Extended or Composite Systems
Center of Mass

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Last time

- perfectly inelastic collisions
- the ballistic pendulum
- 2D collisions

Overview

- center of mass
- finding the center of mass

Center of Mass

The center of mass of a system is the average position of the system's mass.

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The translational motion is independent of all these other motions.

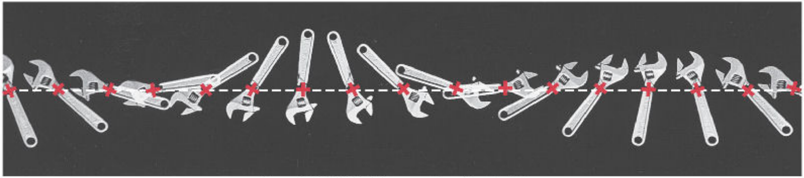
Center of Mass

For a solid, rigid object:

center of mass

the point on an object where we can model all the mass as being, in order to find the object's trajectory; a freely moving object rotates about this point

The center of mass of the wrench follows a straight line as the wrench rotates about that point.



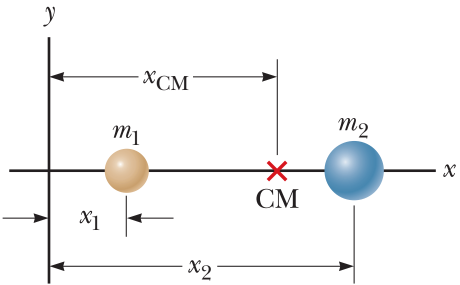
Center of Mass

[1]Figure from
http://www4.uwsp.edu/physastr/kmenning/Phys203/Lect18.html

Center of Mass

For that system of two particles:

$$x_{\mathrm{CM}} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$



Center of Mass

For that system of two particles:

$$x_{\mathrm{CM}} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

For more particles in 1 dimension:

$$x_{\mathrm{CM}} = \frac{\sum_i m_i x_i}{\sum_i m_i}$$

or

$$x_{\mathrm{CM}} = \frac{1}{M} \sum_i m_i x_i$$

where M is the total mass of all the particles in the system.

Center of Mass

This expression also gives us the x coordinate of the center of mass when we have more dimensions.

$$x_{\text{CM}} = \frac{1}{M}\sum_i m_i x_i$$

Likewise for y:

$$y_{\text{CM}} = \frac{1}{M}\sum_i m_i y_i$$

and z:

$$z_{\text{CM}} = \frac{1}{M}\sum_i m_i z_i$$

where M is the total mass of all the particles in the system.

Center of Mass

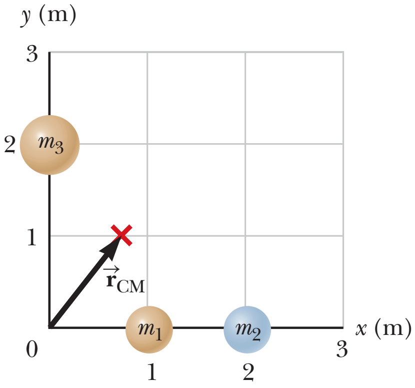
Therefore, we can condense all three expressions into a single vector expression.

$$\vec{\mathbf{r}}_{\mathrm{CM}} = \frac{1}{M} \sum_i m_i \vec{\mathbf{r}}_i$$

where $\vec{\mathbf{r}}_i = x_i \hat{\mathbf{i}} + y_i \hat{\mathbf{i}} + z_i \vec{\mathbf{k}}$ is the displacement of particle i from the origin.

Example 9.10 - Three Particles in a plane

A system consists of three particles located as shown. Find the center of mass of the system. The masses of the particles are $m_1 = m_2 = 1.0$ kg and $m_3 = 2.0$ kg.



[1]Serway & Jewett, page 269.

Example 9.10

2 dimensions: can find the x and y coordinates of the center of mass separately.

