

Collisions in 2 Dimensions

The conservation of momentum equation is a vector equation.

It will apply for any number of dimensions that are relevant in a question.

$$\vec{\mathbf{p}}_i = \vec{\mathbf{p}}_f \quad \Rightarrow \quad m_1 \vec{\mathbf{v}}_{1i} + m_2 \vec{\mathbf{v}}_{2i} = m_1 \vec{\mathbf{v}}_{1f} + m_2 \vec{\mathbf{v}}_{2f}$$

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In particular, we can write equations for each component of the momentum. In 2-d, with x and y components:

$$\begin{aligned} \mathbf{x}: \;\; m_1 v_{1ix} + m_2 v_{2ix} &= m_1 v_{1fx} + m_2 v_{2fx} \\ \mathbf{y}: \;\; m_1 v_{1iy} + m_2 v_{2iy} &= m_1 v_{1fy} + m_2 v_{2fy} \end{aligned}$$

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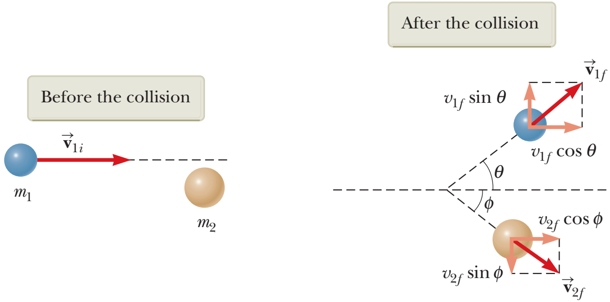
\end{aligned}$$

If it is an elastic collision:

$$K_i = K_f \quad \Rightarrow \quad \frac{1}{2} m_1 (v_{1i})^2 + \frac{1}{2} m_2 (v_{2i})^2 = \frac{1}{2} m_1 (v_{1f})^2 + \frac{1}{2} m_2 (v_{2f})^2$$

Collisions in 2 Dimensions

As an example, consider the case of a *glancing* collision.



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Collisions in 2 Dimensions

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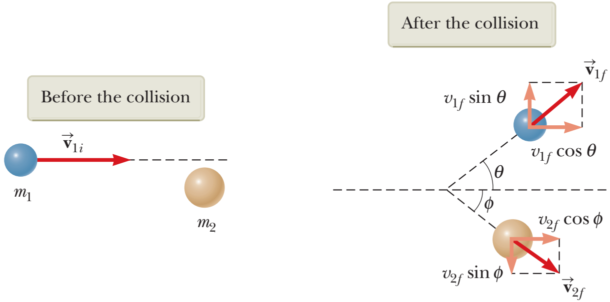


The velocity of particle 1 is in the x-direction.

x-components:

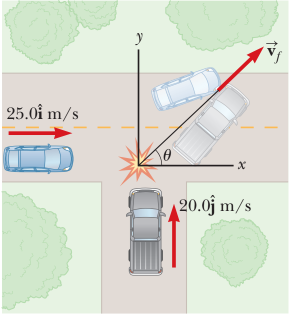
$$m_1 v_{1i} = m_1 v_{1f} \cos\theta + m_2 v_{2f} \cos\phi$$

y-components:

$$0 = m_1 v_{1f} \sin\theta - m_2 v_{2f} \sin\phi$$

Example 9.8 - Car collision

A 1500 kg car traveling east with a speed of 25.0 m/s collides at an intersection with a 2500 kg truck traveling north at a speed of 20.0 m/s. Find the direction and magnitude of the velocity of the wreckage after the collision, assuming the vehicles stick together after the collision.



[1] Serway & Jewett, page 265.

Example 9.8 - Car collision

This is an inelastic collision.

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x-components:

$$m_1 v_{1i} = (m_1 + m_2) v_f \cos\theta \qquad (1)$$

y-components:

$$m_2 v_{2i} = (m_1 + m_2) v_f \sin\theta \qquad (2)$$

Dividing (2) by (1):

$$\frac{m_2 v_{2i}}{m_1 v_{1i}} = \tan\theta$$

$$\theta = \tan^{-1}\left(\frac{m_2 v_{2i}}{m_1 v_{1i}}\right) = 53.1^\circ$$

and

$$v_f = \frac{m_2 v_{2i}}{(m_1 + m_2)\sin(53.1)} = 15.6 \text{ m/s}$$

Example 9.14 - Exploding Rocket

A rocket is fired vertically upward. At the instant it reaches an altitude of 1000 m and a speed of $v_i = 300$ m/s, it explodes into three fragments having equal mass.

One fragment moves upward with a speed of $v_1 = 450$ m/s following the explosion. The second fragment has a speed of $v_2 = 240$ m/s and is moving east right after the explosion.

What is the velocity of the third fragment immediately after the explosion?

(What is the sign of the change in kinetic energy of the system of the rocket parts?)

Example 9.14 - Exploding Rocket

$$\vec{\mathbf{p}}_i = \vec{\mathbf{p}}_f \;\Rightarrow\; M\,\vec{\mathbf{v}}_i = \frac{M}{3}(\vec{\mathbf{v}}_1 + \vec{\mathbf{v}}_2 + \vec{\mathbf{v}}_3)$$

Let $\hat{\mathbf{j}}$ point in the the upward vertical direction, and $\hat{\mathbf{i}}$ point east.

$$\begin{aligned}\vec{\mathbf{v}}_3 &= 3\vec{\mathbf{v}}_i - \vec{\mathbf{v}}_1 - \vec{\mathbf{v}}_2 \\ &= 3\times 300\,\hat{\mathbf{j}} - 450\,\hat{\mathbf{j}} - 240\,\hat{\mathbf{i}} \\ &= (-240\,\hat{\mathbf{i}} + 450\,\hat{\mathbf{j}})\ \text{m/s}\end{aligned}$$

Or, <u>510 m/s at an angle of 62° above the horizontal to the west</u>.

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Or, <u>510 m/s at an angle of 62° above the horizontal to the west</u>.

($\Delta K = K_f - K_i = \frac{1}{2}M(450^2 + 240^2 + 510^2) - \frac{1}{2}(3M)(300^2) = +1.25 \times 10^5 M$, a positive number)

Summary

- ballistic pendulum
- collisions in 2 dimensions

Quiz Monday.

3rd Collected Homework will be posted today, 1st question could use momentum / collisions.

(Uncollected) Homework Serway & Jewett,

- Look at example 9.9 on page 266.
- Ch 9, onward from page 275. Probs: 35, 37, 41, 43, 67, 71, 81