Extended or Composite Systems: Deformation
Rotational Motion: Rotational Kinematics

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Last Time

- center of mass tricks and examples
- systems of many particles

Overview

- deforming systems
- rotation
- rotational kinematics

Deformable Systems

Some systems will change the distribution of their mass during their motion.

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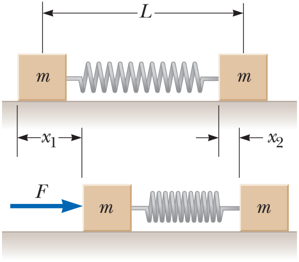
[1]Image from http://northdallasgazette.com

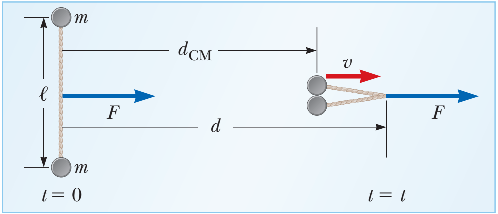
Deformable Systems

In a system that is deformed, the positions of the masses of particles may change relative to the center of mass, but we can still study the system by considering what happens to the center of mass.



Deformable Systems Problem, pg 288, #59

Below is an overhead view of the initial configuration of two pucks of mass m on frictionless ice. The pucks are tied together with a string of length ℓ, and negligible mass. At time $t = 0$, a constant force of magnitude F begins to pull to the right on the center point of the string. At time t, the moving pucks strike each other and stick together. At this time, the force has moved through a distance d, and the pucks have attained a speed v. (a) What is v in terms of F, d, ℓ, and m? (b) How much of the energy transferred into the system by work done by the force has been transformed to internal energy?



Deformable Systems

(a) Speed of pucks, v, after collision?

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$$a_{\mathrm{CM}} = \frac{F}{2m}$$

$$\begin{aligned} v_f^2 &= v_i^2 + 2a_{\mathrm{CM}}\,\Delta x_{CM} \\ v_f^2 &= 0 + 2\left(\frac{F}{2m}\right)\left(d - \frac{\ell}{2}\right) \\ v_f &= \sqrt{\frac{2Fd - F\ell}{2m}} \end{aligned}$$

Deformable Systems

(b) Increase in internal energy?

system: pucks + pucks' and ice's internal degrees of freedom

$$W = \Delta K + \Delta E_{\text{int}}$$

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system: pucks + pucks' and ice's internal degrees of freedom

$$W = \Delta K + \Delta E_{\text{int}}$$

$$\begin{aligned}
\Delta E_{\text{int}} &= W - K_f \\
&= Fd - \frac{1}{2}(2m)v^2 \\
&= Fd - \frac{1}{2}(2m)\frac{2Fd - F\ell}{2m} \\
&= Fd - Fd + F\frac{\ell}{2} \\
&= \frac{1}{2}F\ell
\end{aligned}$$

Rotation of Rigid Objects

Now we understand that while we *can* treat a collection of particles as a single point particle at the center of mass, we do not have to do that.

This will allow us to describe another important kind of motion: *rotation*.

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Begin with rotational kinematics.

Rotation of Rigid Objects

To begin, consider a rotating disc.

$$s = r\theta \; ; \quad \theta = \frac{s}{r}$$

r is constant in time for a rigid object.

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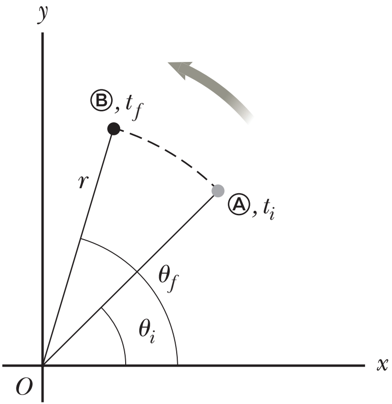
Units for θ: radians. Often written as "rad". But notice, that a dimensional analysis gives $\frac{[\mathrm{m}]}{[\mathrm{m}]} = 1$, unitless! The radian is an artificial unit. In fact, angles given in radians are dimensionless.

Rotation of Rigid Objects

How does the angle advance in time?



$$\Delta\theta = \theta_f - \theta_i$$

Angular Speed

Rate at which the angle advances is a speed: the angular speed, ω.

Average angular speed:

$$\omega_{\text{avg}} = \frac{\theta_f - \theta_i}{t_f - t_i} = \frac{\Delta\theta}{\Delta t}$$

Instantaneous angular speed:

$$\omega = \frac{\mathrm{d}\theta}{\mathrm{d}t}$$

Angular Acceleration

Rate at which the angular speed changes: the angular acceleration, α.

Average angular acceleration:

$$\alpha_{\mathrm{avg}} = \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta\omega}{\Delta t}$$

Instantaneous angular acceleration:

$$\alpha = \frac{\mathrm{d}\omega}{\mathrm{dt}}$$

Rotation of Rigid Objects and Vector Quantities

We can also define these quantities as vectors! (Provided we fix the axis of rotation.)