Total mass, $M = 2(1.0 \text{ kg}) + 2.0 \text{ kg} = 4 \text{ kg}$.
x-direction:

$$\begin{aligned} x_{\text{CM}} &= \frac{1}{M}\sum_i m_i x_i \\ &= \frac{1}{M}(m_1x_1 + m_2x_2 + m_3x_3) \\ &= \frac{1}{4}(1\times 1 + 1\times 2 + 2\times 0) \\ &= 0.75 \text{ m} \end{aligned}$$

Example 9.10

x-direction: $x_{\mathrm{CM}} = 0.75$ m

y-direction:

$$\begin{aligned} y_{\mathrm{CM}} &= \frac{1}{M}\sum_i m_i y_i \\ &= \frac{1}{M}(m_1 y_1 + m_2 y_2 + m_3 y_3) \\ &= \frac{1}{4}(1 \times 0 + 1 \times 0 + 2 \times 2) \\ &= 1.0 \text{ m} \end{aligned}$$

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&= 1.0 \text{ m}
\end{aligned}
$$

Putting both components together:

$$\vec{\mathbf{r}}_{CM} = (0.75\,\hat{\mathbf{i}} + 1.0\,\hat{\mathbf{j}}) \text{ m}$$

Center of Mass

For discrete particles the center of mass is the average point of the mass:

$$\vec{\mathbf{r}}_{\mathrm{CM}} = \frac{1}{M}\sum_i m_i \vec{\mathbf{r}}_i$$

What if we have a solid object?

Continuous mass distribution

On a microscopic scale we don't really believe mass is continuously distributed. (Atoms, etc.!)

However, on a macroscopic scale it sure seems like it is, and it's a very good approximation. It is also *much* easier than trying to describe every last atom / subatomic particle in a solid.

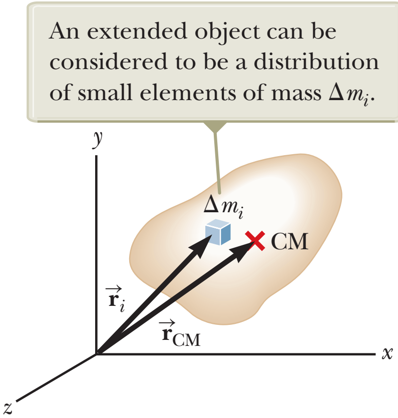
We need to revise our previous center of mass (CM) definition to suit this case:

$$\vec{\mathbf{r}}_{\mathrm{CM}} = \lim_{\Delta m_i \to 0} \frac{1}{M} \sum_i \vec{\mathbf{r}}_i \, \Delta m_i$$

$$\vec{\mathbf{r}}_{\mathrm{CM}} = \frac{1}{M} \int \vec{\mathbf{r}} \, \mathrm{dm}$$

Continuous mass distribution

$$\vec{\mathbf{r}}_{CM} = \frac{1}{M} \int \vec{\mathbf{r}}\, dm$$

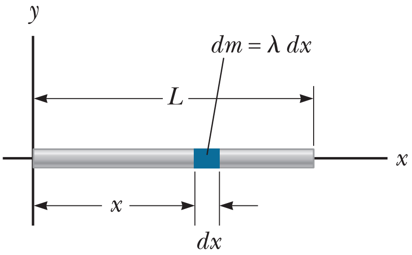


Discrete vs Continuous mass distribution

Center of mass of a rod of uniform density.

Density, $\rho = \frac{M}{V}$, is mass per unit volume. Here however, we will consider a rod that can be treated as 1 dimensional.

We need the mass per unit length: $\lambda = \frac{M}{L}$.



Where is the center of mass?

Discrete vs Continuous mass distribution

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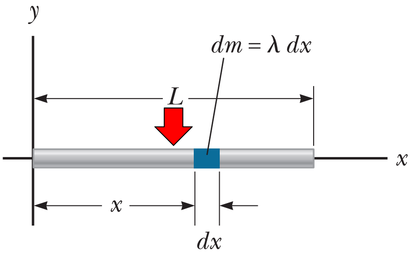

Where is the center of mass?