This might seem a bit strange because the direction of the motion any point (other than the axis) on a rotating disc is always changing.

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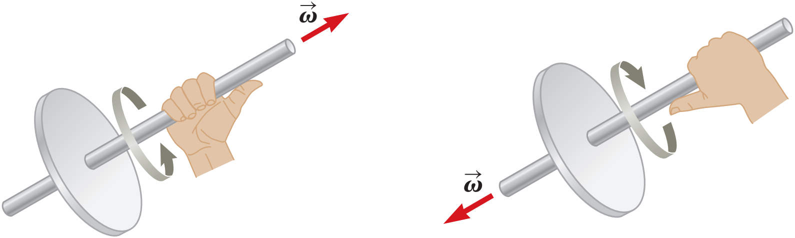
This might seem a bit strange because the direction of the motion any point (other than the axis) on a rotating disc is always changing.

However, the angle can be positive or negative depending on whether it is clockwise or counterclockwise from the reference point.

Rotation of Rigid Objects and Vector Quantities

By convention, we define the counterclockwise direction to be positive. The vector itself is drawn along the axis of rotation.



Then we can write:

$$\vec{\boldsymbol{\theta}} = \frac{s}{r}\,\hat{\mathbf{n}} \quad ; \quad \vec{\boldsymbol{\omega}} = \frac{\mathrm{d}\vec{\boldsymbol{\theta}}}{\mathrm{dt}} \quad ; \quad \vec{\boldsymbol{\alpha}} = \frac{\mathrm{d}\vec{\boldsymbol{\omega}}}{\mathrm{dt}}$$

where $\vec{\hat{\mathbf{n}}}$ is a unit vector perpendicular to the plane of rotation. (Assuming we fix the axis of rotation.)

Comparison of Linear and Rotational quantities

Linear Quantities	Rotational Quantities
$\vec{\mathbf{x}}$	$\vec{\theta}$
$\vec{\mathbf{v}} = \frac{d\vec{\mathbf{x}}}{dt}$	$\vec{\omega} = \frac{d\vec{\theta}}{dt}$
$\vec{\mathbf{a}} = \frac{d\vec{\mathbf{v}}}{dt}$	$\vec{\alpha} = \frac{d\vec{\omega}}{dt}$

Rotational Kinematics

If α is constant, we have basically the same kinematics equations as before, but the relations are between the new quantities.

$$\vec{\omega}_f = \vec{\omega}_i + \vec{\alpha} t$$

$$\vec{\theta}_f = \vec{\theta}_i + \vec{\omega}_i t + \frac{1}{2}\vec{\alpha} t^2$$

$$\omega_f^2 = \omega_i^2 + 2\vec{\alpha} \cdot \overrightarrow{\Delta\theta}$$

$$\vec{\omega}_{\text{avg}} = \frac{1}{2}(\vec{\omega}_i + \vec{\omega}_f)$$

$$\vec{\theta}_f = \vec{\theta}_i + \frac{1}{2}(\vec{\omega}_i + \vec{\omega}_f) t$$

Kinematics Comparison

Linear Quantities	Rotational Quantities
$\vec{\mathbf{v}}_f = \vec{\mathbf{v}}_\mathbf{i} + \vec{\mathbf{a}}\,t$	$\vec{\omega}_f = \vec{\omega}_i + \vec{\alpha}\,t$
$\vec{\mathbf{x}}_f = \vec{\mathbf{x}}_\mathbf{i} + \vec{\mathbf{v}}_\mathbf{i}\,t + \frac{1}{2}\vec{\mathbf{a}}\,t^2$	$\vec{\theta}_f = \vec{\theta}_i + \vec{\omega}_i t + \frac{1}{2}\vec{\alpha}\,t^2$
$v_f^2 = v_i^2 + 2\vec{\mathbf{a}} \cdot \boldsymbol{\Delta x}$	$\omega_f^2 = \omega_i^2 + 2\boldsymbol{\alpha} \cdot \boldsymbol{\Delta\theta}$
$\vec{\mathbf{v}}_{avg} = \frac{1}{2}(\vec{\mathbf{v}}_\mathbf{i} + \vec{\mathbf{v}}_\mathbf{f})$	$\vec{\omega}_{\mathrm{avg}} = \frac{1}{2}(\vec{\omega}_i + \vec{\omega}_f)$
$\vec{\mathbf{x}}_\mathbf{f} = \vec{\mathbf{x}}_\mathbf{i} + \frac{1}{2}(\vec{\mathbf{v}}_\mathbf{i} + \vec{\mathbf{v}}_\mathbf{f})t$	$\vec{\theta}_f = \vec{\theta}_i + \frac{1}{2}(\vec{\omega}_i + \vec{\omega}_f)t$

(The rotational equations can all be proved in an an analogous way to the linear ones.)