Discrete vs Continuous mass distribution

How to do the calculation:

$$
\begin{aligned}
x_{\mathrm{CM}} &= \frac{1}{M}\int x\,\mathrm{dm} \\
&= \frac{1}{M}\int x\,\mathrm{dm}
\end{aligned}
$$

Observe that $\mathrm{dm} = \lambda\,\mathrm{dx}$:

$$
\begin{aligned}
x_{\mathrm{CM}} &= \frac{1}{M}\int x(\lambda\,\mathrm{dx}) \\
&= \frac{1}{M}\,\lambda\,\int_0^L x\,\mathrm{dx}
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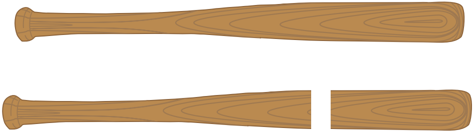
&= \frac{1}{M}\,\lambda\left(\frac{L^2}{2} - 0\right) \\
&= \frac{1}{M}\left(\frac{M}{L}\right)\frac{L^2}{2} \\
&= \frac{L}{2}
\end{aligned}
$$

In this case (uniform density) the center of mass is at the center of the rod.

Center of Mass Intuition Question

Quick Quiz 9.7[1] A baseball bat of uniform density is cut at the location of its center of mass as shown. Which piece has the smaller mass?



(A) the piece on the right

(B) the piece on the left

(C) both pieces have the same mass

(D) impossible to determine

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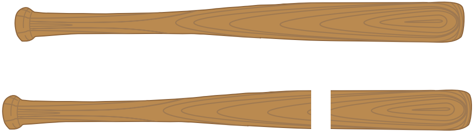

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Center of Mass of Continuous Objects

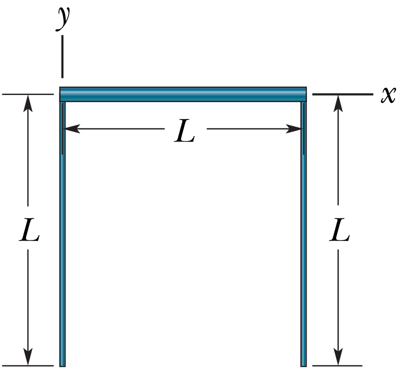
In solid objects, it is possible to calculate the center of mass by evaluating integrals.

However, we can use symmetry arguments to find the position of the center of mass in many cases.

(Ideally) this meterstick has reflection symmetry through the 50 cm mark. That transformation must not change the location of the CM.

Center of Mass of Continuous Objects

If an object is made up of several different regular shapes, you can find the center of mass by treating each part as point mass at the center of mass of that shape.



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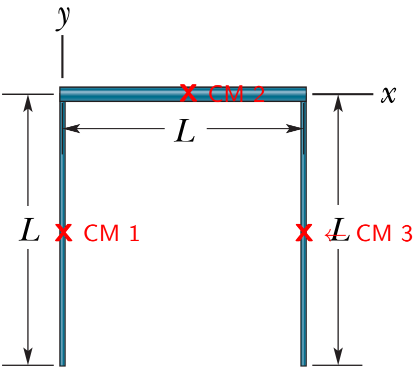

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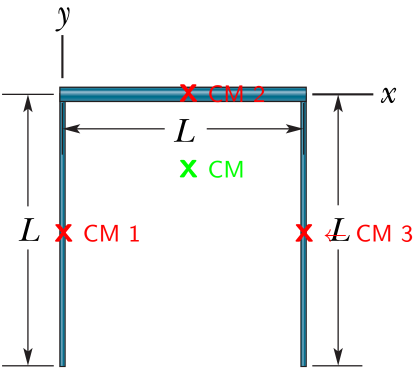
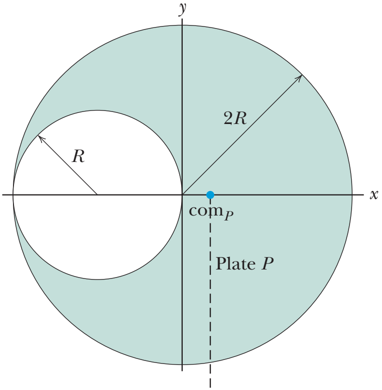


Center of Mass of Continuous Objects

Suppose this 2-D object, plate P, has uniform mass-per-unit-area.

How can we find its center of mass without integrating?



Summary

- center of mass

Quiz on Friday.

(Uncollected) Homework Serway & Jewett,

- Ch 9, onward from page 288. Probs: 45, 49, 48
- Read Chapter 9, if you haven't already.