Using Rotational Kinematics Equations: Example 1

Page 325, #5

$\boxed{\textbf{5.}}$ **W** A wheel starts from rest and rotates with constant angular acceleration to reach an angular speed of 12.0 rad/s in 3.00 s. Find (a) the magnitude of the angular acceleration of the wheel and (b) the angle in radians through which it rotates in this time interval.

Example 1

Known: ω_i, ω_f, t

(a) Angular accel., α?

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$$\omega_f = \omega_i + \alpha t$$

Example 1

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(a) Angular accel., α?

$$\begin{aligned}
\omega_f &= \omega_i + \alpha t \\
\alpha &= \frac{\omega_f - \omega_i}{t} \\
\alpha &= \frac{12.0 \text{ rad/s} - 0}{3.00 \text{ s}} \\
\alpha &= 4.00 \text{ rad s}^{-2}
\end{aligned}$$

Example 1

Known: ω_i, ω_f, t, α

(b) Angle in radians, $\Delta\theta$?

Example 1

Known: ω_i, ω_f, t, α

(b) Angle in radians, $\Delta\theta$?

Either use

$$\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2 = 0 + \frac{1}{2}(4)(3)^2 = 18 \text{ radians}$$

or use

$$\begin{aligned}
\Delta\theta &= \frac{1}{2}(\omega_i + \omega_f)t \\
\Delta\theta &= \frac{1}{2}(0 + 12.0)(3.00) \\
&= 18.0 \text{ radians}
\end{aligned}$$

Using Rotational Kinematics Equations: Example 2

Page 325, #8

8. A machine part rotates at an angular speed of 0.060 rad/s; its speed is then increased to 2.2 rad/s at an angular acceleration of 0.70 rad/s^2. (a) Find the angle through which the part rotates before reaching this final speed. (b) If both the initial and final angular speeds are doubled and the angular acceleration remains the same, by what factor is the angular displacement changed? Why?

Example 2

Known: ω_i, ω_f, α

(a) $\Delta\theta$?

Example 2

Known: ω_i, ω_f, α

(a) $\Delta\theta$?

$$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$$

Example 2

Known: ω_i, ω_f, α

(a) $\Delta\theta$?

$$\begin{aligned}
\omega_f^2 &= \omega_i^2 + 2\alpha\Delta\theta \\
\Delta\theta &= \frac{\omega_f^2 - \omega_i^2}{2\alpha} \\
&= \frac{(2.2)^2 - (0.060)^2}{2 \times 0.70} \\
&= 3.5 \text{ rad}
\end{aligned}$$

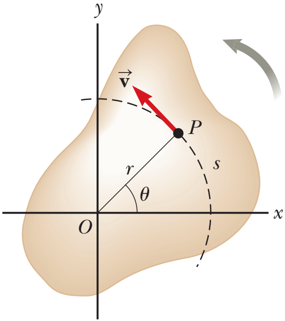
Example 2

(b) If both ω_i and ω_f are doubled, α kept constant, what happens to $\Delta\theta$?

$$
\begin{aligned}
\Delta\theta' &= \frac{(2\times\omega_f)^2 - (2\times\omega_i)^2}{2\alpha} \\
&= 4\frac{\omega_f^2 - \omega_i^2}{2\alpha} \\
&= 4\,\Delta\theta
\end{aligned}
$$

Relating Rotational Quantities to Translation of Points

Consider a point on the rotating object. How does its speed relate to the angular speed?



We know $s = r\theta$, so since the object's speed is its speed along the path s,

$$v = \frac{ds}{dt} = r\frac{d\theta}{dt}$$

Relating Rotational Quantities to Translation of Points

Since $\omega = \frac{d\theta}{dt}$, that gives us and expression for (tangential) speed

$$v = r\omega$$

And differentiating both sides with respect to t again:

$$a_t = r\alpha$$

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Notice that the above equation gives the rate of change of speed, which is the tangential acceleration.

Centripetal Acceleration

Remember:

$$a_t = \frac{\mathrm{dv}}{\mathrm{dt}}$$

where v is the speed, not velocity.

So,

$$a_t = r\alpha$$

But of course, in order for a mass at that point, radius r, to continue moving in a circle, there must be a centripetal component of acceleration also.

$$a_c = \frac{v^2}{r} = \omega^2 r$$

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For a rigid object, the force that supplies this acceleration will be some internal forces between the mass at the rotating point and the other masses in the object. Those are the forces that hold the object together.

Summary

- deformations
- rotation
- rotational kinematics

Quiz Friday (tomorrow).

(Uncollected) Homework Serway & Jewett,

- Look at example 9.15 on page 275.
- Ch 9, onward from page 288. Probs: 51, 55, 57, 92
- Ch 10, Probs: 7, 11, 15, 17, 19, 21, 25 (will not be on quiz)